Unit I - Static Electric Field

The static electric field could be termed as Electrostatic.

An electrostatic field is produced by a static charge distribution.

A typical example of such a field is found in a cathode-ray tube.

Applications of electrostatics:

- Electric power transmission
- X-ray machines
- lightning protection.

Vector Algebra :- Objective : To know the mathematical tool that helps to realize the Electromagnetic concepts

- Vector analysis is mathematical tool with which electromagnetic concepts are most conveniently expressed.

- A quantity can be either a scalar or a vector.

A scalar is a quantity that has only magnitude

Eg: Time, mass, distance, temperature, electric potential.

- A vector is a quantity that has both magnitude and direction.

Eg: Velocity, force, displacement electric field intensity.

- A field is a function that specifies a particular quantity everywhere in the region.

Example for scalar fields ⇒ electric potential in a region.

Vector field ⇒ Gravitational force on a body in Space & velocity of raindrops in the atmosphere.

Unit Vector :-

A unit vector along $\vec{A}$ is defined as a vector whose magnitude is unity, and its direction is along A.

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$

or $\vec{A} = |A| \, \hat{a}$

A vector in Cartesian (rectangular) coordinates may be represented as

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z \quad \text{or} \quad (A_x, A_y, A_z)$$

A_x, A_y, A_z are called components of $\vec{A}$ in the x, y, z coordinates. a_x, a_y, a_z are unit vectors.

Magnitude of the vector A is given by

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

and the unit vector along A is given by

$$\hat{a} = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{|A|}$$

Vector addition and subtraction

- vector addition is :

$$\vec{C} = \vec{A} + \vec{B}$$

If $\vec{A} = (A_x, A_y, A_z)$ and $B = (B_x, B_y, B_z)$

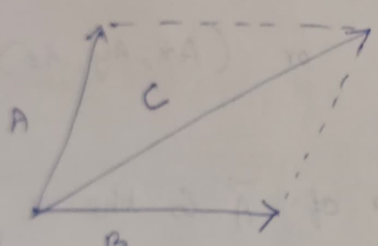
$$\vec{C} = (A_x + B_x)\,\hat{a}_x + (A_y + B_y)\,\hat{b}_y + (A_z + B_z)\,\hat{a}_z$$

- Vector subtraction is

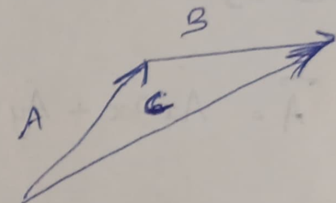
$$\vec{D} = \vec{A} - \vec{B}$$

$$\vec{D} = (A_x - B_x)\,\hat{a}_x + (A_y - B_y)\,\hat{a}_y + (A_z - B_z)\,\hat{a}_z$$

Vector addition. $\vec{C} = \vec{A} + \vec{B}$

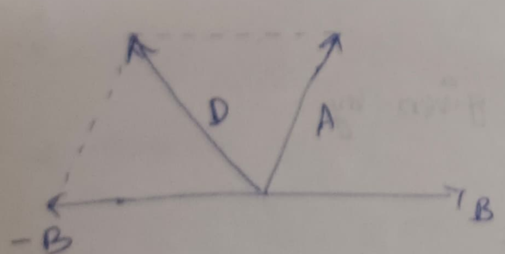


a) Parallelogram rule

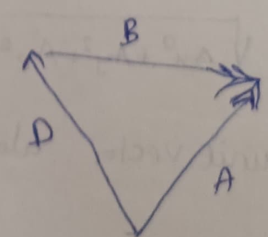


b) head-to-tail rule

vector subtraction : $\vec{D} = \vec{A} - \vec{B}$



a) Parallelogram ~~system~~



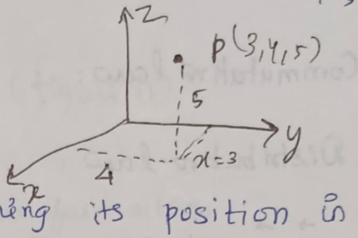
b) head-to-tail rule

Position and Distance vectors

A point P in cartesian coordinates may be represented by (x, y, z).

- A position vector r_p (or radius vector) of point P is defined as the directed distance from the origin O to P.

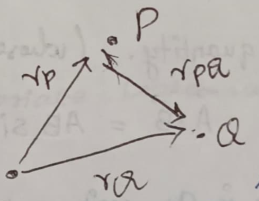
$$\vec{r}_p = \vec{OP} = x\,\hat{a_x} + y\,\hat{a_y} + z\,\hat{a_z}$$



- A position vector is useful in defining its position in space.

- A distance vector is the displacement from one point to another.

If two points P and Q are given by (x_p, y_p, z_p) (x_1, y_1, z_1) and (x_Q, y_Q, z_Q) (x_2, y_2, z_2) the distance vector (or separation vector) is the displacement from P to Q.



$$\vec{r}_{PQ} = r_Q - r_P$$

$$= (x_Q - x_P)\,\hat{a_x} + (y_Q - y_P)\,\hat{a_y} + (z_Q - z_P)\,\hat{a_z}$$

$$= (x_2 - x_1)\,\hat{a_x} + (y_2 - y_1)\,\hat{a_y} + (z_2 - z_1)\,\hat{a_z}$$

Vector Multiplication:

1. Scalar (or) Dot Product $A \cdot B$
2. Vector (or) Cross Product $A \times B$
3. Scalar triple product $A \cdot (B \times C)$
4. Vector triple Product $A \times (B \times C)$

Scalar Product

Dot Product: The dot product of two vectors A & B,

$$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

A and B are said to be orthogonal (or) perpendicular with each other if $\vec{A}\cdot\vec{B} = 0$.

- Commutative law: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- Distributive law: $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- $\vec{A}\cdot\vec{A} = |A|^2 = A^2$

$$a_x\cdot a_y = a_y\cdot a_z = a_z\cdot a_x = 0$$

$$a_x\cdot a_x = a_y\cdot a_y = a_z\cdot a_z = 1$$

Cross Product: Vector Product

Cross product of two vectors A and B, $A\times B$ is a vector quantity. (whose magnitude in the area of the Parallelogram) & direction of advance of a right handed screw)

$$\vec{A}\times\vec{B} = AB\sin\theta_{AB}\cdot a_n$$

figure(i)

a_n is a unit vector normal to the plane containing A and B.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- It is not commutative: $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$
- It is anticommutative: $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- It is not associative: ~~A x~~

$$\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$$

- It is distributive:

$$A \times (B + C) = A \times B + A \times C$$

- $A \times A = 0$

$a_x \times a_y = a_z$ $\quad$ $a_y \times a_x = -a_z$ $\quad$ $a_x \times a_x = 0$

$a_y \times a_z = a_x$ $\quad$ $a_z \times a_y = -a_x$

$a_z \times a_x = a_y$ $\quad$ $a_x \times a_z = -a_y$ (figure ii)

Coordinate Systems and Transformation

Objectives: Importance of coordinate system for the analysis of Electromagnetic fields.

Why coordinate systems?

Physical quantities dealing with Electro-magnetics are functions of space and time. In order to describe the spatial variations of the quantities, we must be able to define all the points uniquely in space.

This requires an appropriate coordinate system.

It may be orthogonal or nonorthogonal.

An orthogonal system is one in which the coordinates are mutually perpendicular.

Examples of Orthogonal C.S:

1. Cartesian (or rectangular)
2. Circular Cylindrical
3. Spherical
4. Elliptic Cylindrical.

Cylindrical etc.,