Control Engineering I

Signal Flow Graphs

Introduction

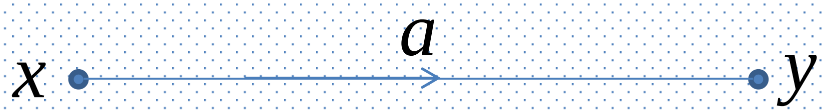
- Alternative method to block diagram representation, developed by Samuel Jefferson Mason.
- Advantage: the availability of a flow graph gain formula, also called Mason's gain formula.
- A signal-flow graph consists of a network in which nodes are connected by directed branches.
- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

- The signal flow graph of the equation is shown below;

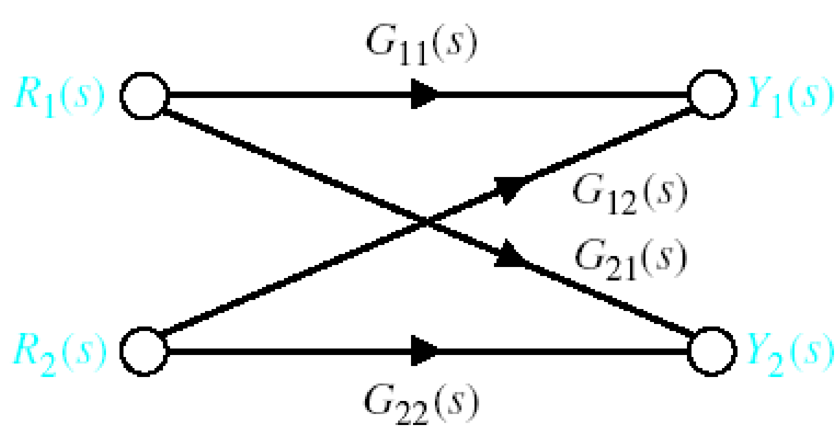


- Every variable in a signal flow graph is designed by a **Node**.
- Every transmission function in a signal flow graph is designed by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal flow.

Signal-Flow Graph Models

$$Y_1(s) = G_{11}(s)\cdot R_1(s) + G_{12}(s)\cdot R_2(s)$$

$$Y_2(s) = G_{21}(s)\cdot R_1(s) + G_{22}(s)\cdot R_2(s)$$

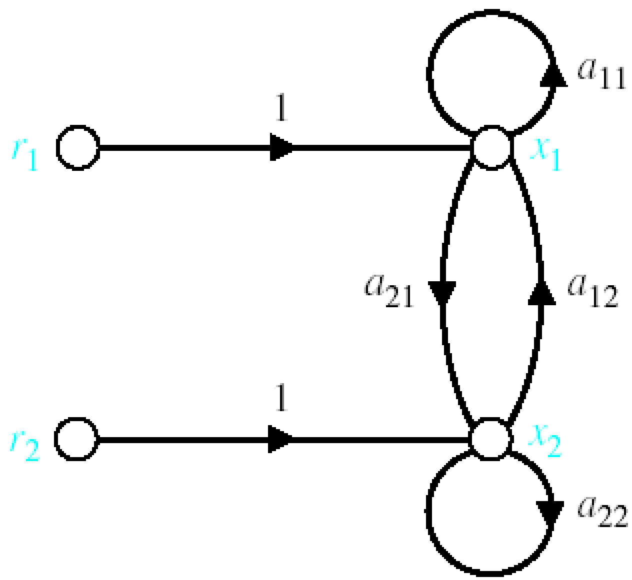


Signal-Flow Graph Models

r_1 and r_2 are inputs and x_1 and x_2 are outputs

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + r_1 = x_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + r_2 = x_2$$



