# Coulomb's law & Electric field intensity

Objectives:

* To learn the basic law used for finding force and electric field due to different charge configurations.

* Two fundamental laws governing electrostatic fields:

  1) Coulomb's law (2) Gauss's law.

Although coulomb's law is applicable in finding the electric field due to any charge configuration, it is easier to use gauss's law for symmetrical charge distribution.

* Assume electricfield is in a vacuum or free space.

* Based on coulomb's law, the concept of electric field intensity is introduced and applied to point, line, surface and volume charges.

## Coulomb's law: ( Inverse square law)

Is an experimental law formulated in 1785 by Charles Augustin de coulomb.

It deals with the force a point charge exerts on another point charge.

point charge ⇒ smaller dimensions.

Charge ⇒ Coulombs (C) ⇒ One Coulomb = $6 \times 10^{18}$ elect

one electron charge $e = -1.6019 \times 10^{-19}$ C

Coulomb's law states that the force F between two point charges $Q_1$ and $Q_2$ is :

along the line joining them

directly proportional to the product $Q_1 Q_2$ of the charges

inversely proportional to the square of distance R between them.

Expressed mathematically,

$$F = k \frac{Q_1 Q_2}{R^2} \quad \text{Newtons}$$

where k is the proportionality constant

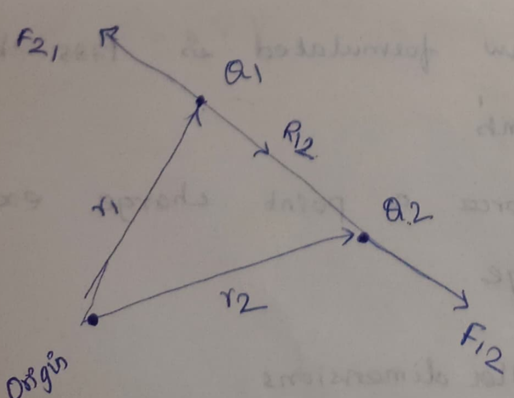
$$k = \frac{1}{4\pi\varepsilon_0}$$

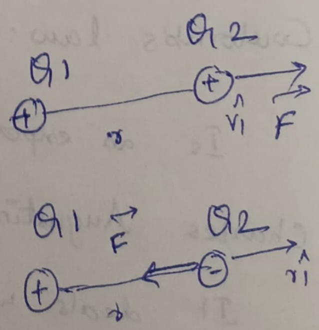
$\varepsilon_0$ is the permittivity of free space. ( farads per meter)

$$\varepsilon_0 = 8.854 \times 10^{-12} \; F/m$$

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \; m/F$$

Coulomb vector force on point charges $Q_1$ and $Q_2$



$$F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2}$$

Force on $Q_2$ due to $Q_1$,

$$\boxed{\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R_{12}^2} \hat{a}_{R12}}$$

where $\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$

$R = |\vec{R}_{12}|$ $\qquad |\vec{R}| = R$

$$\hat{a}_{R12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}$$

$$\therefore \boxed{\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^3} \vec{R}_{12}}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi \varepsilon_0 |r_2 - r_1|^3}$$

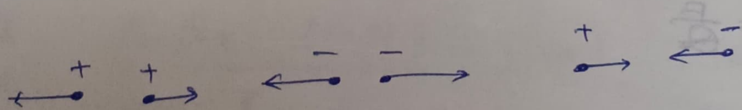
Force on $Q_1$ due to $Q_2$

$$\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2} \hat{a}_{R21}$$

$$= |\vec{F}_{12}| (-\hat{a}_{R12})$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

* Like charges repel each other, while unlike charges attract.




* $Q_1$ & $Q_2$ must be point charges
* $Q_1$ and $Q_2$ must be static (rest)
* For like charges $Q_1 Q_2 > 0$, for unlike charges $Q_1 Q_2 < 0$.

## Principle of Superposition

To determine the force on a particular charge, when we have more than two point charges.

