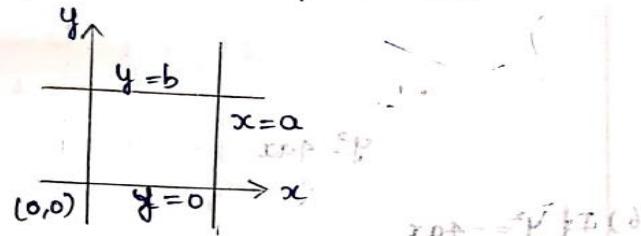


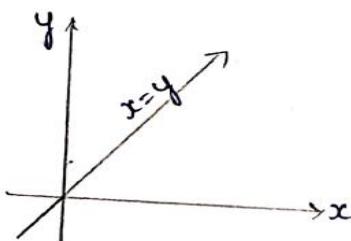


change of order of Integration  
Basic formulae

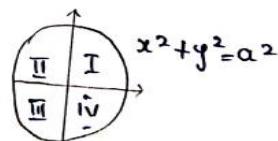
- 1) If  $x=0, y=0, x=a$  and  $y=b$ , then



- 2) If  $x=y$ , then



- 3) If  $x^2 + y^2 = a^2$





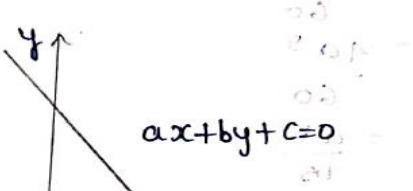
# SNS COLLEGE OF TECHNOLOGY

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Coimbatore-641035.

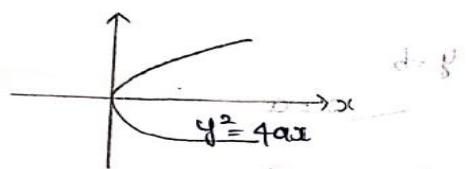


4) If  $ax+by+c=0$  then

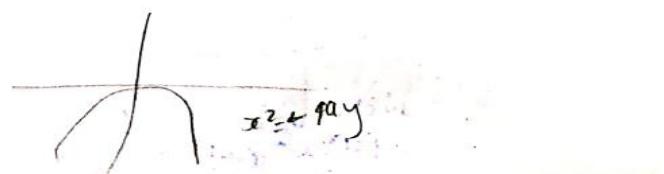
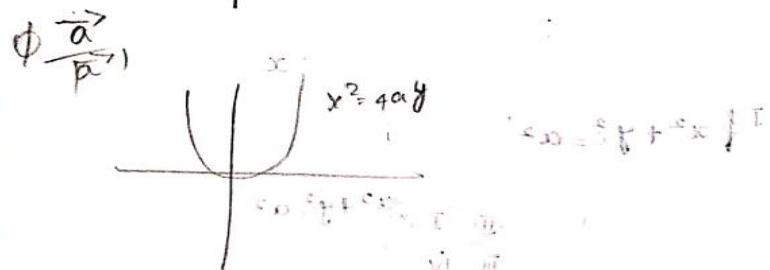
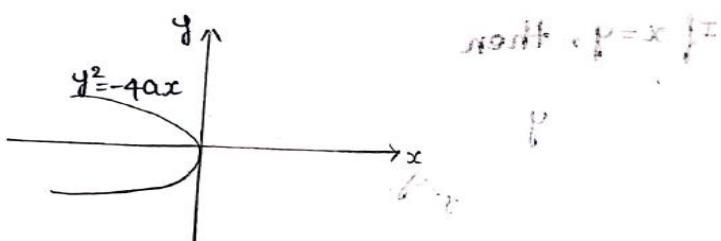


coincident for zero slope  
columns of signs

5) (a) If  $y^2 = 4ax$  then  $x = \frac{y^2}{4a}$   $\Rightarrow x = f(y)$



6) If  $y^2 = -4ax$





Example-1

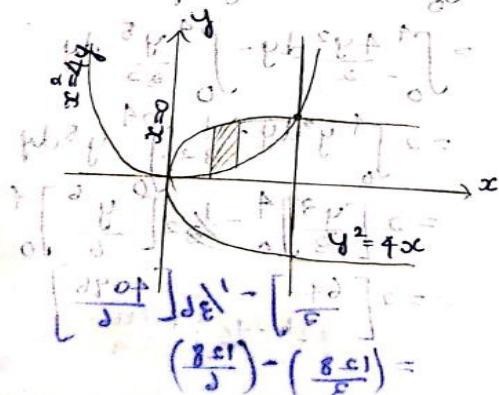
Change the order of integration and  
Evaluate for  $\int_0^4 \int_{x^2}^{2\sqrt{x}} xy \, dy \, dx$

$$I = \int_0^4 \int_{x^2}^{2\sqrt{x}} xy \, dy \, dx$$

Given limits

x limits varies from 0 to 4  
ie,  $x=0$  to  $x=4$

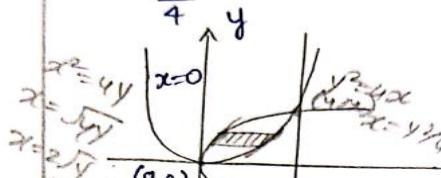
y limits varies from  $x^2$  to  $2\sqrt{x}$   
ie,  $y = x^2$  to  $y = 2\sqrt{x}$





change the order of limits

$$\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{y}} xy \, dx \, dy$$



Help me

area under the curve for x varies from 0 to 4  
y varies from 0 to sqrt(x)