Signal-Flow Graph Models

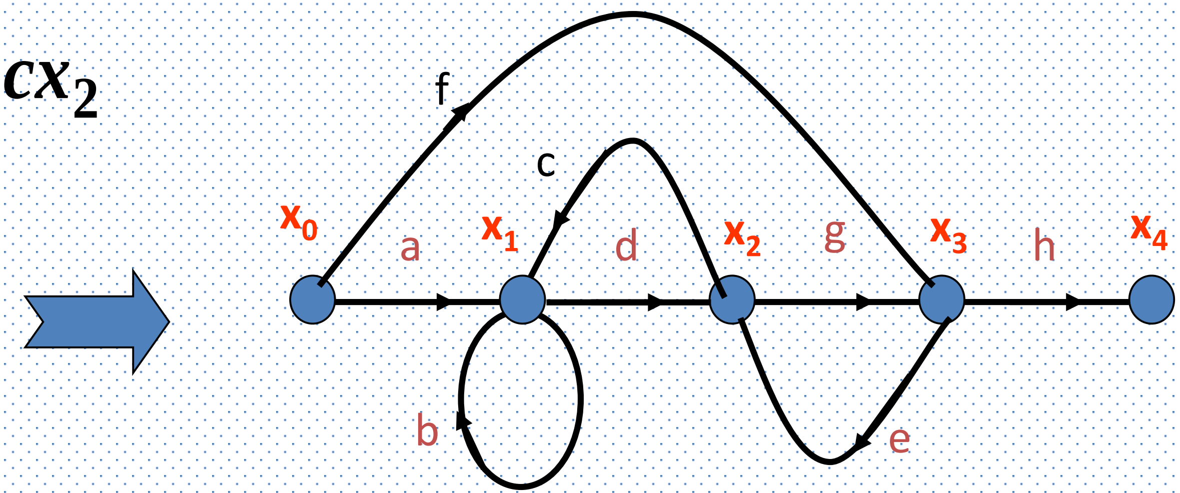
x_o is input and x_4 is output

$$x_1 = ax_0 + bx_1 + cx_2$$

$$x_2 = dx_1 + ex_3$$

$$x_3 = fx_0 + gx_2$$

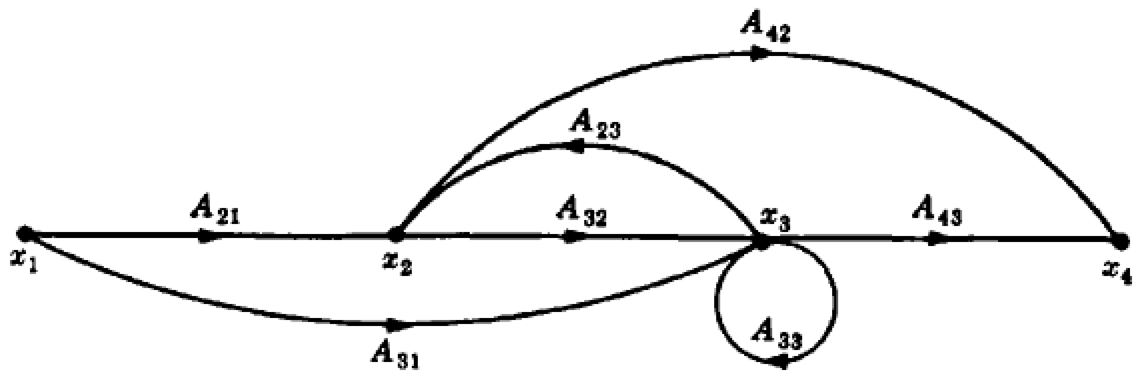
$$x_4 = hx_3$$



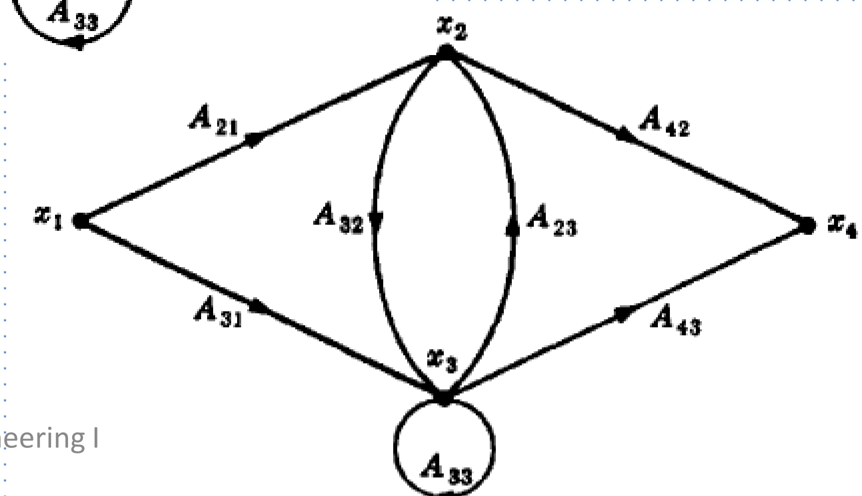
Construct the signal flow graph for the following set of simultaneous equations.

$$x_2 = A_{21}x_1 + A_{23}x_3 \qquad x_3 = A_{31}x_1 + A_{32}x_2 + A_{33}x_3 \qquad x_4 = A_{42}x_2 + A_{43}x_3$$

- There are four variables in the equations (i.e., x_1,x_2,x_3,and x_4) therefore four nodes are required to construct the signal flow graph.
- Arrange these four nodes from left to right and connect them with the associated branches.



- Another way to arrange this graph is shown in the figure.

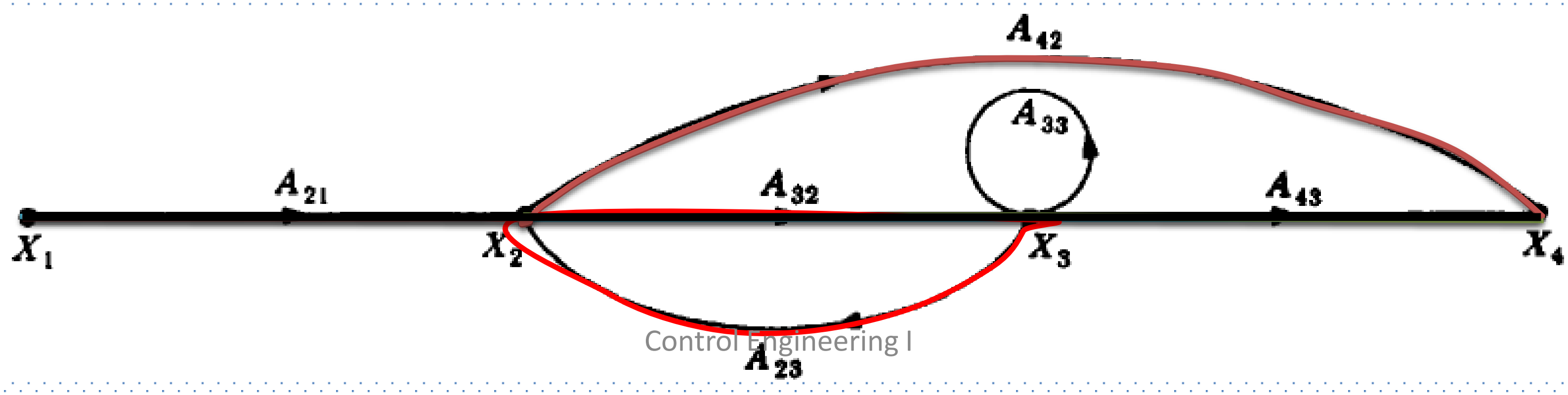


Terminologies

- An **input node** or source contain only the outgoing branches. i.e., X_1
- An **output node** or sink contain only the incoming branches. i.e., X_4
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e.,

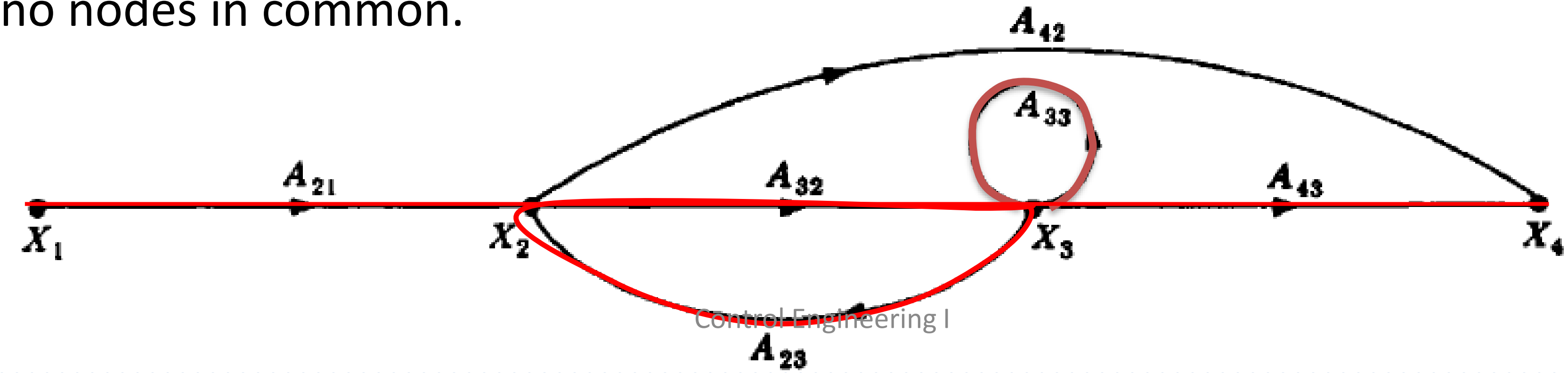
  X_1 *to* X_2 *to* X_3 *to* X_4 X_1 *to* X_2 *to* X_4 X_2 *to* X_3 *to* X_4
- A **forward path** is a path from the input node to the output node. i.e.,

  X_1 *to* X_2 *to* X_3 *to* X_4, and X_1 *to* X_2 *to* X_4, are forward paths.
- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; X_2 *to* X_3 **and back to** X_2 is a feedback path.

