SNS COLLEGE OF TECHNOLOGY
COIMBATORE-35
DEPARTMENT OF MECHATRONICS ENGINEERING

23MCT205 MECHANICS OF MACHINES

UNIT – I

KINEMATICS OF MECHANISMS

Methods for Determining the Velocity of a Point on a Link

Though there are many methods for determining the velocity of any point on a link in a mechanism whose direction of motion (*i.e.* path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods are important from the subject point of view.

1. Instantaneous centre method, and **2.** Relative velocity method.

The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configuration diagram

Velocity of a Point on a Link by Instantaneous Centre Method

The instantaneous centre method of analysing the motion in a mechanism is based upon the concept that any displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

Consider two points *A* and *B* on a rigid link. Let v_A and v_B be the velocities of points *A* and *B*, whose directions are given by angles α and β as shown in Fig.1. If v_A is known in magnitude and direction and v_B in direction only, then the magnitude of v_B may be determined by the instantaneous centre method as discussed below:

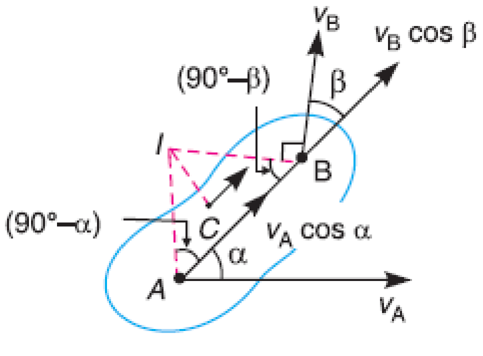


Fig.1. Velocity of a point on a link.

Draw *AI* and *BI* perpendiculars to the directions v_A and v_B respectively. Let these lines intersect at *I*, which is known as instantaneous centre or virtual centre of the link. The complete rigid link is to rotate or turn about the centre *I*. Since *A* and *B* are the points on a rigid link, therefore there cannot be any relative motion between them along the line *AB*.

Properties of the Instantaneous Centre:

The following properties of the instantaneous centre are important from the subject point of view:

1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.

2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (*i.e.* instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link.

Number of Instantaneous Centres in a Mechanism

The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres,

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

Types of Instantaneous Centres

The instantaneous centres for a mechanism are of the following three types:

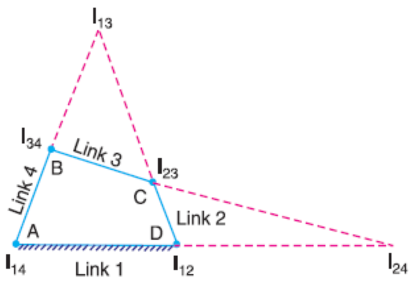
1. Fixed instantaneous centres, **2.** Permanent instantaneous centres, and **3.** Neither fixed nor permanent instantaneous centres.

The first two types *i.e.* fixed and permanent instantaneous centres are together known as ***primary instantaneous centres*** and the third type is known as ***secondary instantaneous centres***.

Consider a four bar mechanism *ABCD* as shown in Fig.2. The number of instantaneous centres (N) in a four bar mechanism is given by

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

The instantaneous centres I_{12} and I_{14} are called the ***fixed instantaneous centres*** as they remain in the same place for all configurations of the mechanism. The



instantaneous centres I_{23} and I_{34} are the ***permanent instantaneous centres*** as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I_{13} and I_{24} are ***neither fixed nor permanent instantaneous centres*** as they vary with the configuration of the mechanism.

Fig. 2. Types of instantaneous centres.

Aronhold Kennedy (or Three Centres in Line) Theorem

The Aronhold Kennedy's theorem states that ***if three bodies move relatively to each other***, ***they have three instantaneous centres and lie on a straight line.***

Rubbing Velocity at a Pin Joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links *OA* and *OB* connected by a pin joint at *O* as shown in Fig.3.

Let ω_1 = Angular velocity of the link *OA* or the angular velocity of the point *A* with respect to *O*.

ω_2 = Angular velocity of the link *OB* or the angular velocity of the point *B* with respect to *O*, and

r = Radius of the pin.

