



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT 4 Lagrange's Method of Undermined Multipliers

2. A rectangular box open at the top is to have a volume of 32 cc. Find the dimension of the box requiring the least material for the construction minima

$f(x, y, z) = xy + 2yz + 2zx \rightarrow$ Area of rectangle except the top area
 $g(x, y, z) = xyz - 32$ } condition is $V = 32$
 volume of $\square = xyz$

$F = f + \lambda g$
 $= xy + 2yz + 2zx + \lambda (xyz - 32) \quad \dots (1)$

$\frac{\partial F}{\partial x} = y + 2z + \lambda yz$
 $\frac{\partial F}{\partial y} = x + 2z + \lambda xz$
 $\frac{\partial F}{\partial z} = 2y + 2x + \lambda xy$
 $\frac{\partial F}{\partial \lambda} = xyz - 32$

$\frac{\partial F}{\partial x} = 0 \Rightarrow y + 2z + \lambda yz = 0$
 $-\lambda yz = y + 2z$
 $-\lambda = \frac{y}{yz} + \frac{2z}{yz}$
 $-\lambda = \frac{1}{z} + \frac{2}{y} \quad \dots (2)$

$\frac{\partial F}{\partial y} = 0 \Rightarrow x + 2z + \lambda xz = 0$
 $-\lambda xz = x + 2z$
 $-\lambda = \frac{x}{xz} + \frac{2z}{xz}$
 $-\lambda = \frac{1}{z} + \frac{2}{x} \quad \dots (3)$



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$$\frac{\partial F}{\partial z} = 0 \Rightarrow 2y + 2x + \lambda xy = 0$$

$$+\lambda xy = 2x + 2y$$

$$-\lambda = \frac{2x}{xy} + \frac{2y}{xy}$$

$$-\lambda = \frac{2}{y} + \frac{2}{x} \quad \text{--- (3)}$$

$$-\lambda = \frac{2}{x} + \frac{2}{y} \quad \text{--- (4)}$$

$$\frac{\partial F}{\partial x} = 0 \Rightarrow xyz - 32 = 0$$

$$xyz = 32 \quad \text{--- (5)}$$

(2) & (3) $\Rightarrow \frac{1}{z} + \frac{2}{y} = \frac{1}{z} + \frac{2}{x}$

$$\frac{2}{y} = \frac{2}{x}$$

$$\frac{1}{y} = \frac{1}{x}$$

$$x = y$$

(3) & (4) $\Rightarrow \frac{1}{z} + \frac{2}{x} = \frac{2}{x} + \frac{2}{y}$

$$\frac{1}{z} = \frac{2}{y}$$

$$y = 2z$$

$$\Rightarrow x = y = 2z$$

(5) $\Rightarrow (x)(x)\left(\frac{x}{2}\right) = 32$

$$x^3 = 64$$

$$x = 4; y = 4; z = 2$$



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Least material for construction is.

$$f = 16 + 2(4)(2) + 2(2)(4)$$

$$= 16 + 16 + 16 = 48$$

3. Find the volume of the largest rectangular solid which can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Let dimensions be $2x, 2y, 2z$

$$f(x, y, z) = 8xyz$$

$$g(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$F = f + \lambda g$$

$$= 8xyz + \lambda \left[\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 \right] \quad \text{--- (1)}$$

$$\frac{\partial F}{\partial x} = 8yz + \lambda \left(\frac{2x}{a^2} \right)$$

$$\frac{\partial F}{\partial y} = 8xz + \lambda \left(\frac{2y}{b^2} \right)$$

$$\frac{\partial F}{\partial z} = 8xy + \lambda \left(\frac{2z}{c^2} \right)$$

$$\frac{\partial F}{\partial \lambda} = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1$$

$$\frac{\partial F}{\partial x} = 0 \rightarrow 8yz + 2\lambda \frac{x}{a^2} = 0 \quad \text{--- (2)}$$



