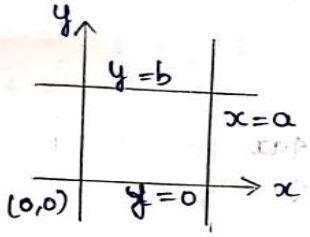
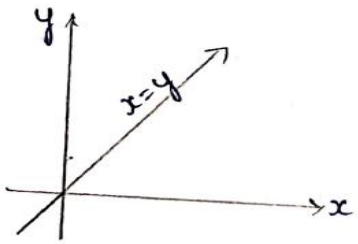
Change of order of Integration

Basic formulae

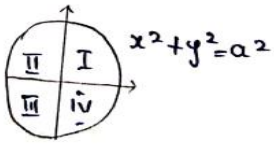
1) If $x=0$, $y=0$, $x=a$ and $y=b$, then



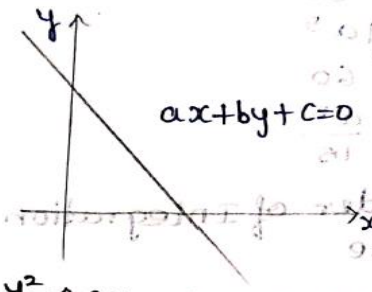
2) If $x=y$, then



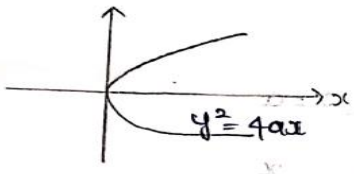
3) If $x^2+y^2=a^2$



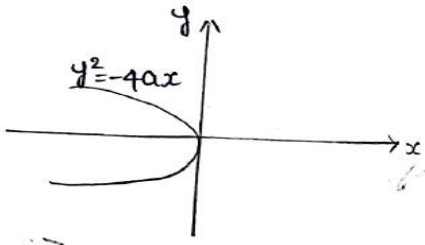
4) If $ax+by+c=0$ then



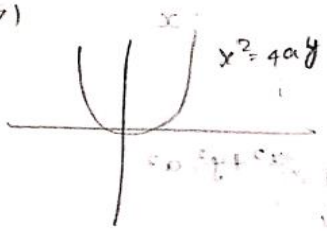
5) (a) If $y^2=4ax$

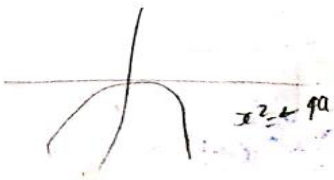


6) If $y^2=-4ax$



ϕ $\frac{\vec{a}}{\vec{b}}$





Example:-1

Change the order of integration and Evaluate for $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} xy \, dy dx$

$$I = \int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} xy \, dy dx$$

Given limits

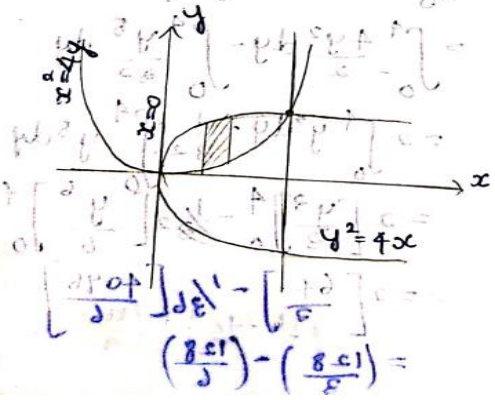
x limits varies from 0 to 4

ie, $x=0$ to $x=4$

y limits varies from $\frac{x^2}{4}$ to $2\sqrt{x}$

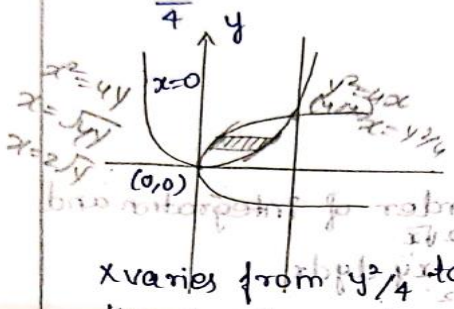
ie, $y = \frac{x^2}{4}$ to $y = 2\sqrt{x}$

