- It is distributive:

$$A \times (B+C) = A \times B + A \times C$$

- $A \times A = 0$

$a_x \times a_y = a_z$ $\quad a_y \times a_x = -a_z$ $\quad a_x \times a_x = 0$

$a_y \times a_z = a_x$ $\quad a_z \times a_y = -a_x$

$a_z \times a_x = a_y$ $\quad a_x \times a_z = -a_y$ (figure ii)

Coordinate Systems and Transformation

Objectives: Importance of coordinate system for the analysis of Electromagnetic fields.

Why coordinate systems?

Physical quantities dealing with Electro-magnetics are functions of space and time. In order to describe the spatial variations of the quantities, we must be able to define all the points uniquely in space.

This requires an appropriate coordinate system.

It may be orthogonal or nonorthogonal.

An orthogonal system is one in which the coordinate are mutually perpendicular

Examples of Orthogonal C.S:

1. Cartesian (or rectangular)
2. Circular Cylindrical
3. Spherical
4. Elliptic Cylindrical.

Cylindrical etc.,

Cartesian Coordinates (x, y, z)

A point P can be represented as (x, y, z).

The ranges of the variables are

$$-\infty < x < \infty$$

$$-\infty < y < \infty$$

$$-\infty < z < \infty$$

A vector in cartesian is written as

(A_x, A_y, A_z) or $\boxed{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}$

Circular Cylindrical Coordinate (ρ, ϕ, z)

A point P is represented as (ρ, ϕ, z). ρ is the radius of the cylinder passing through P or the radial distance from the z-axis.

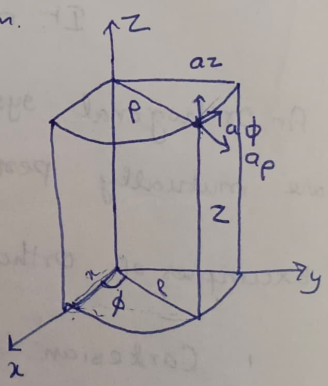
ϕ is the azimuthal angle, measured from the x axis in the xy plane.

z is same as in the cartesian system.

$$0 \leq \rho \leq \infty$$

$$0 \leq \phi \leq 2\pi$$

$$-\infty \leq z < \infty$$



A vector A in cylindrical system

(A_ρ, A_ϕ, A_z) or $\boxed{A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z}$

a_ρ, a_ϕ, a_z are unit vectors in the ρ, ϕ, z directions.

$a_\rho \cdot a_\rho = a_\phi \cdot a_\phi = a_z \cdot a_z = 1$

$a_\rho \cdot a_\phi = a_\phi \cdot a_z = a_z \cdot a_\rho = 0$

$a_\rho \times a_\phi = a_z$

$a_\phi \times a_z = a_\rho$

$a_z \times a_\rho = a_\phi$

Relationship between the variables (x, y, z) of the Cartesian & (ρ, ϕ, z) of Cylindrical.

$\rho = \sqrt{x^2+y^2}$ $\quad\quad$ $x = \rho\cos\phi$

$\phi = \tan^{-1}(y/x)$ $\quad\quad$ $y = \rho\sin\phi$

$z = z$ $\quad\quad$ $z = z$

Relationship between (A_x, A_y, A_z) and (A_ρ, A_ϕ, A_z)

$A_\rho = A \cdot a_\rho$

$A_\rho = A_x\cos\phi + A_y\sin\phi$

$A_\phi = A \cdot a_\phi$

$A_\phi = -A_x\sin\phi + A_y\cos\phi$

$A_z = A \cdot a_z$

$A_z = A_z$

$\cdot$	a_ρ	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$\sin\phi$	$\cos\phi$	0
a_z	0	0	1

In matrix form

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_\rho = A \cdot \hat{a}_\rho = (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) \cdot \hat{a}_\rho$$
$$= A_x \hat{a}_x \cdot \hat{a}_\rho + A_y \hat{a}_y \cdot \hat{a}_\rho + A_z \hat{a}_z \cdot \hat{a}_\rho$$
$$= A_x \cos\phi + A_y \sin\phi + 0$$

$(A_\rho, A_\phi, A_z) \longrightarrow (A_x, A_y, A_z)$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ +\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Spherical coordinates (r, θ, ϕ)

Point P is represented as (r, θ, ϕ)

r is the distance from the origin to Point P or the radius of a sphere.

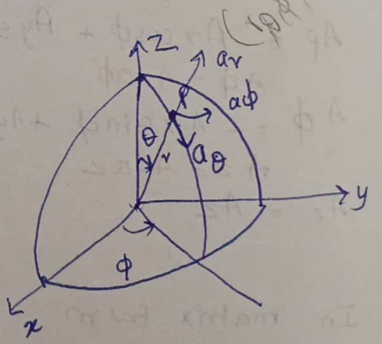
θ (called as colatitude) is the angle between Z axis and position vector of P.

ϕ is measured from the x axis (same azimuthal angle in cylindrical coordinates)

$0 \le r \le \infty$

$0 \le \theta \le \pi$

$0 \le \phi < 2\pi$



A vector A in spherical

(A_r, A_θ, A_ϕ) or $\boxed{A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}}$

$a_r \cdot a_r = a_\theta \cdot a_\theta = a_\phi \cdot a_\phi = 1$

$a_r \cdot a_\theta = a_\theta \cdot a_\phi = a_\phi \cdot a_r = 0$

$a_r \times a_\theta = a_\phi$

$a_\theta \times a_\phi = a_r$

$a_\phi \times a_r = a_\theta$

Relationship between cartesian and spherical.

$r = \sqrt{x^2+y^2+z^2}$ $\qquad x = r\sin\theta\cos\phi$

$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$ (or) $\cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$ $\qquad y = r\sin\theta\sin\phi$

(r)

$\phi = \tan^{-1}(y/x)$ $\qquad z = r\cos\theta$

$A_r = \vec{A}\cdot\hat{a_r}$ $\qquad A_\phi = \vec{A}\cdot\hat{a}_\phi$ $\qquad A_z = \vec{A}\cdot\hat{a_z}$

$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$

$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$

$A_\phi = -A_x\sin\phi + A_y\cos\phi$

In matrix form

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Inverse transformation

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$\cdot$	a_r	a_θ	a_ϕ
a_x	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
a_y	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
a_z	$\cos\theta$	$-\sin\theta$	0