$$= \cancel{10} \frac{10\times10^{-9}}{(1/9\times10^{9})}\left[\frac{10^{-3}(-3,1,2)}{14\sqrt{14}} + \frac{2\times10^{-3}(1,4,-3)}{26\sqrt{26}}\right]$$

$$\vec{F} = (-6.507\,\hat{a_x} - 3.817\,\hat{a_y} + 7.506\,a_z)\,mN$$

At that point,

$$\vec{E} = \vec{F}/Q$$

$$= \frac{(-6.507\,\hat{a_x} - 3.817\,\hat{a_y} + 7.506\,\hat{a_z})\times10^{-3}}{10\times10^{-9}}$$

$$\vec{E} = -650.7\,\hat{a_x} - 381.7\,\hat{a_y} + 750.6\,\hat{a_z}\ \ kV/m$$

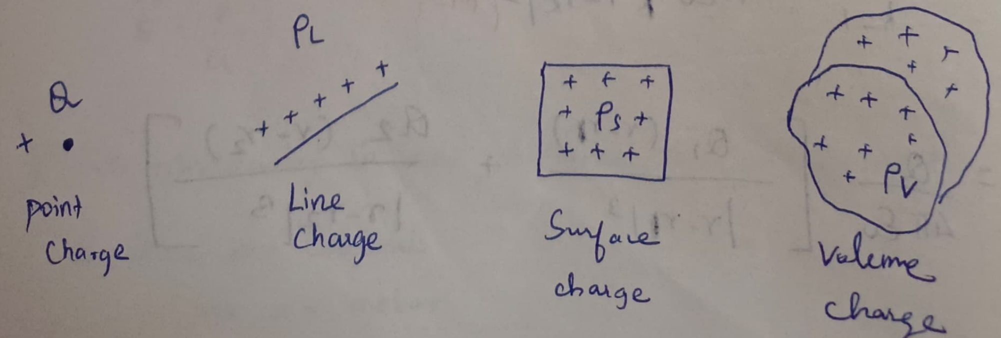
Electric Fields due to continuous charge distributions:

* So far we have considered only forces & electric fields due to point charges,
* Continuous charge distributions along a line, on a surface or in a volume are also possible.

Line charge density : ρ_L in C/m

Surface charge density : ρ_s in C/m2

Volume charge density : ρ_v in C/m^3.



$dQ = \rho_L \, dL$

$Q = \int_L \rho_L \, dl$ (line charge)

$dQ = \rho_s \, ds$

$Q = \int_S \rho_s \, ds$ (surface charge)

$dQ = \rho_v \, dv$

$Q = \int_V \rho_v \, dv$ (volume charge)

Electric field intensity

$$E = \int_L \frac{\rho_L \, dl}{4\pi \varepsilon_0 R^2} \hat{a_R} \quad \text{(line charge)}$$

$$E = \int_S \frac{\rho_s \, ds}{4\pi \varepsilon_0 R^2} \hat{a_R} \quad \text{(surface charge)}$$

$$E = \int_V \frac{\rho_v \, dv}{4\pi \varepsilon_0 R^2} (\hat{a_R}) \quad \text{(volume charge)}$$

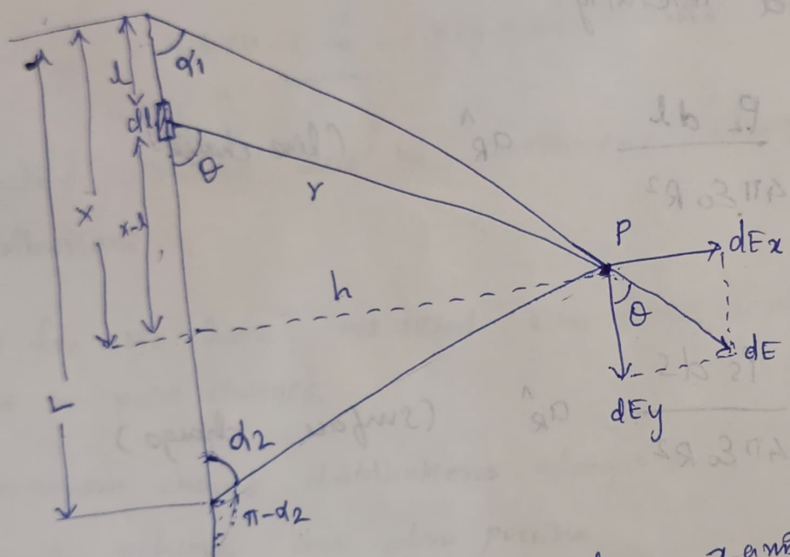
$$\rho_L = \frac{Q}{L}$$

$\rho_L . L = Q$

Electric field intensity due to line charge

Assume a straight uniformly charged wire of length L at a linear density of ρ_L Coulombs/m. Let P be any point at which Field intensity has to be determined.