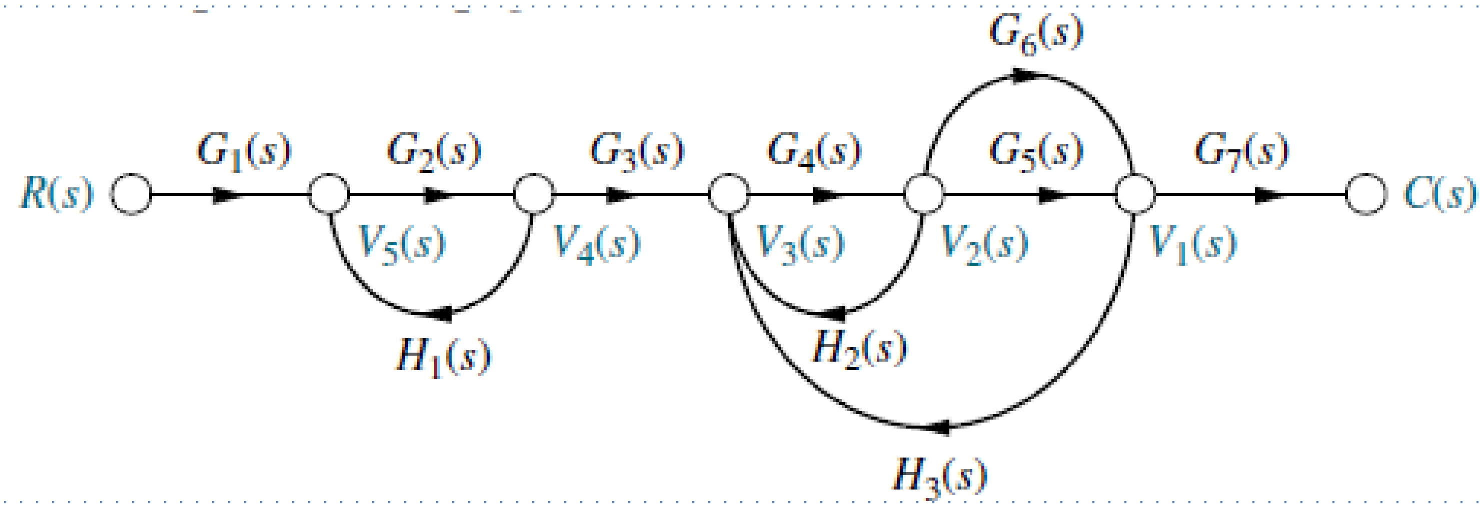


Terminologies

- A **self-loop** is a feedback loop consisting of a single branch. i.e.; A_{33} is a self loop.
- The **gain** of a branch is the transmission function of that branch.
- The **path gain** is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X_1 *to* X_2 *to* X_3 *to* X_4 is $A_{21}A_{32}A_{43}$
- The **loop gain** is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.

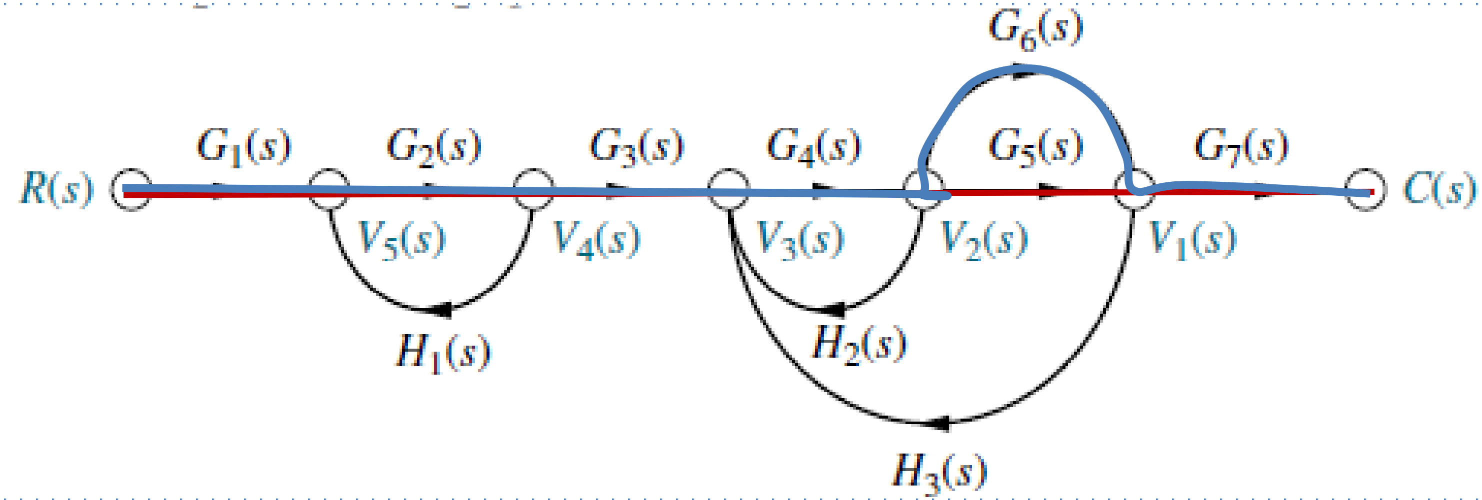


Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths (loops).
e) Determine the loop gains of the feedback loops.
f) Determine the path gains of the forward paths.
g) Non-touching loops

Consider the signal flow graph below and identify the following



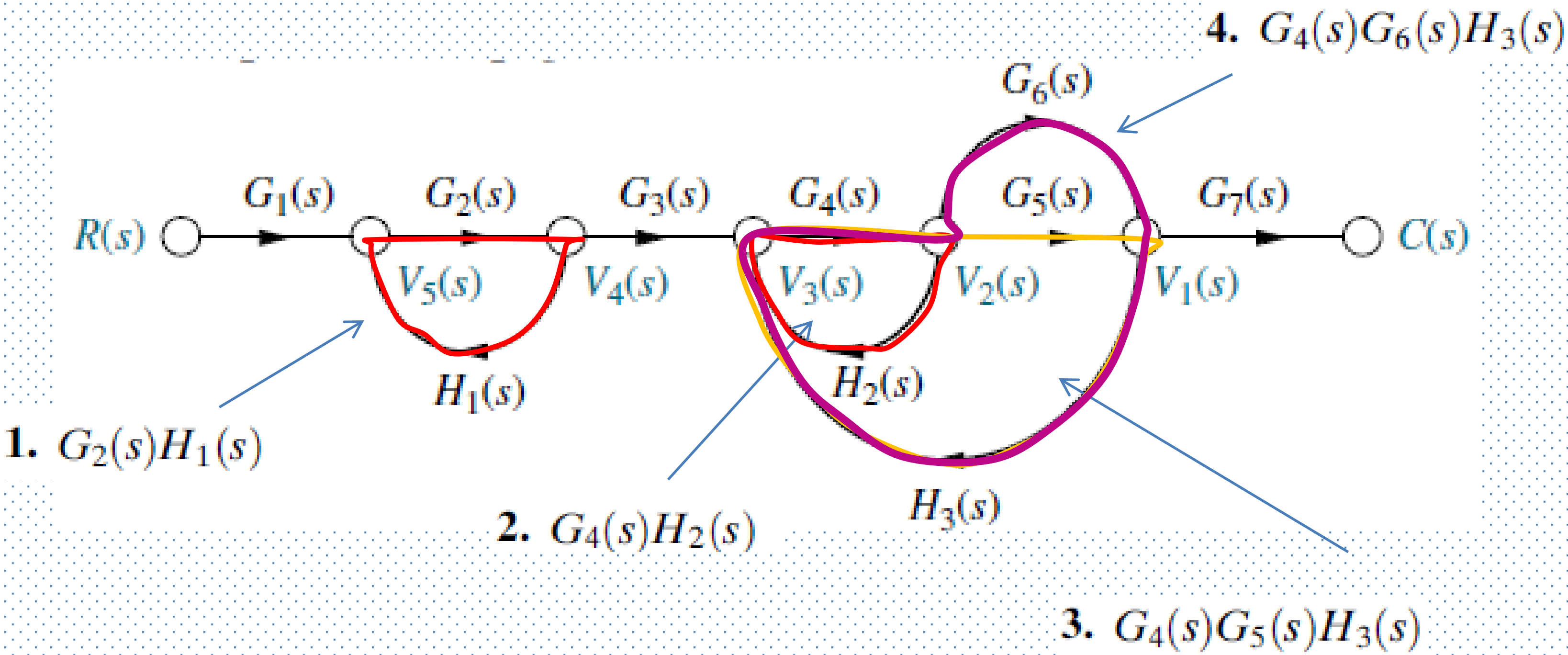
- There are two forward path gains;

