TAYLOR's SERIES EXPANSION.

point (a,b); $h = x - a$; $k = y - b$

$$f(x,y) = f(a,b) + \frac{1}{1!}[h f_x + k f_y]$$

$$+ \frac{1}{2!}[h^2 f_{xx} + k^2 f_{yy} + 2hk f_{xy}]$$

$$+ \frac{1}{3!}[h^3 f_{xxx} + k^3 f_{yyy} + 3h^2k f_{xxy} + \underbrace{3k^2h}_{3hk^2} f_{xyy}]$$

1. Expand $e^x \cos y$ as tailer series in powers of x and y upto third degree

$f(x,y) = e^x \cos y$ point is not given directly

$(a,b) = (0,0)$

$h = x$; $k = y$. but $h = x - a$, $k = y - b$

$f(a,b) = e^a \cos b$

$f(0,0) = e^0 \cos 0 = 1$

$f(a,b) = 1$

$f(x,y) = e^x \cos y$	At pt $(0,0)$
$f_x = \frac{\partial f}{\partial x} = e^x \cos y$	$e^0 \cos 0 = 1$
$f_y = \frac{\partial f}{\partial y} = -e^x \sin y$	$-e^0 \sin 0 = 0$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$

$= \frac{\partial}{\partial x}(e^x \cos y)$ $\qquad e^0 \cos 0 = 1.$

$= e^x \cos y$

$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$

$= \frac{\partial}{\partial y}(-e^x \sin y)$ $\qquad -e^0 \cos 0 = 1$

$= -e^x \cos y.$

$f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial y}\right]$ $\qquad -e^0 \sin 0 = 0$

$= \frac{\partial}{\partial x}(-e^x \sin y)$

$= -e^x \cancel{\cos y} \sin y.$

$f_{xxx} = \frac{\partial^3 f}{\partial x \partial x \partial x} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x^2}\right)$

$= \frac{\partial}{\partial x}(e^x \cos y)$ $\qquad e^0 \cos 0 = 1$

$= e^x \cos y.$

$f_{yyy} = \frac{\partial^3 f}{\partial y \partial y \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)$

$= \frac{\partial}{\partial y}(-e^x \cos y)$ $\qquad e^0 \sin 0 = 0$

$= e^x \sin y.$

$$f_{xxy} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x \partial y}\right) = \frac{\partial}{\partial x}(-e^x \sin y)$$

$$= -e^x \sin y \qquad -e^0 \sin 0 = 0.$$

$$f_{xyy} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)$$

$$= \frac{\partial}{\partial x}(-e^x \cos y) \qquad -e^0 \cos 0 = -1$$

$$= -e^x \cos y.$$

$$f(x,y) = f(a,b) + \frac{1}{1!}[h f_x + k f_y] + \frac{1}{2!}[h^2 f_{xx} + k^2 f_{yy} + 2hk f_{xy}] + \frac{1}{3!}[h^3 f_{xxx} + k^3 f_{yyy} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy}]$$

$$= 1 + \frac{1}{1!}[x(1) + y(0)] + \frac{1}{2!}[x^2(1) + y^2(-1) + 2xy(0)] + \frac{1}{3!}[x^3(1) + y^3(0) + 3x^2 y(0) + 3xy^2(-1)]$$

$$= 1 + x + \frac{1}{2}(x^2 - y^2) + \frac{1}{6}(x^3 - 3xy^2) + \ldots$$

2. $e^x \sin y$ at $(0,0)$

$$f(x,y) = e^x \sin y$$

$$(a,b) = (0,0)$$

$$h = x;\ k = y$$

$$f(a,b) = f(0,0) = e^0 \sin 0 = 0$$

$f(x,y) = e^x \sin y$	At pt $(0,0)$
$f_x = e^x \sin y$	$e^0 \sin 0 = 0$
$f_y = e^x \cos y$	$e^0 \cos 0 = 1$
$f_{xx} = e^x \sin y$	$e^0 \sin 0 = 0$
$f_{yy} = -e^x \sin y$	$-e^0 \sin 0 = 0$
$f_{xy} = \dfrac{\partial^2 f}{\partial x \partial y}$	$e^0 \cos 0 = 1$

$$= \frac{\partial}{\partial x}(e^x \cos y)$$

$$= e^x \cos y$$

$$f_{xxx} = e^x \sin y \qquad e^0 \sin 0 = 1$$

$$f_{yyy} = -e^x \cos y \qquad -e^0 \cos 0 = -1$$

$$f_{xxy} = \frac{\partial^3 f}{\partial x^2 \partial y} \qquad e^0 \cos 0 = 1$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x \partial y}\right)$$

$$= \frac{\partial}{\partial x}(e^x \cos y) = e^x \cos y$$

$$f_{xyy} = \frac{\partial^3 f}{\partial x \partial^2 y}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial^2 y}\right) \qquad -e^0 \sin 0 = 0$$

$$= \frac{\partial}{\partial x}(-e^x \sin y)$$

$$= -e^x \sin y$$

$$f(x,y) = f(a,b) + \frac{1}{1!}[hf_x + kf_y] + \frac{1}{2!}[h^2 f_{xx} + 2hx f_{xy} + k^2 f_{yy}] + \frac{1}{3!}[h^3 f_{xxx} + 3h^2k f_{xxy} + 3hk^2 f_{xyy} + k^3 f_{yyy}]$$

