17-03

1) The following table gives the aptitude test scores and productivity indices of 10 workers selected at random.

Aptitude scores (x)	60	62	65	70	72	48	53	73	65	82
productivity index (y)	68	60	62	80	85	40	52	62	60	81

Find the two regression equations and estimate:
(i) the productivity index of a worker whose test score is 92.
(ii) the test score of a worker whose productivity index is 75.

Soln: $\bar{x} = \frac{650}{10} = 65$; $\bar{y} = \frac{650}{10} = 65$

x	y	$x = x - \bar{x}$	$y = y - \bar{y}$	x^2	y^2	xy
60	68	-5	3	25	9	-15
62	60	-3	-5	9	25	15
65	62	0	-3	0	9	0
70	80	5	15	25	225	75
72	85	7	20	49	400	140
48	40	-17	-25	289	625	425
53	52	-12	-13	144	169	156
73	62	8	-3	64	9	-24
65	60	0	-5	0	25	0
82	81	17	16	289	256	272
				$\Sigma x^2 = 894$	$\Sigma y^2 = 1752$	$\Sigma xy = 1044$

The regression line x on Y is

$(x - \bar{x}) = b_{xy}(y - \bar{y})$ where $b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{1044}{1752} = 0.596$

$(x - 65) = 0.596(y - 65)$

$x - 65 = 0.596y - 38.74$

$x = 0.596y - 38.74 + 65$

$x = 0.596y + 26.26$

ii) Estimate value of x when $y = 75$

$x = 0.596(75) + 26.26$

$x = 70.96$

Regression equation of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.8 \times \frac{12}{3} = 3.2$$

$$y - 90 = 3.2x - 32 + 90$$

$$y = 3.2x + 58$$

(ii) when advertisement expenditure is Rs.15 lakhs,

$$y = 3.2(15) + 58$$

$$= 46 + 58$$

$$y = 106$$

Sales = 106 lakhs

(iii) when sales target is 120 lakhs, the advertisement expenditure is,

$$x = (0.2)(120) - 8$$

$$= 24 - 8$$

$$x = 16$$

Advertisement expenditure = 16 lakhs

3) A panel of Judges A & B graded seven debators and independently awarded the following marks.

Debators	1	2	3	4	5	6	7
Marks by A (x)	40	34	28	30	44	38	31
Marks by B (y)	32	39	26	30	38	34	28

Soln:

$$\bar{x} = \frac{\Sigma x}{N} = \frac{245}{7} = 35$$

$$\bar{y} = \frac{\Sigma y}{N} = \frac{227}{7} = 32.43$$

The regression line y on x is

$y - \bar{y} = b_{yx}(x - \bar{x})$ where $b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{1044}{894} = 1.167$

$(y - 65) = 1.167(x - 65)$

$y - 65 = 1.167x - 75.855$

(i) Estimate value of y when $x = 92$

$y - 65 = 1.167(92) - 75.855$

$y = 107.364 - 75.855 + 65$

$y = 107.364 - 10.855$

$\boxed{y = 96.509}$

2) You are given below the following information about advertising and sales.

	Adv. Exp (x) (Rs. Lakhs)	Sales (y) (Rs. Lakhs)
Mean	10	90
S.D	3	12

Correlation coefficient = 0.8 (7)

(i) obtain the two regression lines

(ii) Find the likely sales when advertisement expenditure is Rs. 15 lakhs.

(iii) what should be advertisement expenditure if the company wants to attain sales target of Rs. 120 lakhs?

Soln:

(i) Regression equation of x on y

$x - \bar{x} = b_{xy}(y - \bar{y})$

$b_{xy} = r\,\sigma_x/\sigma_y$

$r = 0.8$, $\bar{x} = 10$, $\bar{y} = 90$, $\sigma_x = 3$, $\sigma_y = 12$

$b_{xy} = 0.8 \times \frac{3}{12} = 0.2$

$x - 10 = 0.2(y - 90)$

$x = 0.2y - 18 + 10$

$\boxed{x = 0.2y - 8}$

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	xy	x^2	y^2
40	32	5	-0.43	-2.15	25	0.185
34	39	-1	6.57	-6.57	1	43.165
28	26	-7	-6.43	45.01	49	41.345
30	30	-5	-2.43	12.15	25	5.905
44	38	9	5.57	50.13	81	31.025
38	34	3	1.57	4.71	9	2.465
31	28	-4	-4.43	17.72	16	19.625
				$\Sigma xy = 121$	$\Sigma x^2 = 206$	$\Sigma y^2 = 143.715$

The regression line of Y on x,

$$x - \bar{Y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{121}{206} = 0.587$$

$$x - 32.43 = 0.587(x - 35)$$

$$Y = 0.587x - 20.545 + 32.480$$

$$Y = 0.587x + 11.885$$

when $x = 36$

$$Y = 0.587(36) + 11.885$$

$$Y = 33.017 \simeq 33.000$$

∴ If judge B were also present, he would have awarded 33 marks for 8th debator.