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$

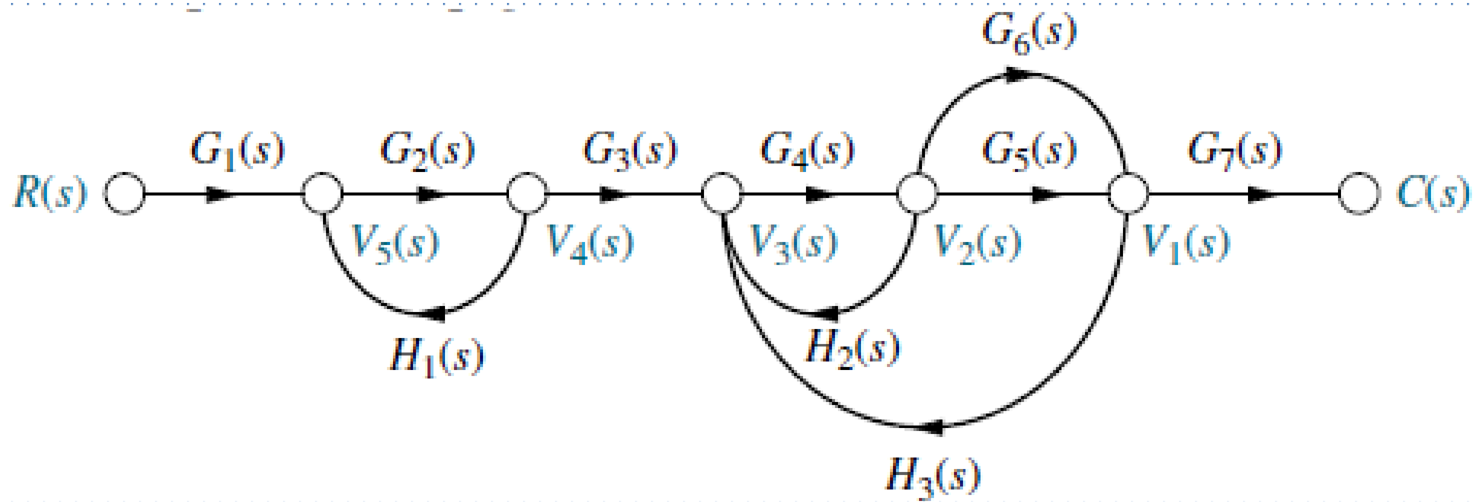
2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

Consider the signal flow graph below and identify the following

- There are four loops



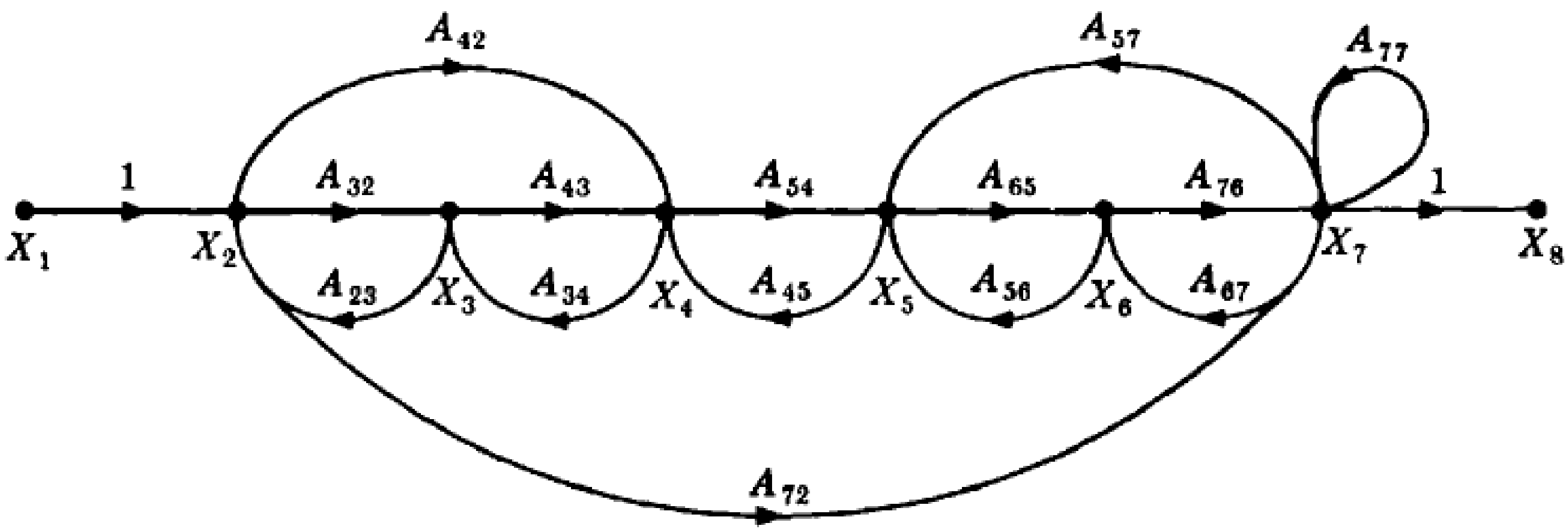
Consider the signal flow graph below and identify the following



- Nontouching loop gains;