$x^2 = 4y$ to $y^2 = 4x$



Change the order of limits

$$\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} xy\,dx\,dy$$



x varies from $y^2/4$ to $2\sqrt{y}$

y varies from 0 to 4

$$I = \int_0^4 \left(\int_{y^2/4}^{2\sqrt{y}} xy\,dx \right) dy$$

$$= \int_0^4 y \left[\int_{y^2/4}^{2\sqrt{y}} x\,dx \right] dy$$

$$= \int_0^4 y \left[\frac{x^2}{2} \right]_{y^2/4}^{2\sqrt{y}} dy$$

$$= \int_0^4 y \left[\frac{4y}{2} - \frac{y^4}{32} \right] dy$$

$$= \int_0^4 \frac{4y^2}{2}\,dy - \int_0^4 \frac{y^5}{32}\,dy$$

$$= 2\int_0^4 y^2\,dy - \frac{1}{32}\int_0^4 y^5\,dy$$

$$= 2\left[\frac{y^3}{3} \right]_0^4 - \frac{1}{32}\left[\frac{y^6}{6} \right]_0^4$$

$$= 2\left[\frac{64}{3} \right] - \frac{1}{96}\left[\frac{4096}{6} \right]$$

$$= \left(\frac{128}{3} \right) - \left(\frac{128}{6} \right)$$

$$= \frac{256-128}{6}$$

$$= \frac{128}{6}$$

$$= \frac{64}{3}$$

Example: 3

Change the order of integration in $\int_0^a \int_x^a (x^2+y^2)\,dy\,dx$

$\Rightarrow$ $I = \int_0^a \int_x^a (x^2+y^2)\,dy\,dx$

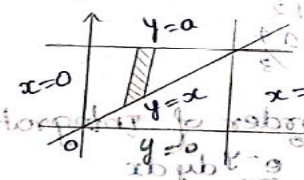
Given limits

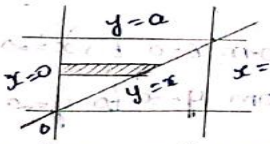
x varies from 0 to a

ie, $x=0$; $x=a$

Y varies from x to a

ie, $y=x$, $y=a$





Change the order of limits $\int_0^a \int_y^0 (x^2+y^2)\,dx\,dy$

x varies from 0 to y

Y varies from 0 to a

$I = \int_0^a \left(\int_0^y (x^2+y^2)\,dx \right) dy$

Example: 3

change the order of integration in

$$\int_0^a \int_x^a (x^2+y^2)\,dy\,dx$$

$\Longrightarrow$

$$I = \int_0^a \int_x^a (x^2+y^2)\,dy\,dx$$

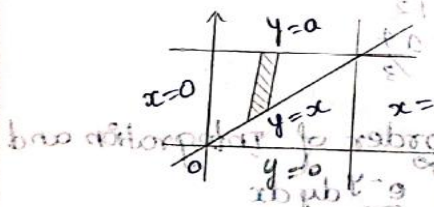
Given limits

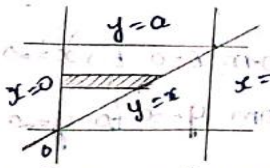
x varies from 0 to a

ie, $x=0$; $x=a$

Y varies from x to a

ie, $y=x$, $y=a$





Change the order of limits $\int_0^a \int_A^0 (x^2+y^2)\,dx\,dy$

x varies from 0 to y

Y varies from 0 to a

$$I = \int_0^a \left(\int_0^{A} (x^2+y^2)\,dx \right) dy$$

$$= \int_0^a \left(\int_0^y x^2 dx + \int_0^y y^2 dx \right) dy$$

$$= \int_0^a \left(\left[\frac{x}{3} \right]_0^y + \left[y^2 \right]_0^y \right] dy$$

$$= \int_0^a \left(\left[\frac{y^3}{3} \right] + y^3 \right) dy$$

$$\Rightarrow \int_0^a \frac{y^3}{3} dy + \int_0^a y^3 dy$$

$$= \frac{1}{3} \int_0^a y^3 dy + \int_0^a y^3 dy$$

$$= \frac{1}{3} \left[\frac{y^4}{4} \right]_0^a + \left[\frac{y^4}{4} \right]_0^a$$

$$= \frac{1}{3} \left[\frac{a^4}{4} \right] + \left[\frac{a^4}{4} \right]$$

$$= \frac{a^4}{12} + \frac{a^4}{4}$$

$$= \frac{a^4 + 3a^4}{12}$$

$$= \frac{4a^4}{12}$$

$$= \frac{a^4}{3}$$