Consider a small elemental length dl. Let dE be the Electric field due to charge element $\rho_L\, dl$.



dE has two components along x axis and y axis.

$$dE = \frac{\rho_L\, dl}{4\pi\varepsilon_0 r^2} \rightarrow (1)$$

$$\sin\theta = \frac{dE_x}{dE}$$

$$dE_x = dE\sin\theta$$

$$\cos\theta = \frac{dE_y}{dE}$$

$$dE_y = dE\cos\theta$$

$$\therefore dE_x = \frac{\rho_L\, dl}{4\pi\varepsilon_0 r^2}\sin\theta \rightarrow (2)$$

from figure

$$\tan\theta = \frac{h}{x-l}$$

$$x-l = \frac{h}{\tan\theta} = h\cot\theta$$

diff wrt θ

$$0 - \frac{dl}{d\theta} = h(-\text{cosec}^2\theta)$$

$$\boxed{dl = h\,\text{cosec}^2\theta\, d\theta}$$

$$\sin\theta = \frac{h}{r}$$

$$r = \frac{h}{\sin\theta}$$

$$\boxed{r = h\,\text{cosec}\,\theta}$$

Sub in (2)

$$dEx = \frac{\rho_l \cdot h\,\text{cosec}^2 d\theta}{4\pi\varepsilon_0 r^2} \sin\theta$$

$$dEx = \frac{\rho_l\, \cancel{h}\,\cancel{\text{cosec}^2\theta}\, d\theta\, \sin\theta}{4\pi\varepsilon_0\, h^{\cancel{2}}\, \cancel{\text{cosec}^2\theta}}$$

$$\boxed{dEx = \frac{\rho_l \sin\theta\, d\theta}{4\pi\varepsilon_0 h}}$$

Integrating

$$Ex = \frac{\rho_l}{4\pi\varepsilon_0 h} \int_{\alpha_1}^{\pi-\alpha_2} \sin\theta\, d\theta$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h} \left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$= \frac{-\rho_l}{4\pi\varepsilon_0 h} [\underbrace{\cos(\pi-\alpha_2)}_{-\cos\alpha_2} - \cos\alpha_1]$$

$$E_x = \frac{-\rho l}{4\pi\varepsilon_0 h}\left[-\cos\alpha_2 - \cos\alpha_1\right]$$

$$E_x = \frac{\rho l}{4\pi\varepsilon_0 h}\left(\cos\alpha_1 + \cos\alpha_2\right)$$

Similarly

$$dE_y = \frac{\rho l}{4\pi\varepsilon_0 h}\cos\theta \, d\theta$$

$$E_y = \frac{\rho l}{4\pi\varepsilon_0 h}\int_{\alpha_1}^{\pi-\alpha_2}\cos\theta \, d\theta$$

$$= \frac{\rho l}{4\pi\varepsilon_0 h}\left[\sin\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$= \frac{\rho l}{4\pi\varepsilon_0 h}\left(\sin(\pi-\alpha_2) - \sin\alpha_1\right)$$

$$E_y = \frac{\rho l}{4\pi\varepsilon_0 h}\left(\sin\alpha_2 - \sin\alpha_1\right)$$

Total electric field intensity at point 'P' is

$$E = E_x \hat{a_x} + E_y \hat{a_y}$$

$$E = \frac{\rho l}{4\pi\varepsilon_0 h}\left(\cos\alpha_1 + \cos\alpha_2\right)\hat{a_x} + \frac{\rho l}{4\pi\varepsilon_0 h}\left(\sin\alpha_2 - \sin\alpha_1\right)\hat{a_y}$$

If the point 'p' is along the perpendicular bisector of wire then $\alpha_1 = \alpha_2$

Let $\alpha_1 = \alpha_2 = \alpha$

$$\therefore E = \frac{\rho_l}{4\pi\epsilon_0 h} 2\cos\alpha \, \hat{a_x}$$

$$\vec{E} = \frac{\rho_l}{2\pi\epsilon_0 h} \cos\alpha \, \hat{a_x}$$

If the line is infinitely long, then $\alpha = 0$

$$\therefore E = \frac{\rho_l}{4\pi\epsilon_0 h} (2) \, \hat{a_x}$$

$$\boxed{\vec{E} = \frac{\rho_l}{2\pi\epsilon_0 h} \hat{a_x}}$$

— x —

Electric field due to circular sheet of charge:

Consider a circular sheet of charge of radius 'a' with charge density ρ_s C/m². Let 'p' be a point, 'h' meters from the disc along its axis at which field has to be determined.

Consider a small elemental area.

$$ds = 2\pi r \, dr.$$

Let dE be the field at Point 'p' due to small area ds.

Owing to the symmetry of the charge distribution, for every element 1, there is corresponding element 2