



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

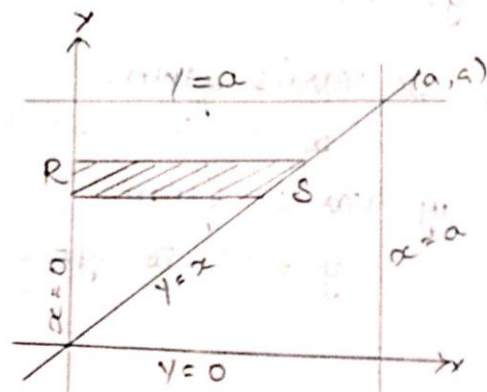
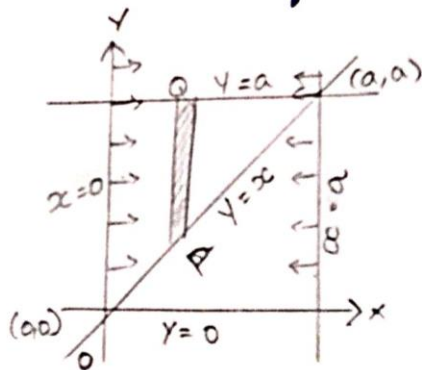
Change of order of integration

1. Change the order of integration in $\int_0^a \int_x^a f(x,y) dy dx$

Sol

Ans: x varies from $x=0$ to $x=a$

y varies from $y=x$ to $y=a$



The region of integration is bounded by $x=0$, $x=a$, $y=x$ and $y=a$.

Here, x varies from $x=0$ to $x=a$

y varies from $y=x$ to $y=a$



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Our region of integration is a triangle.

Here,

$x=0$ to $x=a$ represents vertical path.

$y=x$ to $y=a$ represents vertical strip PQ
sliding area.

Changing the order of integration is
nothing but to change the vertical path
into horizontal path and to change
the vertical strip to horizontal strip RS

Hence, by changing the order, we get

$$= \int_0^a \int_0^y f(x,y) dx dy.$$

H.W

2. Change the order of integration and hence
evaluate.



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Sol

The region of integration is bounded by.

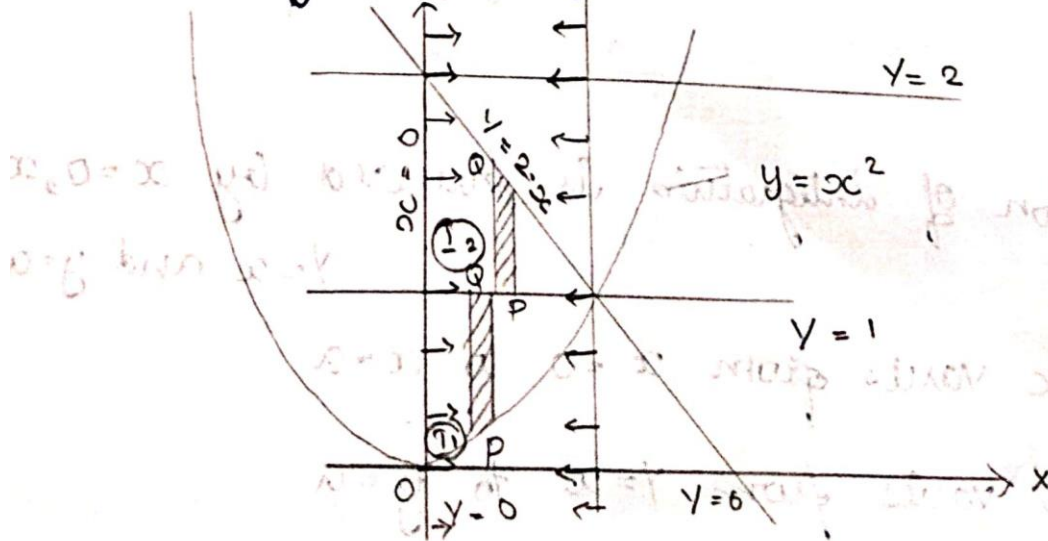
$$x = 0, x = 1, y = x^2 \text{ and } y = 2 - x.$$