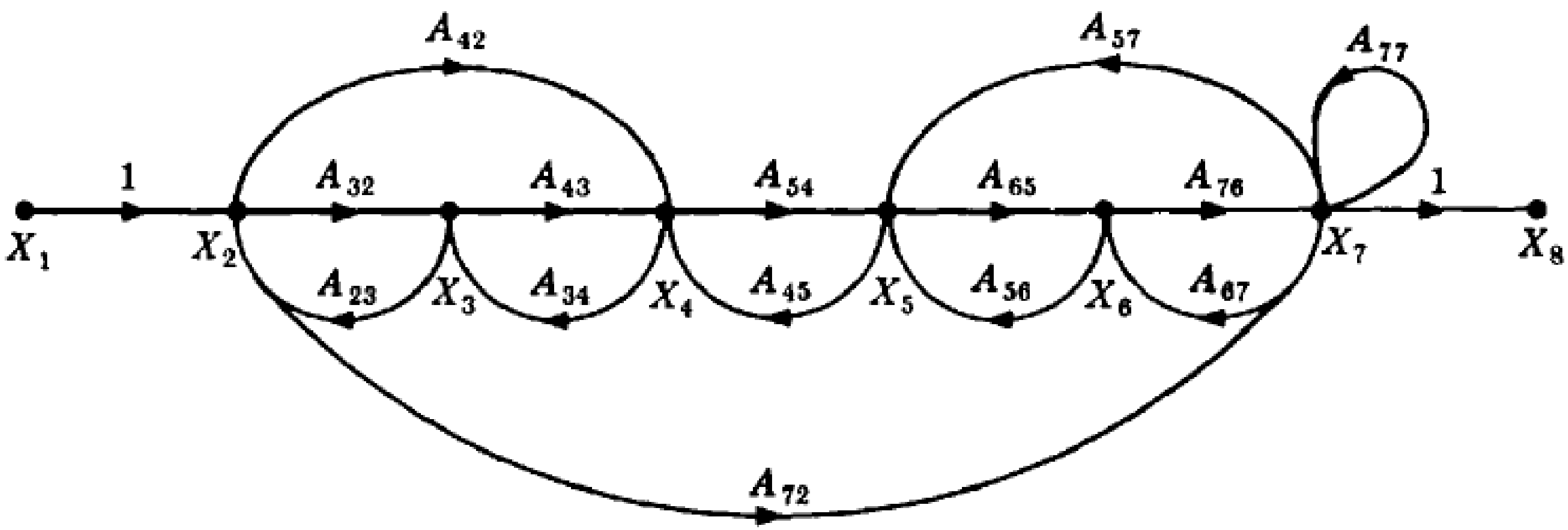
1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths.
e) Self loop.
f) Determine the loop gains of the feedback loops.
g) Determine the path gains of the forward paths.