It states that if there are N charges $Q_1, Q_2, \ldots Q_N$ located respectively, at points with position vectors $r_1, r_2 \ldots r_N$, the resultant force F on a charge Q located at point r is the vector sum of the forces exerted on Q by each of the charges $Q_1, Q_2 \ldots Q_N$.

$$\vec{F} = \frac{Q Q_1 \vec{R_1}}{4\pi\varepsilon_0 |\vec{R_1}|^3} + \frac{Q Q_2 (\vec{R_2})}{4\pi\varepsilon_0 |\vec{R_2}|^3} + \cdots + \frac{Q Q_N (\vec{R_N})}{4\pi\varepsilon_0 |\vec{R_N}|^3}$$

or

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k (\vec{R_k})}{|\vec{R_k}|^3}$$

$$\boxed{\frac{(r - r_k)}{|r - r_k|^3}}$$

## Electric field Intensity

Electric field intensity (or electric field strength) E is the force per unit charge when placed in an electric field.

$$E = \frac{F}{Q}$$

$$\vec{E} = \frac{\vec{F}}{Q}$$

$\vec{E}$ is obviously in the direction of the force $\vec{F}$.

<u>Unit:</u> Newtons per Coulomb
or Volts per meter

Electric field intensity at point $r$ due to a point charge located at $r'$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R \quad \text{or} \quad \frac{Q\,(r-r')}{4\pi\varepsilon_0 |r-r'|^3}$$

For N point charges $Q_1, Q_2 \ldots Q_N$ located at $r_1, r_2 \ldots r_N$, the electric field intensity at point $r$ is

$$\vec{E} = \frac{Q_1 \vec{R_1}}{4\pi\varepsilon_0 |\vec{R_1}|^3} + \frac{Q_2 \vec{R_2}}{4\pi\varepsilon_0 |\vec{R_2}|^3} + \cdots + \frac{Q_N (\vec{R_N})}{4\pi\varepsilon_0 |\vec{R_N}|^3}$$

(or)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{K=1}^{N} \frac{Q_K \vec{R_K}}{|\vec{R_K}|^3}$$

$$\boxed{\frac{(r-r_k)}{|r-r_k|^3}}$$

Problems:

1. Point charges 1 mC and −2 mC are located at (3, 2, −1) and (−1, −1, 4) respectively. Calculate the electric force on a 10 nC charge located at (0, 3, 1) and the electric field intensity at that point.

Solution:

$$\vec{F} = \sum_{k=1}^{2} \frac{Q\,Q_K\,\overbrace{(r-r_k)}^{\vec{R_{12}}}}{4\pi\varepsilon_0 |r-r_k|^3\ |R_{12}|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{Q_1 (r-r_1)}{|r-r_1|^3} + \frac{Q_2 (r-r_2)}{|r-r_2|^3}\right]$$

$$= \frac{10\times10^{-9}}{(1/9\times10^{9})}\left[\frac{10^{-3}(-3,1,2)}{14\sqrt{14}} + \frac{2\times10^{-3}(1,4,-3)}{26\sqrt{26}}\right]$$

$$\vec{F} = (-6.507\,\hat{a_x} - 3.817\,\hat{a_y} + 7.506\,a_z)\,mN$$

At that point,

$$\vec{E} = \vec{F}/Q$$

$$= \frac{(-6.507\,\hat{a_x} - 3.817\,\hat{a_y} + 7.506\,\hat{a_z})\times10^{-3}}{10\times10^{-9}}$$

$$\vec{E} = -650.7\,\hat{a_x} - 381.7\,\hat{a_y} + 750.6\,\hat{a_z} \quad kV/m$$

Electric Fields due to continuous charge distributions:

* So far we have considered only forces & electric fields due to point charges,

* Continuous charge distributions along a line, on a surface or in a volume are also possible.

Line charge density : $\rho_L$ in C/m

Surface charge density : $\rho_s$ in C/m2

Volume charge density : $\rho_v$ in C/m$^3$.