

Flux density at any point on the surface,  $\vec{D}$  is

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r \text{ C/m}^2$$

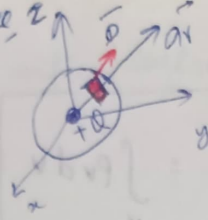
$$S = 4\pi r^2$$

Relation between  $\vec{D}$  and  $\vec{E}$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

$$\vec{E} = \vec{D}/\epsilon$$

$$\boxed{\vec{D} = \epsilon \vec{E}}$$



vector form

$$\vec{D} = \frac{d\psi}{ds} \hat{a}_n \text{ C/m}^2$$

$d\psi \Rightarrow$  Total flux lines crossing normal thro the diff surface area  $dS$

Electric flux density is also called as electric displacement.

## Gauss's law

Objective:- To learn one of the fundamental law of electromagnetics

To find fields due to symmetrical charge distributions

Gauss's law constitutes one of the fundamental laws of electromagnetism.

Gauss's law states that the total electric flux  $\psi$  through any closed surface is equal to the total charge enclosed by that surface.

$$\psi = Q_{enc.}$$

$$\psi = \oint_S d\psi$$

$$\psi = \oint_S \vec{D} \cdot d\vec{s} = \text{total charge enclosed}$$

$$Q = \int_V \rho_v dv$$

$$Q = \oint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v dv$$

Integral form

Applying divergence theorem

$$\oint_S \vec{D} \cdot d\vec{s} = \int_V \nabla \cdot \vec{D} dv$$

Comparing two equations

$$\therefore \rho_v = \nabla \cdot \vec{D}$$

Differential or point form.

which is first of the four Maxwell's equations.

Equation states that the volume charge density is the same as the divergence of electric flux density.

(It relates the definition of divergence & also  
(WKT  $\rho_v$  is simply at charge per unit volume))

Advantages :-

Gauss' law provides an easy means of finding  $E$  or  $D$  for symmetrical charge distributions, such as a point charge, an infinite line charge, an infinite cylindrical surface charge & a spherical distribution of charge.

Proof of Gauss's law :-

Consider a point charge  $Q$  kept at the origin.(Consider a small area ' $ds$ ' on the surface of the sphere)Let  $d\psi$  be the flux crossing the surface.

$$d\psi = \vec{D} \cdot d\vec{s}$$

$$= D ds \cos \theta$$

$$d\psi = D ds \cos \theta$$

Total flux leaving the entire surface

$$\Psi = \oint \vec{D} \cdot d\vec{s}$$

$$\Psi = \iint D \, ds$$

$$\Psi = \oint D r \hat{a}_r \cdot \hat{a}_r \, ds$$

$$r^2 \sin \theta d\theta d\phi$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D \cdot (r^2 \sin \theta d\theta d\phi)$$

$$= D r^2 \int_{\phi=0}^{2\pi} (-\cos \theta)_0^{\pi} d\phi$$

$$= D r^2 (-\cos \pi + \cos 0) (2\pi)$$

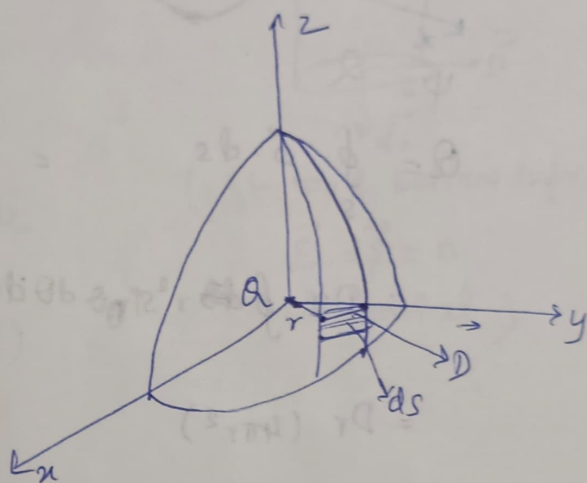
$$= D r^2 (2\pi) (2)$$

$$\Psi = 4\pi r^2 D$$

WKT  $D = \frac{Q}{4\pi r^2}$

$$\Psi = 4\pi r^2 \cdot \frac{Q}{4\pi r^2}$$

$$\underline{\underline{\Psi = Q}}$$



Applications of Gauss's law:

→ To calculate the electric field due to different charge distributions:-

If Symmetric charge distribution exists, gaussian surface is constructed such that  $\vec{D}$  is normal or tangential to the Gaussian surface

When  $D$  is normal to the surface  $\vec{D} \cdot d\vec{s} = D \, ds$

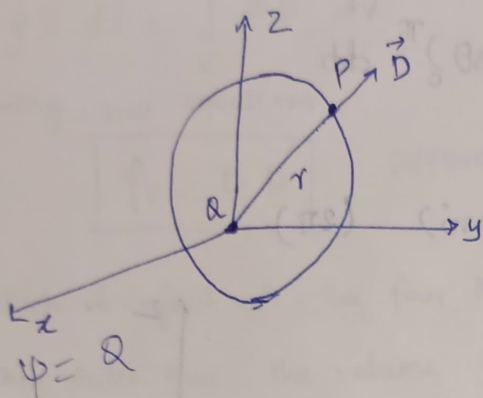
" tangential

$$\vec{D} \cdot d\vec{s} = 0$$

## i) Point Charge

A point charge  $Q$  is located at the origin.

To determine  $\vec{D}$  at a point  $P$ , it is easy to choose a spherical surface with symmetry conditions. (Gaussian surface)



$$\begin{aligned}
 Q &= \oint_S \vec{D} \cdot d\vec{s} = \oint_S D r \hat{a}_r \cdot r^2 \sin\theta d\theta d\phi \hat{a}_r \\
 &= D r \int_0^{2\pi} \int_0^\pi r^2 \sin\theta d\theta d\phi = D r \int_0^{2\pi} \int_0^\pi r^2 \sin\theta d\theta d\phi \\
 &= D r (4\pi r^2)
 \end{aligned}$$

$$Q = D r 4\pi r^2$$

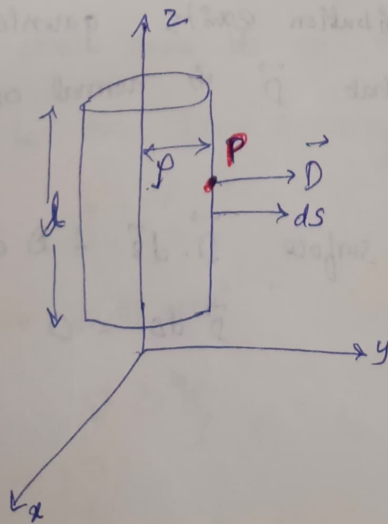
$$D r = Q / 4\pi r^2$$

flux density

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

## ii) Infinite line charge:



Apply Gauss law

$$Q = \oint_S \vec{D} \cdot d\vec{s}$$

$d\rho$

$\rho d\phi$   
 $dz$

$$P_l l = \oint_S \vec{D} \cdot d\vec{s}$$

here  $\vec{D} = D_\rho \hat{a}_\rho$  since  $\vec{D}$  is normal to the cylindrical surface.

$$P_l l = \oint_S D_\rho \hat{a}_\rho \cdot (\rho d\phi dz) \hat{a}_\rho$$

$$= D_\rho \oint_S \rho d\phi dz$$

$$= D_\rho \rho \int_0^{2\pi} d\phi \int_0^l dz$$

$$= D_\rho \rho (2\pi) (l)$$

$$P_l l = D_\rho \rho 2\pi l$$

$$D_\rho = \frac{P_l}{2\pi \rho}$$

$$\vec{D} = D_\rho \hat{a}_\rho$$

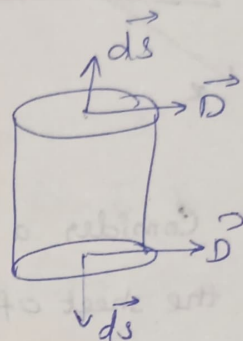
$$\vec{D} = \frac{P_l}{2\pi \rho} \hat{a}_\rho$$

