



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chen

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

Change of order of integration

3) change of the order of integration and hence evaluate it $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \, dy \, dx$.

Sol

The region of integration is bounded by

$$x = 0; x = 4a,$$

$$y = \frac{x^2}{4a} \text{ and } y = 2\sqrt{ax}$$

Here, x varies from

$$x = 0 \text{ to } x = 4a$$

and y varies from

$$y = \frac{x^2}{4a} \text{ to } y = 2\sqrt{ax}$$

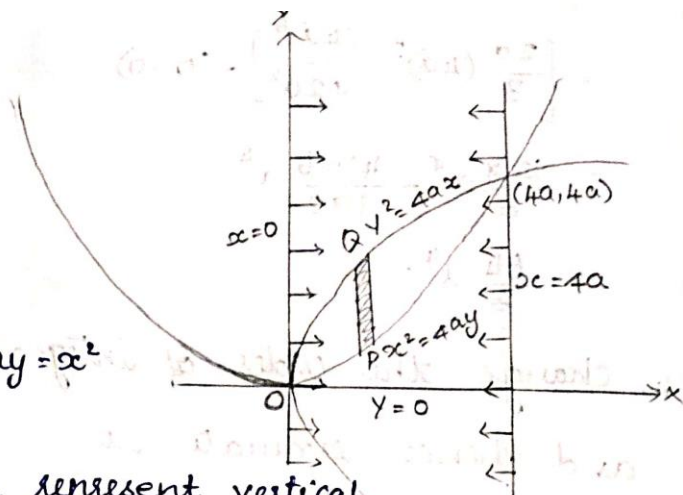
(i.e.) y varies from $ay = x^2$

$$\text{to } y^2 = 4ax$$

Here, $x = 0$ to $x = 4a$ represent vertical path and

$y = \frac{x^2}{4a}$ to $y = 2\sqrt{ax}$ represents vertical strip

PQ sliding area.





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Changing the order of integration is nothing but to change the vertical path into horizontal path and then to change the vertical strip PQ into horizontal strip RS.

Now, $y = 0$ to $y = 4a$ represents horizontal path.

and $x = \frac{y^2}{4a}$ to $x = 2\sqrt{ay}$ represents horizontal

strip RS sliding area.

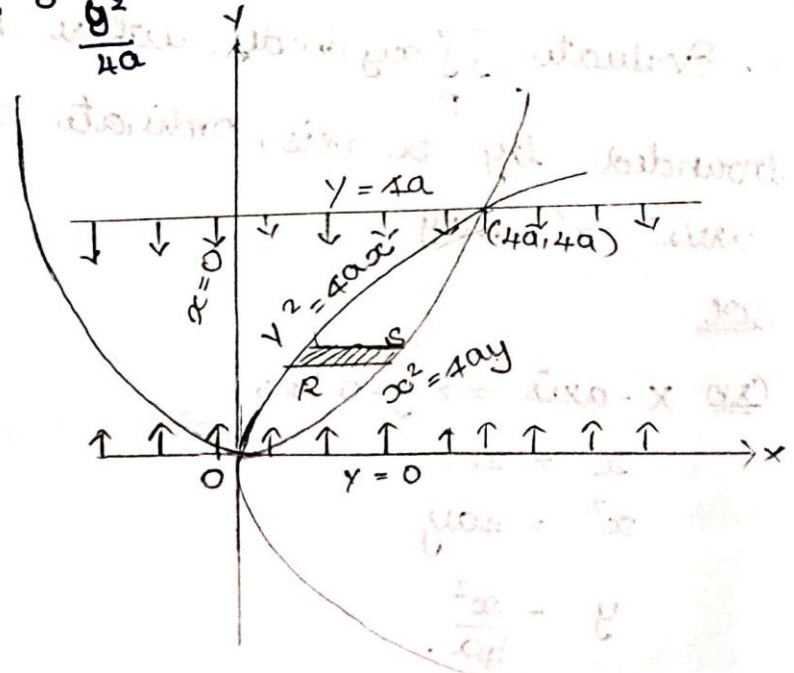
$$\text{Hence, } \int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \, dy \, dx = \int_0^{4a} \int_{\frac{y^2}{4a}}^{2\sqrt{ay}} xy \, dx \, dy.$$

$$= \int_0^{4a} \left[\frac{xc^2}{2} y \right]_{x=\frac{y^2}{4a}}^{x=2\sqrt{ay}}$$

$$= \int_0^{4a} \left[\frac{4ay}{2} y - \frac{y^4}{32a^2} y \right] dy$$

$$= \int_0^{4a} \left[2ay^2 - \frac{y^5}{32a^2} \right] dy$$

$$= \left[2a \frac{y^3}{3} - \frac{y^6}{192a^2} \right]_0^{4a}$$





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$$= \left[\frac{2a}{3} (4a)^3 - \frac{(4a)^6}{192a^2} \right] - (0-0)$$

$$= \frac{128}{3} a^4 - \frac{4096}{192} a^4$$

$$= \frac{64}{3} a^4.$$

4. change the order of integration $\int_0^{\infty} \int_0^{\infty} \frac{e^{-y}}{y} dy dx$
and hence evaluate it