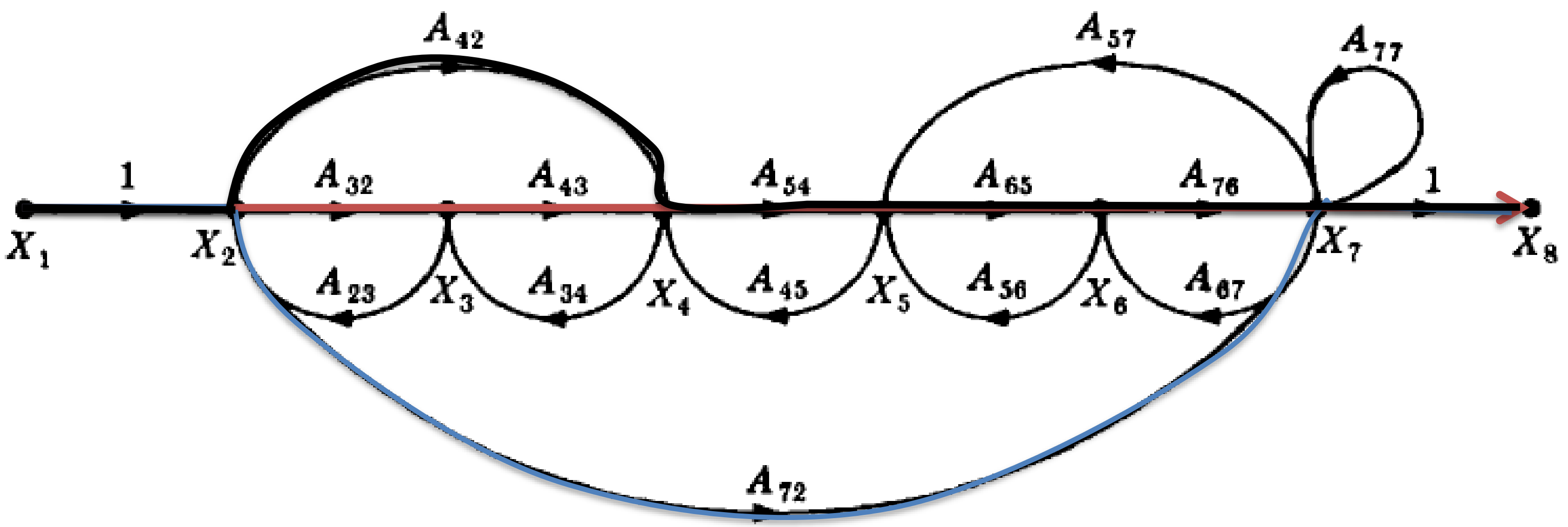
Input and output Nodes



a) Input node X_1

b) Output node X_8

(c) Forward Paths

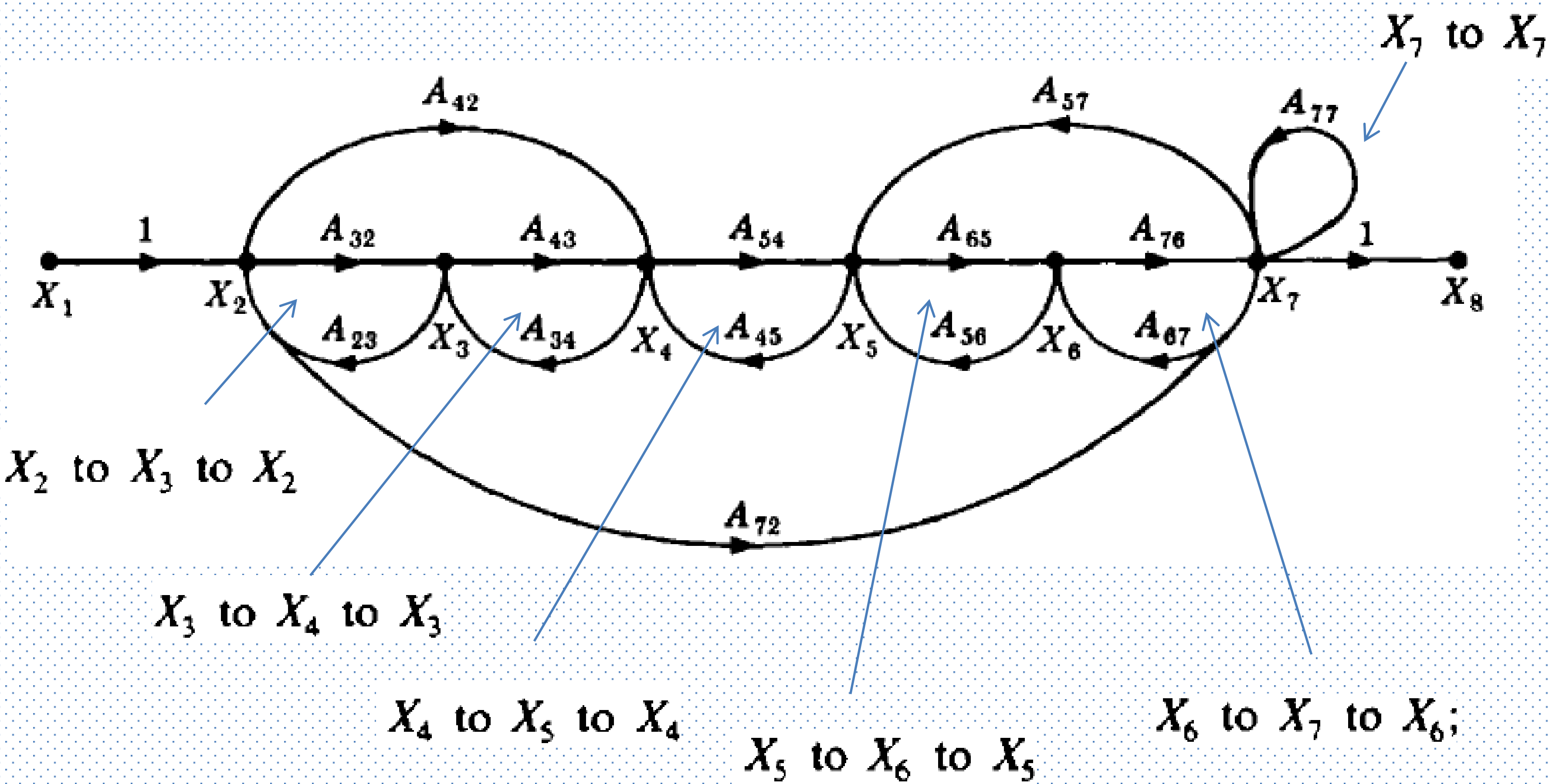


X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8

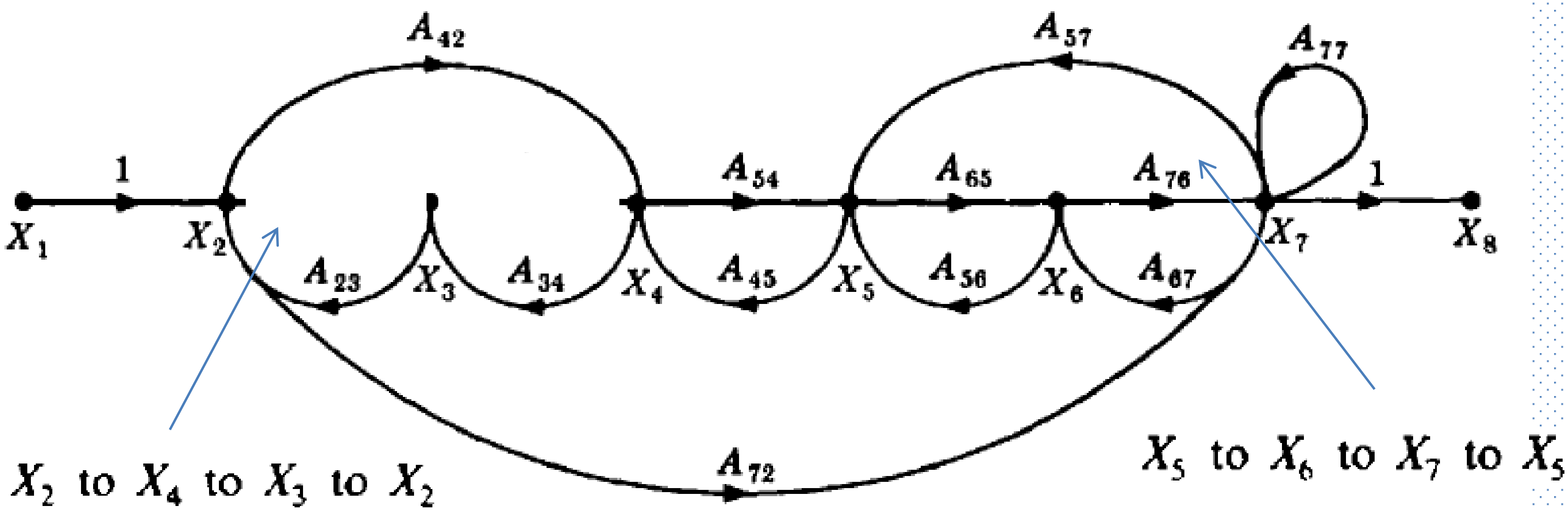
X_1 to X_2 to X_7 to X_8

X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

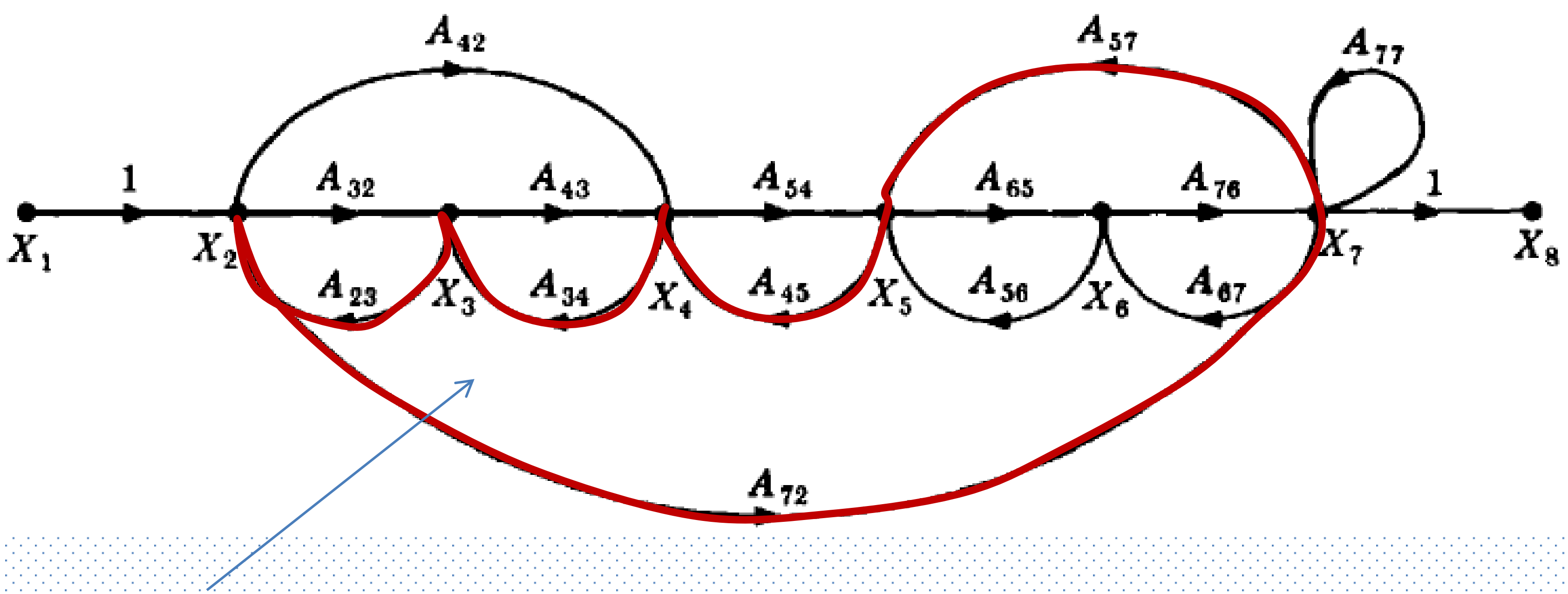
(d) Feedback Paths or Loops



(d) Feedback Paths or Loops

