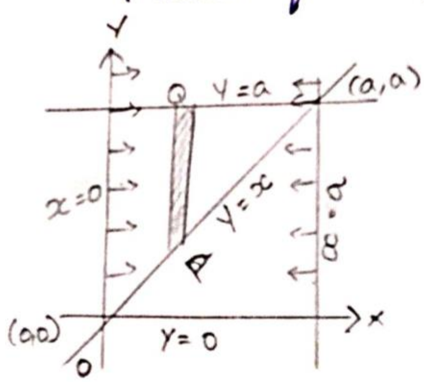
Change of order of integration

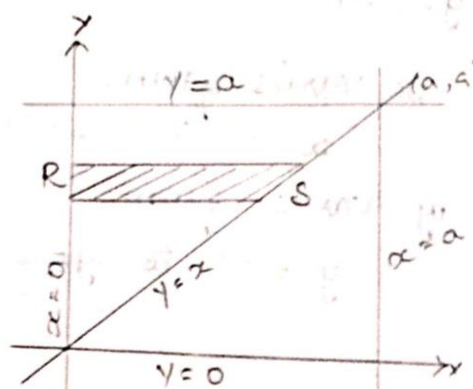
1. Change the order of integration in $\int_0^a \int_x^a f(x,y)\,dy\,dx$

Sol

Given: x varies from $x=0$ to $x=a$.

Y varies from $y=x$ to $y=a$





The region of integration is bounded by $x=0, x=a$

$Y=x$ and $y=a$.

Here, x varies from $x=0$ to $x=a$

Y varies from $Y=x$ to $y=a$

Our region of integration is a triangle.

Here,

$x = 0$ to $x = a$ represents vertical path.

$y = x$ to $y = a$ represents vertical strip PQ sliding area.

Changing the order of integration is nothing but to change the vertical path into horizontal path and to change the vertical strip to Horizontal strip RS

Hence, by changing the order, we get

$$= \int_0^a \int_0^y f(x,y)\,dx\,dy.$$

H.W

2. Change the order of integration and hence evaluate.

Sol

The region of integration is bounded by.

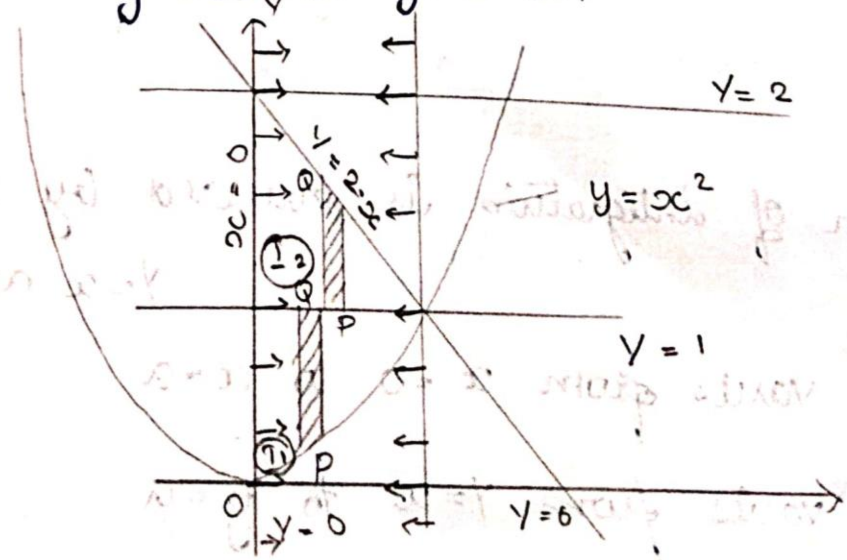
$x = 0$, $x = 1$, $y = x^2$ and
$y = 2 - x$.

Here x varies from.

$x = 0$ to $x = 1$

and y varies from.

$y = x^2$ to $y = 2 - x$.