Sol.

$$= \int_0^{\infty} \int_0^y \frac{e^{-y}}{y} dy dx$$

$$= \int_0^{\infty} \int_0^y \frac{e^{-y}}{y} dx dy$$

$$= \int_0^{\infty} \left[\frac{e^{-y}}{y} x \right]_{x=0}^{x=y} dy$$

$$= \int_0^{\infty} (e^{-y} - 0) dy = \left[\frac{e^{-y}}{-1} \right]_0^{\infty}$$

$$= -[e^{-y}]_0^{\infty} = -[0-1] = 1$$



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Now, we divide the region into

$$I = I_1 + I_2$$
$$= \int_0^1 \int_{x^2}^1 xy \, dy \, dx + \int_0^1 \int_0^{2-x} xy \, dy \, dx.$$

In I_1

x varies from $x=0$ to $x=1$

y varies from $y=x^2$ to $y=1$

Here, $x=0$ to $x=1$ represents vertical path and $y=x^2$ to $y=1$ represents vertical strip sliding area.

Changing the order of integration is nothing but to change the vertical path into Horizontal path and to change the vertical strip PQ into horizontal strip RS.



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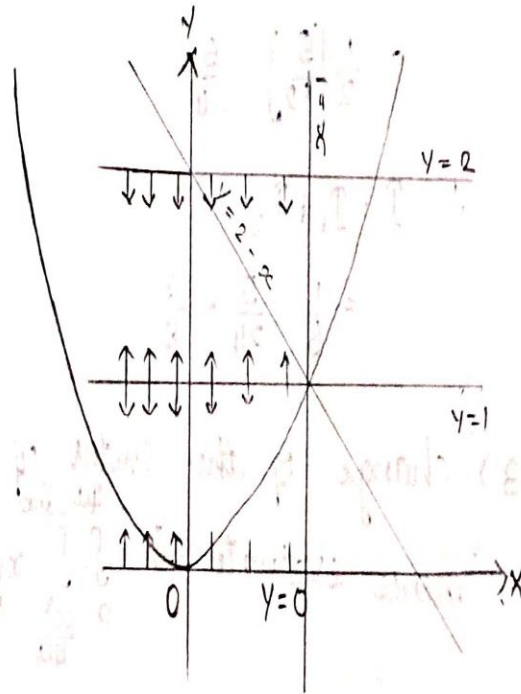
Hence, by changing the order,

we get $\int_0^1 \int_0^{\sqrt{y}} xy \, dx \, dy$.

$$= \int_0^1 \left[y \frac{x^2}{2} \right]_{x=0}^{x=\sqrt{y}} dy$$

$$= \int_0^1 \left[\frac{y^2}{2} - 0 \right] dy = \frac{1}{2} \int_0^1 y^2 dy$$

$$= \frac{1}{2} \left[\frac{y^3}{3} \right]_0^1 = \frac{1}{2} \left[\frac{1}{3} \right] = \frac{1}{6}$$



In T_2

x varies from $x=0$ to $x=1$ and Y axis

$y=1$ to $y=2-x$



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changing the order of integration is nothing but to change the vertical path into horizontal path and to change the vertical strip PQ into horizontal strip RS.

Hence, by changing the order of integration, we get

$$I_2 = \int_1^2 \int_0^{2-y} xy \, dx \, dy = \int_1^2 \left[\frac{x^2}{2} y \right]_{x=0}^{x=2-y} dy$$

$$= \int_1^2 \left[\frac{y(2-y)^2}{2} - 0 \right] dy = \frac{1}{2} \int_1^2 (2-y)^2 y \, dy$$

$$= \frac{1}{2} \int_1^2 (4 + y^2 - 4y) y \, dy = \frac{1}{2} \int_1^2 (4y + y^3 - 4y^2) dy$$

$$= \frac{1}{2} \left[4 \frac{y^2}{2} + \frac{y^4}{4} - 4 \frac{y^3}{3} \right]_1^2$$

$$= \frac{1}{2} \left[2y^2 + \frac{y^4}{4} - \frac{4}{3} y^3 \right]_1^2$$

$$= \frac{1}{2} \left[\left(8 + \frac{16}{4} - \frac{32}{3} \right) - \left(2 + \frac{1}{4} - \frac{4}{3} \right) \right]$$

$$= \frac{1}{2} \left[\frac{5}{12} \right] = \frac{5}{24}$$

$$\therefore I = I_1 + I_2$$

$$= \frac{1}{6} + \frac{5}{24} = \frac{3}{8}$$