$$= 0 + \frac{1}{1!}[x(0) + y(1)] + \frac{1}{2!}[x^2(0) + 2xy(1) + y^2(0)] + \frac{1}{3!}[x^3(0) + 3x^2y(1) + 3xy^2(0) + y^3(-1)] + \cdots$$

$$= y + \frac{1}{2!}(2xy) + \frac{1}{3!}(3x^2y - y^3) + \cdots$$

$$= y + \frac{1}{2}(2xy) + \frac{1}{6}(3x^2y - y^3) + \cdots$$

2. Expand $e^x \log(1+y)$ in powers of x & y upto 2nd degree.

$f(x,y) = e^x \log(1+y)$

$f(a,b) = (0,0)$

$h = x$; $k = y$ $\qquad \log 1 = 0.$

$f(a,b) = f(0,0) = e^0 \log(1+0) = 1(0) = 0$

$f(x,y) = e^x \log(1+y)$	At pt $(0,0)$
$f_x = e^x \log(1+y)$	$e^0 \log(1+0) = 0$, $1(0) = 0$
$f_y = e^x \frac{1}{1+y}(1)$	$e^0\left(\frac{1}{1+0}\right) = 1$
$f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = e^x \frac{1}{1+y}$	

$$f(x,y) = f(a,b) + \frac{1}{1!}[hf_x + kf_y] + \frac{1}{2!}[h^2 f_{xx} + 2hx f_{xy} + k^2 f_{yy}] + \frac{1}{3!}[h^3 f_{xxx} + 3h^2k f_{xxy} + 3hk^2 f_{xyy} + k^3 f_{yyy}]$$

$$= 0 + \frac{1}{1!}[x(0) + y(1)] + \frac{1}{2!}[x^2(0) + 2xy(1) + y^2(0)] + \frac{1}{3!}[x^3(0) + 3x^2y(1) + 3xy^2(0) + y^3(-1)] + \cdots$$

$$= y + \frac{1}{2!}(2xy) + \frac{1}{3!}(3x^2y - y^3) + \cdots$$

$$= y + \frac{1}{2}(2xy) + \frac{1}{6}(3x^2y - y^3) + \cdots$$

2. Expand $e^x \log(1+y)$ in powers of x & y upto 2nd degree.

$f(x,y) = e^x \log(1+y)$

$f(a,b) = (0,0)$

$h = x$; $k = y$ $\qquad \log 1 = 0.$

$f(a,b) = f(0,0) = e^0 \log(1+0)$

$= 1(0) = 0$

$f(x,y) = e^x \log(1+y)$	At pt $(0,0)$
$f_x = e^x \log(1+y)$	$e^0 \log(1+0) = 1(0) = 0$
$f_y = e^x \frac{1}{1+y}(1)$	$e^0\left(\frac{1}{1+0}\right) = 1$
$f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = e^x \frac{1}{1+y}$	

$f_{xx} = e^x \log(1+y)$

$f_{yy} = e^x \left(\frac{-1}{(1+y)^2}\right)(1)$

$f(x,y) = f(a,b) + \frac{1}{1!}[hf_x + kf_y] + \frac{1}{2!}[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}]$

$= 0 + \frac{1}{1!}[x(0) + y(1)] + \frac{1}{2!}[x^2(0) + 2xy(1) + y^2(-1)]$

$= \frac{1}{1}(0+y) + \frac{1}{2}(0 + 2xy - y^2)$

$= y + \frac{1}{2}(2xy - y^2) + \cdots$

4. Expand $xy^2 + 2x - 3y$ in powers of $(x+2)$ and $(y-1)$ upto second degree.

$f(x,y) = xy^2 + 2x - 3y$

$(a,b) = (-2,1)$

$x+2=0$, $x=-2$

$h = x-a = x+2$

$k = y-b = y-1$

$y-1=0$, $y=+1$

$f(a,b) = f(-2,1)$

$= (-2)(1)^2 + 2(-2) - 3(1)$

$= -2 - 4 - 3$

$f(a,b) = -9$

$f(x,y) = xy^2+2x-3y$	At $(-2, 1)$
$f_x = y^2+2$	$(1)^2+2 = 3$
$f_y = 2xy-3$	$2(-2)(1)-3 = -7$
$f_{xx} = 0$	0
$f_{yy} = 2x$	$2(-2) = -4$
$f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = 2y$	$2(1) = 2$

$$f(x,y) = f(a,b) + \frac{1}{1!}[hf_x + kf_y] + \frac{1}{2!}[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}] + \cdots$$

$$= -9 + \frac{1}{1!}[(x+2)3 + (y-1)(-7)] + \frac{1}{2!}[(x+2)^2 0 + 2(x+2)(y-1)2 + (y-1)^2(-4)] + \cdots$$

$$= -9 + 1[3x+6-7y+7] + \frac{1}{2}[x^2+4+4x + 4(xy - x + 2y - 2) + (y^2+1-2)(-4)]$$

$$= -9 + 3x - 7y + 13 + \frac{1}{2}[x^2 + \cancel{4} + \cancel{4x} + 4xy - \cancel{4x} + 8y - \cancel{8} - 4y^2 - \cancel{4} + \cancel{8}]$$

$$= -9 + 3x - 7y + 13 + \frac{1}{2}[x^2 - 4y^2 + 3y + 4xy]$$