Continuity equation for current

Continuity equation of the current is based on the principle of conservation of charge.

It states that the charges can neither be created nor be destroyed.

Consider a closed surface S with a current density $\bar{J}$, then the total current I passing the surface S

$$I = \oint_S \bar{J} \cdot d\bar{S}$$

current I constituted due to outward flow of positive charges from the closed surface S.

According to principle of conservation, there must be decrease of an equal amount of positive charge inside the closed surface.

$$\therefore I = \oint J \cdot ds = -\frac{dQ}{dt}$$

$$\boxed{\oint_S J \cdot ds = -\frac{dQ}{dt}}$$ is the integral form of continuity equation.

using divergence theorem

$$\oint J \cdot ds = \int_V (\nabla \cdot \bar{J})\, dv$$

$$\therefore -\frac{dQ}{dt} = \int_V (\nabla \cdot J)\, dv$$

But $Q = \int_V \rho_v \, dv$.

$$\therefore \int_V (\nabla \cdot J)\,dv = -\frac{d}{dt}\int_V \rho_v\,dv$$

$$\int_V (\nabla \cdot J)\,dv = -\int_V \frac{\partial \rho_v}{\partial t}\,dv$$

$$\therefore \boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

For steady current

$\boxed{\bar{\nabla} \cdot \bar{J} = 0}$

Since not function of time $\partial \rho_v / \partial t = 0$

Rate of flow remains constant.

$\Sigma I = 0$ at a point $\Rightarrow$ Field equivalent of KCL

This is the point form or differential form of the continuity equation of the current.

The equation states that the current or the charge per second, diverging from a small volume is equal to the time rate of decrease of charge at every point.

Properties of Conductor

1. Under static conditions, no charge and no electric field can exist at any point within the conducting material.

2. The charge can exist on the surface of the conductor giving rise to surface charge density.

3. Within a conductor, the charge density is always zero.

4. The charge distribution on the surface depends on the shape of the surface.

5. The conductivity of an ideal conductor is infinite.

6. The conductor surface is an equipotential surface