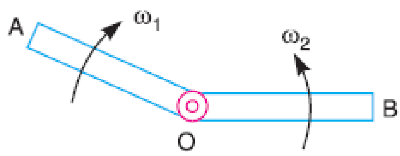


Fig. 3. Links connected by pin joints

According to the definition,

Rubbing velocity at the pin joint *O*

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

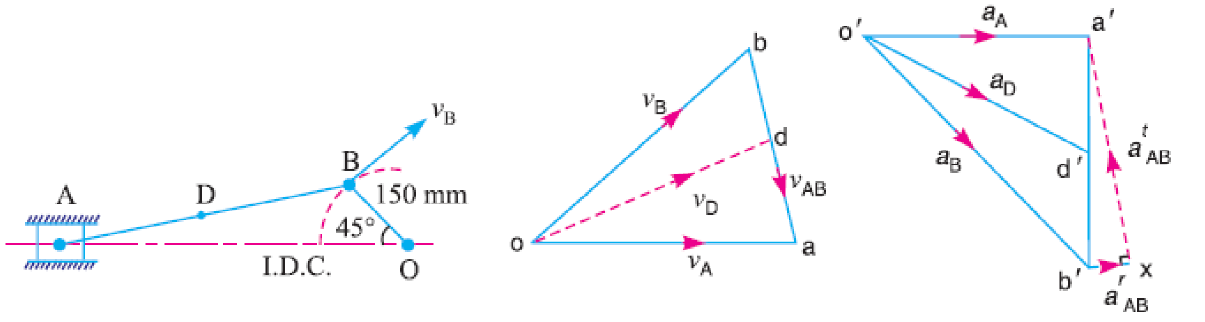
Velocity and Acceleration of a Point on a Link by Relative Velocity Method

Problem 1. *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine:* **1.** *Linear velocity and acceleration of the midpoint of the connecting rod, and* **2.** *angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\,\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of *B* with respect to *O* or velocity of *B*,

$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713$ m/s



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

1. *Linear velocity of the midpoint of the connecting rod*

First of all draw the space diagram, to some suitable scale; as shown in Fig. Now the velocity diagram, as shown in Fig., is drawn as discussed below:

1. Draw vector *ob* perpendicular to *BO*, to some suitable scale, to represent the velocity of *B* with respect to *O* or simply velocity of *B i.e.* v_{BO} or v_B, such that vector $ob = v_{BO} = v_B = 4.713$ m/s

2. From point *b*, draw vector *ba* perpendicular to *BA* to represent the velocity of *A* with respect to *B i.e.* v_{AB} , and from point *o* draw vector *oa* parallel to the motion of *A* (which is along *AO*) to represent the velocity of *A i.e.* v_A. The vectors *ba* and *oa* intersect a

By measurement, we find that velocity of *A* with respect to *B*,

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$

3. In order to find the velocity of the midpoint *D* of the connecting rod *AB*, divide the vector *ba* at *d* in the same ratio as *D* divides *AB*, in the space diagram. In other words, *bd* / *ba* = *BD*/*BA*

Note: Since *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

4. Join *od*. Now the vector *od* represents the velocity of the midpoint *D* of the connecting rod *i.e.* v_D.

By measurement, we find that

v_D = vector *od* = 4.1 m/s **Ans.**

Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of *B* with respect to *O* or the acceleration of *B*,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleraiton of *A* with respect to *B*,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. is drawn as discussed below:

1. Draw vector *o′ b′* parallel to *BO*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *O* or simply acceleration of *B* *i.e.* a^r_{BO} or a_B, such that

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

Note: Since the crank *OB* rotates at a constant speed, therefore there will be no tangential component of the acceleration of *B* with respect to *O*.

2. The acceleration of *A* with respect to *B* has the following two components:

(*a*) The radial component of the acceleration of *A* with respect to *B* *i.e.* a^r_{AB}, and

(*b*) The tangential component of the acceleration of *A* with respect to *B* *i.e.* a^t_{AB}.

These two components are mutually perpendicular.

Therefore from point *b′*, draw vector *b′ x* parallel to *AB* to represent a^r_{AB} = 9.3 m/s^2 and from point *x* draw vector *xa′* perpendicular to vector *b′ x* whose magnitude is yet unknown.

3. Now from *o′*, draw vector *o′ a′* parallel to the path of motion of *A* (which is along *AO*) to represent the acceleration of *A* *i.e.* a_A. The vectors *xa′* and *o′ a′* intersect at *a′*. Join *a′ b′*.

4. In order to find the acceleration of the midpoint *D* of the connecting rod *AB*, divide the vector *a′ b′* at *d′* in the same ratio as *D* divides *AB*. In other words *b′d′* / *b′a′* = *BD* / *BA*

5. Join *o′ d′*. The vector *o′ d′* represents the acceleration of midpoint *D* of the connecting rod *i.e.* a_D.

By measurement, we find that

a_D = vector *o′ d′* = 117 m/s^2 **Ans.**

2. *Angular velocity of the connecting rod*

We know that angular velocity of the connecting rod *AB*,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B) \textbf{ Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a_{AB}^{t} = 103 \text{ m/s}^2$$