x varies from 0 to y/4  
y varies from 0 to 2

$$I = \int_0^4 \left( \int_{y/4}^{2\sqrt{y}} xy \, dx \right) dy$$

$$= \int_0^4 y \left[ \int_{y/4}^{2\sqrt{y}} x \, dx \right] dy$$

$$= \int_0^4 y \left[ \frac{x^2}{2} \Big|_{y/4}^{2\sqrt{y}} \right] dy$$

$$= \int_0^4 y \left[ \frac{4y}{2} - \frac{y^4}{32} \right] dy$$

$$= \int_0^4 \frac{4y^2}{2} dy - \int_0^4 \frac{y^5}{32} dy$$

$$= 2 \int_0^4 y^2 dy - \frac{1}{32} \int_0^4 y^5 dy$$

$$= 2 \left[ \frac{y^3}{3} \right]_0^4 - \frac{1}{32} \left[ \frac{y^6}{6} \right]_0^4$$

$$= 2 \left[ \frac{64}{3} \right] - \frac{1}{32} \left[ \frac{4096}{6} \right]$$

$$= \left( \frac{128}{3} \right) - \left( \frac{128}{6} \right)$$



$$\begin{aligned} &= \frac{256 - 128}{6} \\ &= \frac{128}{6} \\ &= \frac{64}{3} \end{aligned}$$

+ L  
product of 2 rows  
product of 2 rows  
and 1 row } 10 rows  
so  $\int (-p, q)$

Example 3  
change the order of integration in

$$\int_0^a \int_{x^2}^a (x^2 + y^2) dy dx$$
$$\rightarrow I = \int_0^a \int_x^a (x^2 + y^2) dy dx$$

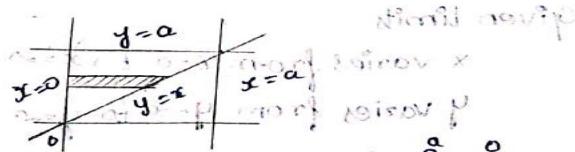
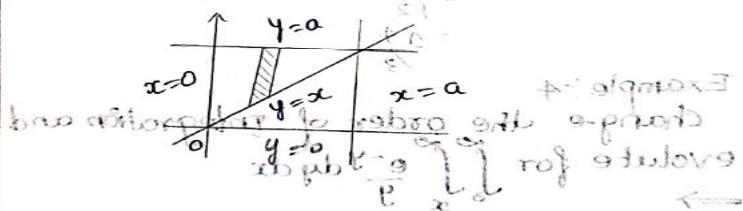
Given limits

x varies from 0 to a

i.e.,  $x=0$ ;  $x=a$

y varies from  $x^2$  to a

i.e.,  $y=x^2$ ,  $y=a$



change the order of limits  $\int_0^a \int_{x^2}^a (x^2 + y^2) dy dx$

x varies from 0 to y

y varies from 0 to a

$$I = \int_0^a \left( \int_{x^2}^a (x^2 + y^2) dy \right) dx$$



Example 3

change the order of integration in

$$\int_0^a \int_x^a (x^2 + y^2) dy dx$$

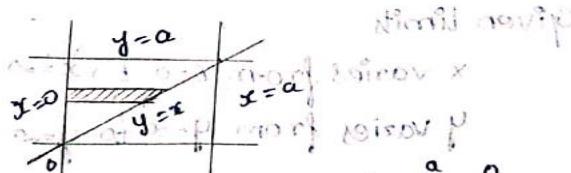
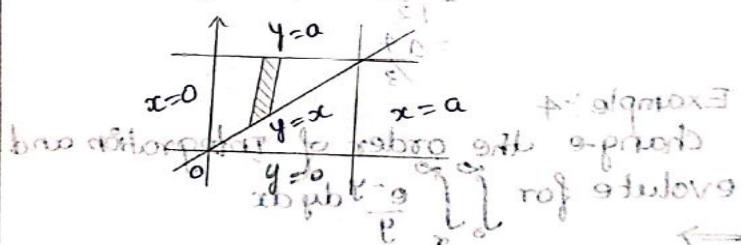
$$\rightarrow I = \int_0^a \int_x^a (x^2 + y^2) dy dx$$

Given limits

x varies from 0 to a

ie,  $x=0$ ;  $x=a$ 

y varies from x to a

ie,  $y=x$ ,  $y=a$ change the order of limits  $\int_0^a \int_0^y (x^2 + y^2) dx dy$ 

x varies from 0 to y

y varies from 0 to a

$$I = \int_0^a \left( \int_0^y (x^2 + y^2) dx \right) dy$$



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$$\begin{aligned} &= \int_0^a \left( \int_0^y x^2 dx + \int_0^y y^2 dx \right) dy \\ &= \int_0^a \left( \left[ \frac{x^3}{3} \right]_0^y + \left[ y^3 \right]_0^y \right) dy \\ &= \int_0^a \left( \left[ \frac{y^3}{3} \right] + y^3 \right) dy \\ &\quad \text{using } \int y^3 dy = \frac{y^4}{4} + C \quad \text{and } \int y^3 dy = \frac{y^4}{4} + C \\ &= \frac{1}{3} \int_0^a y^3 dy + \int_0^a y^3 dy \\ &= \frac{1}{3} \left[ \frac{y^4}{4} \right]_0^a + \left[ \frac{y^4}{4} \right]_0^a \\ &= \frac{1}{3} \left[ \frac{a^4}{4} \right] + \left[ \frac{a^4}{4} \right] \\ &= \frac{a^4}{12} + \frac{a^4}{4} \\ &= \frac{a^4 + 3a^4}{12} \\ &= \frac{4a^4}{12} \\ &= \frac{a^4}{3} \end{aligned}$$