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$$-2\lambda \frac{x}{a^2} = 8yz \quad (2)$$

$$-\lambda = \frac{8yz a^2}{2x}$$

$$-\lambda = \frac{4yz a^2}{x} \quad (3)$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow 8xz + \frac{\lambda(2y)}{b^2} = 0 \quad (4)$$

$$-\lambda = \frac{4xz b^2}{y} \quad (5)$$

$$\frac{\partial F}{\partial z} = 0 \Rightarrow 8xy + \frac{\lambda(2z)}{c^2} = 0 \quad (6)$$

$$-\lambda = \frac{4xy c^2}{z} \quad (7)$$

$$\frac{\partial F}{\partial \lambda} = 0 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (8)$$

multiply (2) with x, (4) with y, (6) with z

$$\frac{8xyz}{a^2} + 2\lambda \frac{x^2}{a^2} + 8yzx + 2\lambda \frac{y^2}{b^2} + 8xyz + 2\lambda \frac{z^2}{c^2} = 0$$

$$24xyz + 2\lambda(x^2 + y^2 + z^2) = 0$$

$$-2\lambda = 24xyz$$

$$-\lambda = 12xyz \quad (9)$$

sub (9) in (3)

$$\sqrt[3]{2}xyz = \frac{4yz a^2}{x}$$

$$3x^2 = a^2$$

$$x^2 = \frac{a^2}{3} \Rightarrow x = \frac{a}{\sqrt{3}}$$

$$x + yz = \frac{76}{\sqrt{3}}$$



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Sub (9) in (b)

$$12xyz = \frac{4xyb^2}{y}$$
$$y^2 = \frac{b^2}{3}$$
$$y = \frac{b}{\sqrt{3}}$$

Sub (9) in (7)

$$12xyz = \frac{4xyz^2}{z}$$
$$z^2 = \frac{c^2}{3}$$
$$z = \frac{c}{\sqrt{3}}$$

Thus the maximum value of $V = 8xyz$

$$= 8 \left[\frac{a}{\sqrt{3}} \right] \left[\frac{b}{\sqrt{3}} \right] \left[\frac{c}{\sqrt{3}} \right]$$

4. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$

$$f(x, y, z) = x^2 + y^2 + z^2$$
$$g(x, y, z) = x + y + z - 3a$$
$$F = f + \lambda g$$
$$= x^2 + y^2 + z^2 + \lambda x + \lambda y + \lambda z - 3\lambda a \quad (1)$$
$$\frac{\partial F}{\partial x} = 2x + \lambda$$
$$\frac{\partial F}{\partial y} = 2y + \lambda$$



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$$\frac{\partial F}{\partial z} = 2z + \lambda$$

$$\frac{\partial F}{\partial \lambda} = x + y + z - 3a$$

$$\frac{\partial F}{\partial x} = 0 \Rightarrow 2x + \lambda = 0$$

$$-\lambda = 2x \quad \text{--- (2)}$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow 2y + \lambda = 0$$

$$-\lambda = 2y \quad \text{--- (3)}$$

$$\frac{\partial F}{\partial z} = 0 \Rightarrow 2z + \lambda = 0$$

$$-\lambda = 2z \quad \text{--- (4)}$$

$$\frac{\partial F}{\partial \lambda} = 0 \Rightarrow x + y + z - 3a = 0$$

$$x + y + z = 3a \quad \text{--- (5)}$$

$$(2) \& (3) \Rightarrow 2x = 2y$$

$$x = y$$

$$(3) \& (4) \Rightarrow 2y = 2z$$

$$y = z$$

$$\Rightarrow x = y = z$$

$$(5) \Rightarrow x + x + x = 3a$$

$$3x = 3a$$

$$x = a ; y = a ; z = a$$

Minimum value is

$$f = x^2 + y^2 + z^2$$

$$= a^2 + a^2 + a^2$$

$$f = 3a^2$$



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5. Find the dimensions of the rectangular box without a top of maximum capacity whose surface is 108 sq. cm.

Volume $\rightarrow xyz$

$$f(x, y, z) = xyz$$

$$g(x, y, z) = xy + 2xz + 2yz - 108$$

$$F = f + \lambda g$$

$$= xyz - 108 + \lambda(xy + 2xz + 2yz - 108) \quad \text{--- (1)}$$

$$\frac{\partial F}{\partial x} = yz + \lambda y + 2z\lambda$$

$$\frac{\partial F}{\partial y} = xz + \lambda x + 2\lambda z$$

$$\frac{\partial F}{\partial z} = xy + \lambda 2x + \lambda 2y$$

$$\frac{\partial F}{\partial \lambda} = xy + 2xz + 2yz - 108$$

$$\frac{\partial F}{\partial x} = 0 \Rightarrow yz + \lambda(y + 2z) = 0$$

$$-\lambda(y + 2z) = yz$$

$$-\lambda = \frac{yz}{y + 2z} \quad \text{--- (2)}$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow xz + \lambda(x + 2z) = 0$$