Now, we divide the region into

$$I = I_1 + I_2$$

$$= \int_0^1 \int_{x^2}^1 xy\,dy\,dx + \int_0^1 \int_1^{2-x} xy\,dy\,dx.$$

In I_1

x varies from $x = 0$ to $x = 1$

y varies from $y = x^2$ to $y = 1$

Here, $x = 0$ to $x = 1$ represents vertical path and $y = x^2$ to $y = 1$ represents vertical strip sliding area.

Changing the order of integration is nothing but to change the vertical path into Horizontal path and to change the vertical strip PQ into horizontal strip RS.

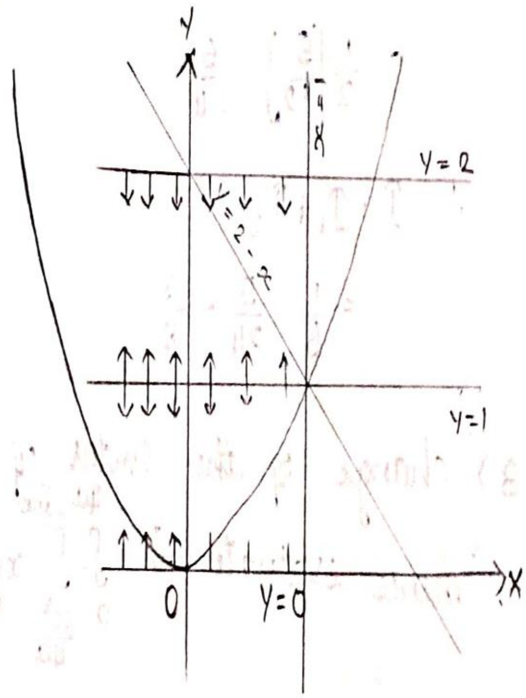
Hence, by changing the order,

we get $\int_0^1 \int_0^{\sqrt{y}} xy\,dx\,dy$.

$$= \int_0^1 \left[y\frac{x^2}{2} \right]_{x=0}^{x=\sqrt{y}} dy$$

$$= \int_0^1 \left[\frac{y^2}{2} - 0\right] dy = \frac{1}{2}\int_0^1 y^2\,dy.$$

$$= \frac{1}{2}\left[\frac{y^3}{3}\right]_0^1 = \frac{1}{2}\left[\frac{1}{3}\right] = \frac{1}{6}.$$



In I_2

x varies from $x=0$ to $x=1$ and Y axis

$y=1$ to $y=2-x$

Changing the order of integration is nothing but to change the vertical path into horizontal path and to change the vertical strip PQ into horizontal strip RS.

Hence, by changing the order of integration, we get

$$I_2 = \int_1^2 \int_0^{2-y} xy \, dx\, dy = \int_1^2 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2-y} dy$$

$$= \int_1^2 \left[\frac{y(2-y)^2}{2} - 0\right] dy = \frac{1}{2}\int_1^2 (2-y)^2 y\, dy$$

$$= \frac{1}{2}\int_1^2 (4+y^2-4y)\, y\, dy = \frac{1}{2}\int_1^2 (4y+y^3-4y^2)\, dy$$

$$= \frac{1}{2}\left[4\frac{y^2}{2} + \frac{y^4}{4} - 4\frac{y^3}{3}\right]_1^2$$

$$= \frac{1}{2}\left[2y^2 + \frac{y^4}{4} - \frac{4}{3}y^3\right]_1^2$$

$$= \frac{1}{2}\left[\left(8 + \frac{16}{4} - \frac{32}{3}\right) - \left(2 + \frac{1}{4} - \frac{4}{3}\right)\right]$$

$$= \frac{1}{2}\left[\frac{5}{12}\right] = \frac{5}{24}.$$

$$\therefore \quad I = I_1 + I_2$$

$$= \frac{1}{6} + \frac{5}{24} = \frac{3}{8}$$