



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

3. Find the volume of that portion of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant using triple integration.

Sol

$$\text{Gn: } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{--- (1)}$$

$$\text{Vol} = \iiint dz \, dy \, dx.$$

To find x limit put $y=0$ and $z=0$ we get,

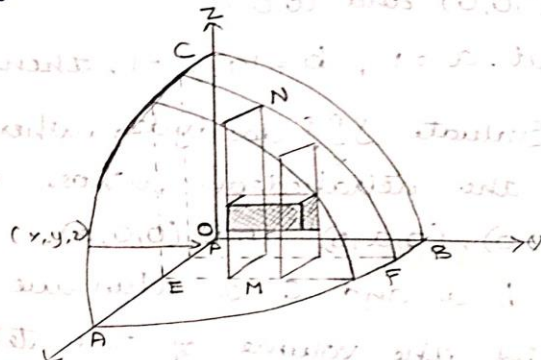
$$\text{(1)} \Rightarrow \frac{x^2}{a^2} = 1$$

$$\therefore x^2 = a^2$$

$$\therefore x = \pm a$$

(i.e.) $x=0$ to $x=a$.

[\because First octant area]



To find y limit put $z=0$ we get [surface integral]

$$\text{(1)} \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \Rightarrow y^2 = b^2 \left(\frac{1-x^2}{a^2} \right)$$

$$y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$$

(i.e.) $y=0$ to $y=b \sqrt{1 - \frac{x^2}{a^2}}$ [\because first octant area]

To find z limit [Volume integral]

$$\text{(1)} \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 ;$$

$$\frac{z^2}{c^2} = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} ; \quad z^2 = c^2 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

$$z = \pm c \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$

(i.e.,) $z = 0$ to $z = c \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$ [\because first octant area]

$$\text{Volume} = \int_0^a \int_0^{b\sqrt{1-x^2/a^2}} \int_0^{c\sqrt{1-x^2/a^2-y^2/b^2}} dz dy dx = \int_0^a \int_0^{b\sqrt{1-x^2/a^2}} [z]_0^{c\sqrt{1-x^2/a^2-y^2/b^2}}$$

$$= \int_0^a \int_0^{b\sqrt{1-x^2/a^2}} c \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} dy dx$$

$$= c \int_0^a \int_0^{b\sqrt{1-x^2/a^2}} \sqrt{\frac{b^2(1-x^2/a^2)-y^2}{b^2}} dy dx$$

Formula: $\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$

$$= \frac{c}{b} \int_0^a \int_0^{b\sqrt{1-x^2/a^2}} \sqrt{(\sqrt{b^2(1-x^2/a^2)})^2 - y^2} dy dx$$

$$= \frac{c}{b} \int_0^a \left[\frac{y \sqrt{b^2(1-x^2/a^2)-y^2}}{2} + \frac{b^2(1-x^2/a^2)}{2} \sin^{-1} \left[\frac{y}{b\sqrt{1-x^2/a^2}} \right] \right]_0^{b\sqrt{1-x^2/a^2}} dx$$

$$= \frac{c}{b} \int_0^a \left(0 + \frac{b^2(1-x^2/a^2)}{2} \right) \left(\frac{\pi}{2} \right) dx$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chen

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

$$= \frac{c}{b} \int_0^a \left(0 + \frac{b^2(1-x^2/a^2)}{2} \right) \left(\frac{\pi}{2} \right) dx.$$

$$= \frac{\pi bc}{4} \int_0^a \left(1 - \frac{x^2}{a^2} \right) dx = \frac{\pi bc}{4} \left[x - \frac{1}{a^2} \frac{x^3}{3} \right]_0^a$$

$$= \frac{\pi bc}{4} \left(a - \frac{a}{3} \right) = \frac{\pi abc}{4} \left(1 - \frac{1}{3} \right)$$

$$= \frac{\pi abc}{4} \left(\frac{2}{3} \right)$$

$$= \frac{\pi abc}{6} \text{ cubic units}$$

Note: The volume of the whole ellipsoid

$$= 8 \left(\frac{\pi abc}{6} \right) = \frac{4}{3} \pi abc$$