$$-\lambda(x + 2z) = xz$$

$$-\lambda = \frac{xz}{x + 2z} \quad \text{--- (3)}$$



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$$\frac{\partial F}{\partial z} = 0 \Rightarrow xy + \lambda(2x + 2y) = 0$$
$$-\lambda = \frac{xy}{2x + 2y} \quad \text{--- (4)}$$

from (2) & (3)

$$\frac{\partial F}{\partial \lambda} = 0 \Rightarrow xy + 2xz + 2yz - 108 = 0$$
$$xy + 2xz + 2yz = 108 \quad \text{--- (5)}$$

from (2) & (3)

$$\frac{yz}{y + 2z} = \frac{xz}{x + 2z}$$
$$y(x + 2z) = x(y + 2z)$$
$$xy + 2zy = xy + 2zx$$
$$y = x$$

(3) & (4) $\Rightarrow \frac{xz}{x + 2z} = \frac{xy}{2x + 2y}$

$$z(2x + 2y) = y(x + 2z)$$
$$2xz + 2zy = xy + 2yz$$
$$2z = y$$
$$\Rightarrow x = y = 2z$$

(5) $\Rightarrow (x)(x) + 2(x)\left[\frac{x}{2}\right] + 2(x)\left[\frac{x}{2}\right] = 108$

$$x^2 + x^2 + x^2 = 108 \Rightarrow 3x^2 = 108$$
$$x^2 = 36$$
$$\Rightarrow x = 6 ; y = 6 ; z = 3$$

Dimensions of box length = 6 cm, breadth = 6 cm ; height = 3 cm.



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6 The temperature T at any point in space is $T = 400xyz^2$. Find the highest temperature in the surface of unit sphere $x^2 + y^2 + z^2 = 1$

$$f(x, y, z) = 400xyz^2$$
$$g(x, y, z) = x^2 + y^2 + z^2 - 1$$
$$F = f + \lambda g$$
$$= 400xyz^2 + \lambda(x^2 + y^2 + z^2 - 1)$$
$$= 400xyz^2 + \lambda x^2 + \lambda y^2 + \lambda z^2 - \lambda \quad (1)$$
$$\frac{\partial F}{\partial x} = 400yz^2 + 2\lambda x$$
$$\frac{\partial F}{\partial y} = 400xz^2 + 2\lambda y$$
$$\frac{\partial F}{\partial z} = 800xyz + 2\lambda z$$
$$\frac{\partial F}{\partial \lambda} = x^2 + y^2 + z^2 - 1$$
$$\frac{\partial F}{\partial x} = 0 \Rightarrow 400yz^2 + 2\lambda x = 0$$
$$-2\lambda x = 400yz^2$$
$$-\lambda = \frac{400yz^2}{2x} \quad (2)$$
$$\frac{\partial F}{\partial y} = 0 \Rightarrow 400xz^2 + 2\lambda y = 0$$
$$-2\lambda y = 400xz^2$$
$$-\lambda = \frac{400xz^2}{2y} \quad (3)$$



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$$\frac{\partial F}{\partial z} = 0 \Rightarrow 800xyz^2 + 2\lambda z = 0$$

$$-2\lambda z = -800xyz^2$$

$$-\lambda = \frac{800xyz^2}{2z}$$

$$-\lambda = \frac{800xy}{2} \quad \text{--- (4)}$$

$$\frac{\partial F}{\partial \lambda} = 0 \Rightarrow x^2 + y^2 + z^2 - 1 = 0$$

$$x^2 + y^2 + z^2 = 1 \quad \text{--- (5)}$$

$$(2) \& (3) \Rightarrow \frac{400yz^2}{2x} = \frac{400xz^2}{2y}$$

$$x^2 = y^2$$

$$(3) \& (4) \Rightarrow \frac{400xz^2}{2y} = \frac{800xy}{2}$$

$$y^2 = \frac{z^2}{2}$$

$$(5) \Rightarrow x^2 + y^2 + z^2 = 1$$

$$x^2 + x^2 + 2x^2 = 1$$

$$4x^2 = 1$$

$$x^2 = \frac{1}{4} \therefore y^2 = \frac{1}{4}; z^2 = \frac{2 \times \frac{1}{4}}{2} = \frac{1}{2}$$

minimum value is

$$f = 400xyz^2$$

$$= 400 \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2}$$

$$= 100 \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2}$$

$$f = 50$$