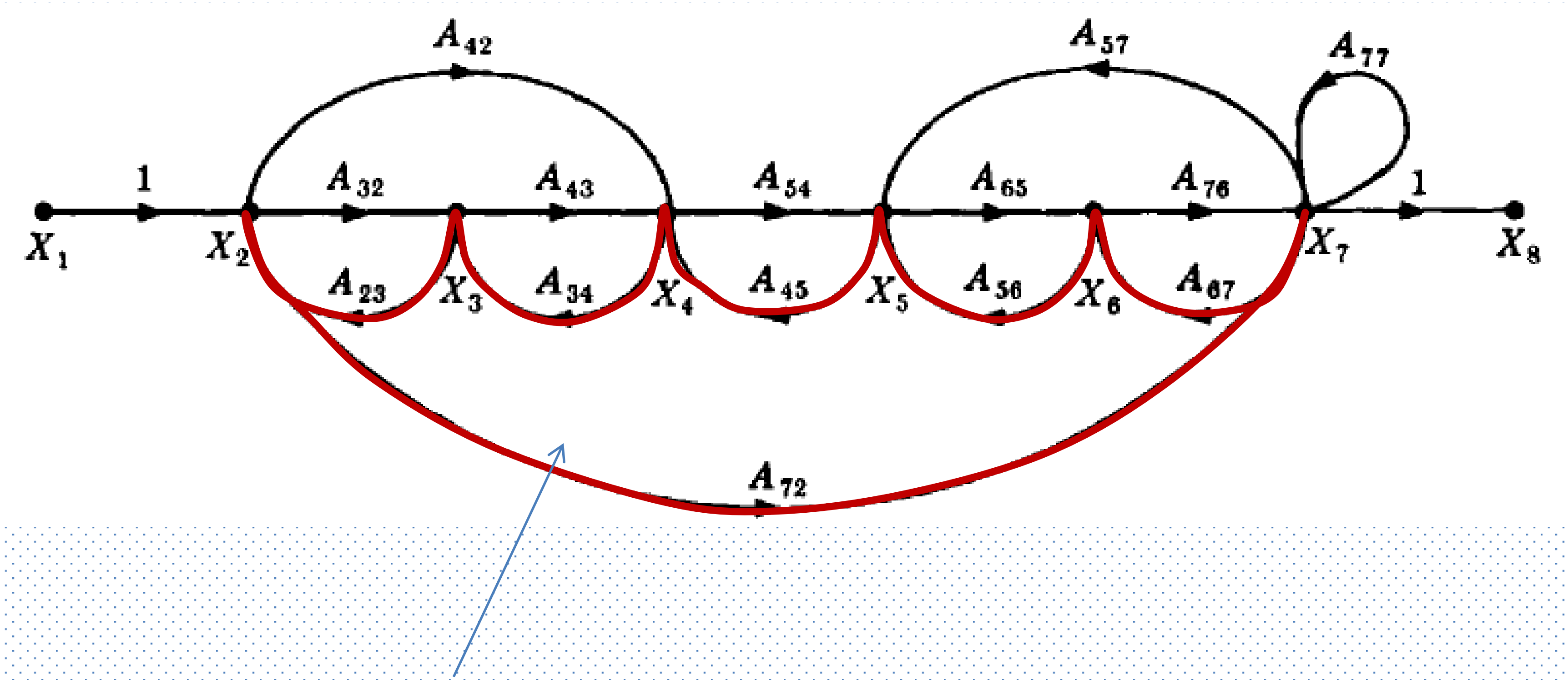


(d) Feedback Paths or Loops



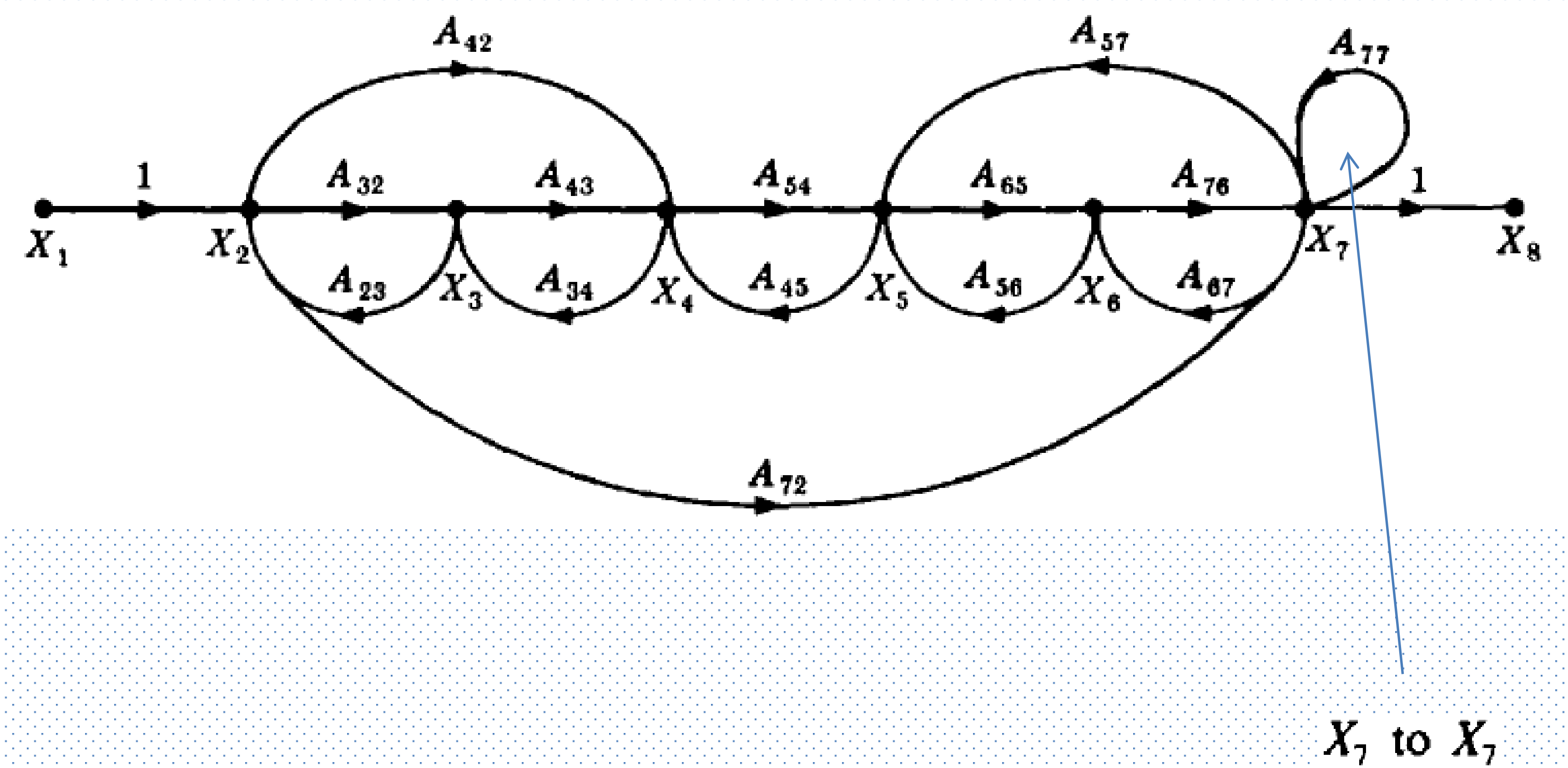
X_2 to X_7 to X_5 to X_4 to X_3 to X_2

(d) Feedback Paths or Loops



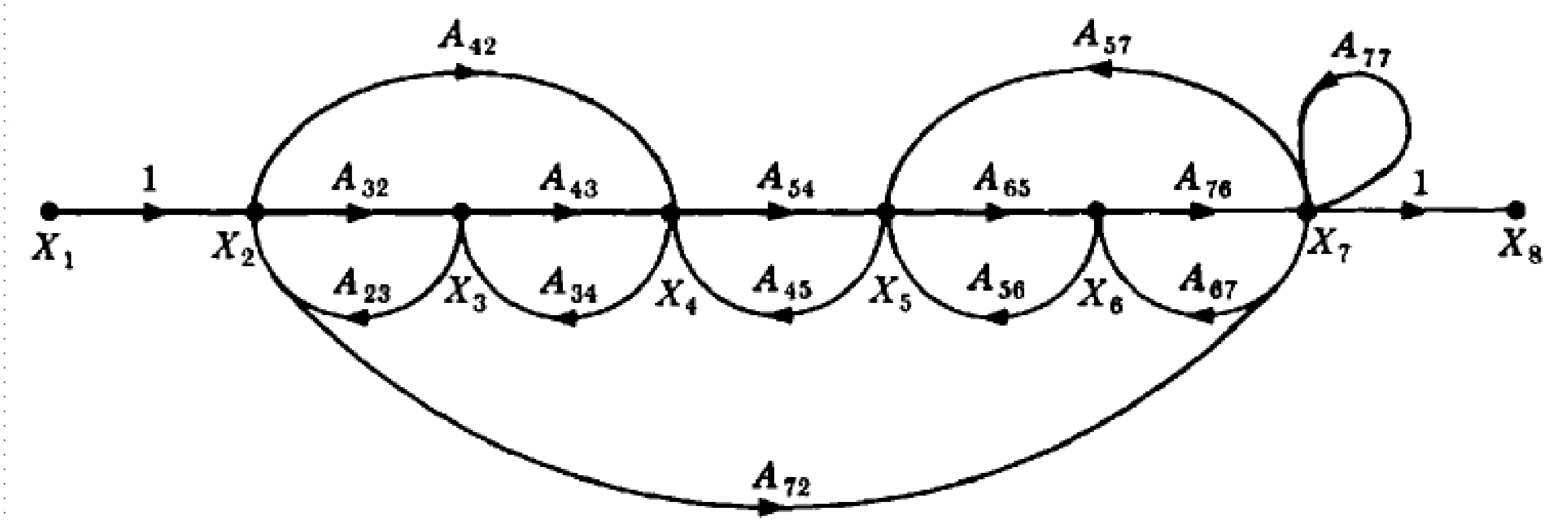
X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X_2

(e) Self Loop(s)



A_{42}
A_{57}
A_{77}
1
A_{32}
A_{43}
A_{54}
A_{65}
A_{76}
1
X_1
X_2
X_7
X_8
A_{23}
X_3
A_{34}
X_4
A_{45}
X_5
A_{56}
X_6
A_{67}
A_{72}
X_7 to X_7