Here x varies from,

$$x = 0 \text{ to } x = 1$$

and y varies from,

$$y = x^2 \text{ to } y = 2 - x.$$





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Now, we divide the region into

$$I = I_1 + I_2$$
$$= \int_0^1 \int_{x^2}^1 xy \, dy \, dx + \int_0^1 \int_0^{2-x} xy \, dy \, dx.$$

In I_1

x varies from $x=0$ to $x=1$

y varies from $y=x^2$ to $y=1$

Here, $x=0$ to $x=1$ represents vertical path and
 $y=x^2$ to $y=1$ represents vertical strip sliding
area.

Changing the order of integration is nothing
but to change the vertical path into Horizontal
path and to change the vertical strip PQ into
horizontal strip RS.



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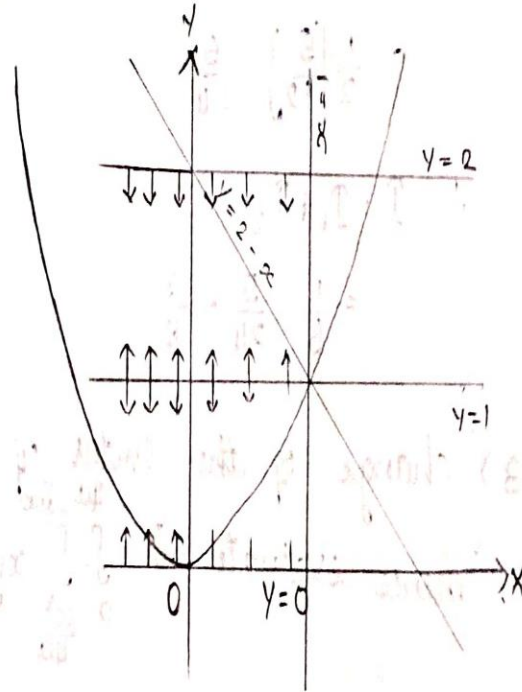
Hence, by changing the order,

we get $\int_0^1 \int_0^{\sqrt{y}} xy \, dx \, dy$.

$$= \int_0^1 \left[y \frac{x^2}{2} \right]_{x=0}^{x=\sqrt{y}} dy$$

$$= \int_0^1 \left[\frac{y^2}{2} - 0 \right] dy = \frac{1}{2} \int_0^1 y^2 dy$$

$$= \frac{1}{2} \left[\frac{y^3}{3} \right]_0^1 = \frac{1}{2} \left[\frac{1}{3} \right] = \frac{1}{6}$$



In I_2

x varies from $x=0$ to $x=1$ and Y axis

$y=1$ to $y=2-x$



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changing the order of integration is nothing but to change the vertical path into horizontal path and to change the vertical strip PQ into horizontal strip RS.

Hence, by changing the order of integration, we get

$$I_2 = \int_1^2 \int_0^{2-y} xy \, dx \, dy = \int_1^2 \left[\frac{x^2}{2} y \right]_{x=0}^{x=2-y} dy$$

$$= \int_1^2 \left[\frac{y(2-y)^2}{2} - 0 \right] dy = \frac{1}{2} \int_1^2 (2-y)^2 y \, dy$$

$$= \frac{1}{2} \int_1^2 (4 + y^2 - 4y) y \, dy = \frac{1}{2} \int_1^2 (4y + y^3 - 4y^2) dy$$

$$= \frac{1}{2} \left[4 \frac{y^2}{2} + \frac{y^4}{4} - 4 \frac{y^3}{3} \right]_1^2$$

$$= \frac{1}{2} \left[2y^2 + \frac{y^4}{4} - \frac{4}{3} y^3 \right]_1^2$$

$$= \frac{1}{2} \left[\left(8 + \frac{16}{4} - \frac{32}{3} \right) - \left(2 + \frac{1}{4} - \frac{4}{3} \right) \right]$$

$$= \frac{1}{2} \left[\frac{5}{12} \right] = \frac{5}{24}$$

$$\therefore I = I_1 + I_2$$

$$= \frac{1}{6} + \frac{5}{24} = \frac{3}{8}$$