∇ in ~~cartesian~~ cylindrical coordinates,

$$\nabla = \hat{a}_\rho \frac{\partial}{\partial \rho} + \frac{1}{\rho}\hat{a}_\phi \frac{\partial}{\partial \phi} + \hat{a}_z \frac{\partial}{\partial z}$$

∇ in spherical coordinates,

$$\nabla = \hat{a}_r \frac{\partial}{\partial r} + \hat{a}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \hat{a}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

Gradient of a scalar

Gradient of a scalar field is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.

$$\text{grad } V = \frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}$$
(or ∇V)

In cylindrical coordinates,

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a_\phi} + \frac{\partial V}{\partial z}\hat{a_z}$$

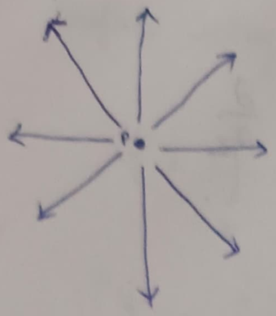
In spherical coordinates

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi$$

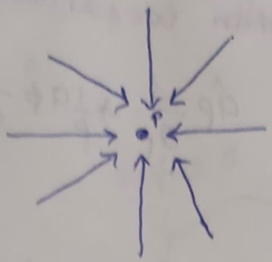
Divergence of a vector.

The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P.

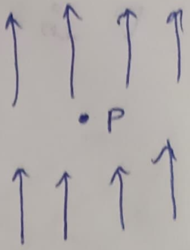
$$\text{div } A = \nabla\cdot\vec{A} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{A}\cdot d\vec{s}}{\Delta v}$$



Positive divergence



Negative divergence



zero divergence

physically, divergence of a vector field is a measure of how much the field diverges or emanates from that point.

or simply it is the limit of the field's source strength per unit volume (or source density).

In cartessian system $\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

In cylindrical $\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

In spherical $\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$

Properties:

* Divergence of a vector field is a scalar.
* $\nabla\cdot(A+B) = \nabla\cdot A + \nabla\cdot B$
* $\nabla\cdot(VA) = V\nabla\cdot A + A\cdot\nabla V$

Divergence theorem:

The divergence theorem states that the total outward flux of a vector field A through the closed surface S is the same as the volume integral of the divergence of A.

$$\oint_S \vec{A}\cdot d\vec{s} = \int_V \vec{\nabla}\cdot\vec{A}\ dv$$

Proof:

WKT $\nabla\cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

Take volume integral on both the sides

$$\iiint_V \nabla\cdot A\ dv = \iiint_V \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) dx\,dy\,dz$$

Consider element volume in x direction

$$\iiint_V \frac{\partial A_x}{\partial x}\ dx\,dy\,dz = \iint \left\{\int \frac{\partial A_x}{\partial x}\ dx\right\} dy\,dz$$

$$= \iint A_x\ dy\,dz$$

$$= \iint A_x\ ds_x$$

$$\iiint_V \frac{\partial A_y}{\partial y}\ dx\,dy\,dz = \iint_S A_y\ ds_y$$

$$\iiint_V \frac{\partial A_z}{\partial z}\ dx\,dy\,dz = \iint_S A_z\ ds_z$$

$$\therefore \iiint \nabla \cdot A \, dv = \iint_S A_x \, ds_x + \iint A_y \, ds_y + \iint_S A_z \, ds_z$$

$$= \oiint_S A \cdot ds$$

$$\therefore \iiint \nabla \cdot A \, dv = \iint A \cdot ds$$

or $$\int_V \vec{\nabla} \cdot \vec{A} \, dv = \oint_S \vec{A} \cdot \vec{ds}$$