(f) Loop Gains of the Feedback Loops



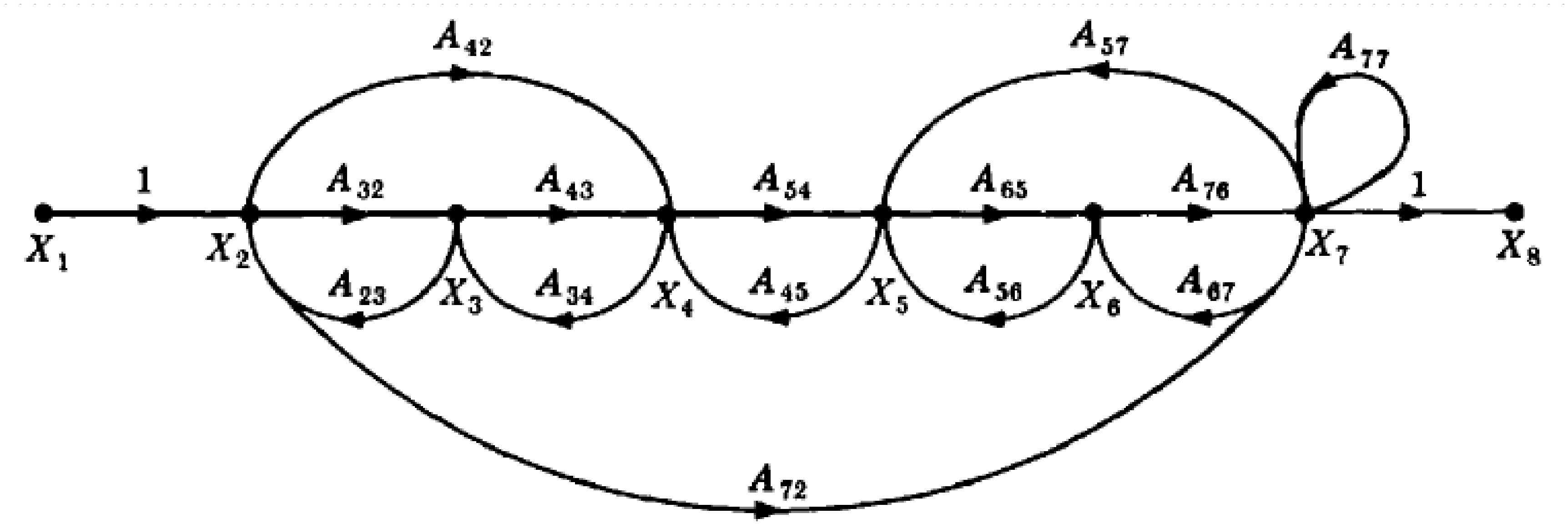
$A_{32}A_{23}$ $\quad$ $A_{76}A_{67}$; $\quad$ $A_{72}A_{57}A_{45}A_{34}A_{23}$;

$A_{43}A_{34}$ $\quad$ $A_{65}A_{76}A_{57}$ $\quad$ $A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$

$A_{54}A_{45}$ $\quad$ A_{77}

$A_{65}A_{56}$ $\quad$ $A_{42}A_{34}A_{23}$

(g) Path Gains of the Forward Paths



$A_{32} A_{43} A_{54} A_{65} A_{76}$

A_{72}

$A_{42} A_{54} A_{65} A_{76}$