Unit-5.

1) The Joint probability mass Function of x and y is given by

x \ Y	-1	1
0	1/8	3/8
1	2/8	2/8

Find the coefficient of correlation

X	0	1
P(x)	4/8	4/8

y	-1	1
P(y)	3/8	5/8

$E(x) = \sum xp(x) = 0 \times \frac{4}{8} + 1 \times \frac{4}{8} = 4/8$

$E(y) = \sum yp(y) = (\frac{-3}{8}) + \frac{5}{8} = 2/8$

$E(x^2) = \sum x^2 p(x) = 0 \times \frac{4}{8} + 1^2 \times 4/8 = 4/8$

$E(y^2) = \sum y^2 p(y) = 1 \times \frac{3}{8} + 1 \times \frac{5}{8} = 1$

$V(x) = E(x^2) - E(x)^2$

$= \frac{4}{8} - (\frac{4}{8})^2 = \frac{4}{8}(1 - 4/8)$

$= \frac{4}{8}(\frac{4}{8}) = \frac{16}{64} = \frac{1}{4}$

$E(xy) = \sum_x \sum_y xy \, p(x,y)$

$= (0)(-1)\frac{1}{8} + (0)(1)\frac{3}{8} + (1)(-1)\frac{2}{8} + (1)(1)\frac{2}{8}$

$= -\frac{2}{8} + \frac{2}{8} = 0$

$Cov(x,y) = E(xy) - E(x)E(y)$

$= 0 - 4/8 \cdot 2/8 = -1/8$

$$r = \frac{cov(x,y)}{\sqrt{v(x)}\sqrt{v(y)}} = \frac{-1/8}{\sqrt{1/4}\sqrt{\frac{15}{16}}}$$

$$\boxed{r = -0.26}$$

② var

Find the covariance matrix for the two random variable x_1 and x_2 when their Joint Probability Function $P(x_1, x_2)$ is given by

x_1\\x_2	0	1	$P(x_1)$
-1	0.24	0.06	0.30
0	0.16	0.14	0.30
1	0.40	0.00	0.40
$P(x_2)$	0.80	0.20	1.00

Soln:-

$$E(x_1) = \sum x_i^2 P(x_i)$$

$$= x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

$$= (-1)(0.3) + 0(0.3) + 1(0.4)$$

$$\boxed{E(x_1) = 0.1}$$

$$E(x_2) = \sum x_2 P(x_2)$$

$$= 0(0.8) + 1(0.2)$$

$$\boxed{E(x_2) = 0.2}$$

$$\text{Cov. Matrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 0.69 & -0.08 \\ -0.08 & 0.16 \end{bmatrix}$$

$$\sigma_{11} = \sum_{x_1} (x_1 - 0.1)^2 . P(x_1)$$

$$= (-1 - 0.1)^2 (0.3) + (0 - 0.1)^2 0.3$$
$$+ (1 - 0.1)^2 \quad 0.4$$

$$= 0.363 + 0.003 + 0.324$$

$$\boxed{\sigma_{11} = 0.69}$$

$$\sigma_{12} = (-1 - 0.1)(0 - 0.2)(0.24) +$$
$$(-1 - 0.1)(1 - 0.2)(0.06) +$$
$$(0 - 0.1)(0 - 0.2)(0.16) +$$
$$(0 - 0.1)(1 - 0.2)(0.14) +$$
$$(1 - 0.1)(0 - 0.2)(0.40) +$$
$$(1 - 0.1)(1 - 0.2)(0.00)$$

$$\boxed{\sigma_{12} = -0.08}$$

$$\sigma_{21} = \sigma_{12} = -0.08$$

$$\sigma_{22} = \sum_{x_2} (x_2 - 0.2)^2 P(x_2)$$

$$= (0 - 0.2)^2 (0.8) + (1 - 0.2)^2 0.20$$

$$\boxed{\sigma_{22} = 0.16}$$

$$\mu = \begin{bmatrix} E(x_1) \\ E(x_2) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$