$\rho \rightarrow$  radius of cylinder

$P_l \rightarrow$  charge density

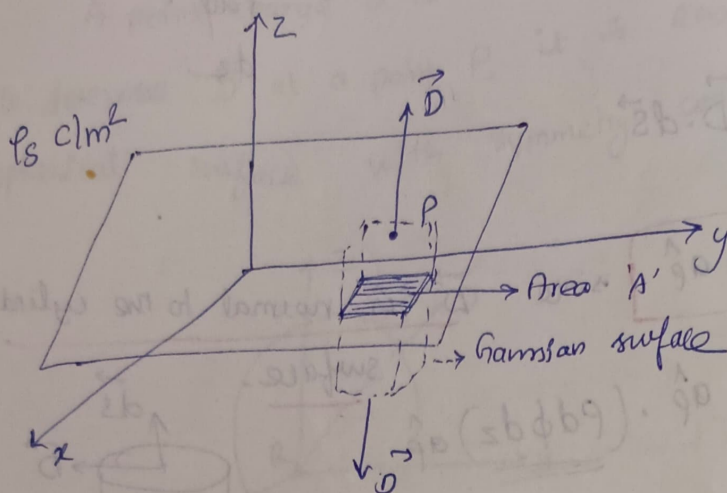
$$\vec{E} = \frac{\vec{D}}{\epsilon_0}$$

$$\vec{E} = \frac{P_l}{2\pi \epsilon_0 \rho} \hat{a}_\rho$$



(At top & bottom surfaces  
 $\vec{D} \cdot d\vec{s} = 0$   
tangential)

### iii) Infinite sheet of charge



Consider a rectangular box that is cut symmetrically by the sheet of charge and has two of its faces parallel to the sheet. Since  $\vec{D}$  is normal to the sheet,

$$\vec{D} = D_z \hat{a}_z$$

Applying Gauss law,

$$Q = \oint_S \vec{D} \cdot d\vec{s}$$

$$\rho_s \cdot A = \oint_S D_z \hat{a}_z \cdot d\vec{s} \hat{a}_z$$

$$= D_z \oint d\vec{s}$$

$$\rho_s A = D_z \left( \oint_{\text{top}} d\vec{s} + \oint_{\text{bottom}} d\vec{s} \right)$$

along  $a_x$  &  $a_y$ ,  $\vec{D}$  has no component  
 $\vec{D} \cdot d\vec{s} = 0$

$$\rho_s A = D_z (A + A)$$

$$\rho_s A = D_z \cdot 2A$$

$$D_z = \frac{\rho_s}{2}$$

$$\vec{D} = D_z \hat{a}_z$$

$$\therefore \boxed{\vec{D} = \frac{\rho_s}{2} \hat{a}_z}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0}$$

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z$$

Gauss Divergence theorem

From gauss law

$$\iint \vec{D} \cdot d\vec{s} = Q$$

$$Q = \iiint \rho_v dv$$

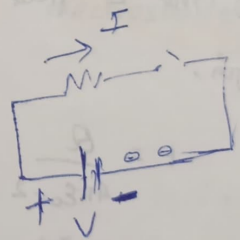
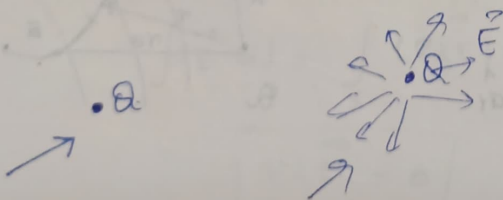
$$\iint \vec{D} \cdot d\vec{s} = \iiint \rho_v dv$$

$$\boxed{\iint \vec{D} \cdot d\vec{s} = \iiint \nabla \cdot \vec{D} dv}$$

from point form of gauss law,

$$\nabla \cdot \vec{D} = \rho_v$$

Electric Potential:



We wish to move a point charge  $Q$  from Point A to Point B in an electric field  $E$ . From Coulomb's law, force on  $Q$  is  $F = QE$ , so that the work done by external source in displacing the charge by  $d\vec{l}$

$$dw = -F \cdot d\vec{l}$$

$$= -QE \cdot d\vec{l}$$