We know that angular acceleration of the connecting rod *AB*,

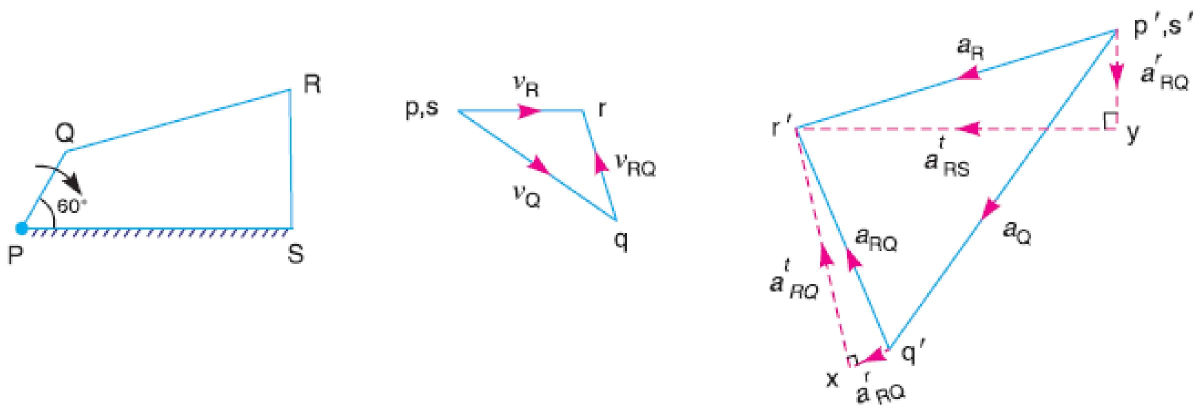
$$\alpha_{AB} = \frac{a_{AB}^{t}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B\text{)}$$ **Ans.**

Problem 2. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm; QR = 175 mm; RS = 112.5 mm; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

Solution. Given: ω_{QP} = 10 rad/s; *PQ* = 62.5 mm = 0.0625 m; *QR* = 175 mm = 0.175 m; *RS* = 112.5 mm = 0.1125 m; *PS* = 200 mm = 0.2 m

We know that velocity of *Q* with respect to *P* or velocity of *Q*,

$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625$ m/s



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Angular velocity of links QR and RS

vector $pq = v_{QP} = v_Q = 0.625$ m/s

v_{RQ} = vector qr = 0.333 m/s, and $v_{RS} = v_R$ = vector sr = 0.426 m/s

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)}$$ **Ans.**

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise)}$$ **Ans.**

$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Angular acceleration of links QR and RS

$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

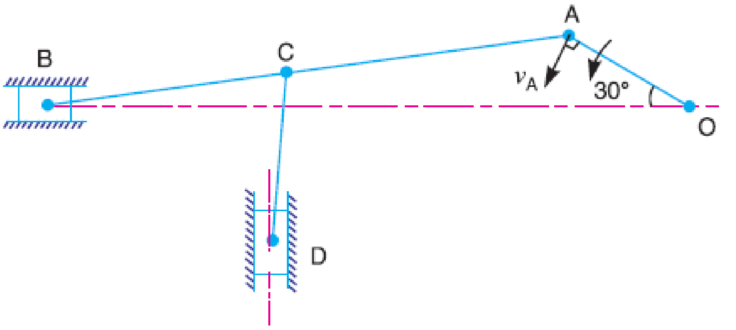
a^t_{RQ} = vector xr' = 4.1 m/s^2 and a^t_{RS} = vector yr' = 5.3 m/s^2

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

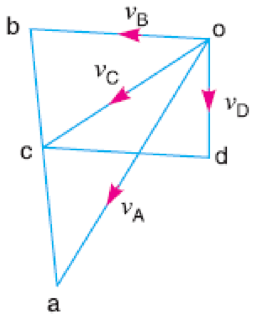
$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

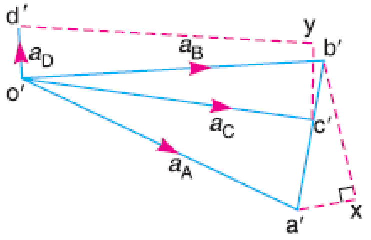
Problem 3. *In the mechanism, as shown in Fig., the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm. For the given configuration, determine:* **1.** *velocities of sliding at B and D,* **2.** *Angular velocity of CD,* **3.** *linear acceleration of D, and* **4.** *angular acceleration of CD.*

Solution. Given : N_{AO} = 20 r.p.m. or $\omega_{AO} = 2\pi \times 20/60 = 2.1$ rad/s ; OA = 300 mm = 0.3 m ; AB = 1200 mm = 1.2 m ; $BC = CD$ = 450 mm = 0.45 m



(*a*) Space diagram.





(*b*) Velocity diagram. (*c*) Acceleration diagram.

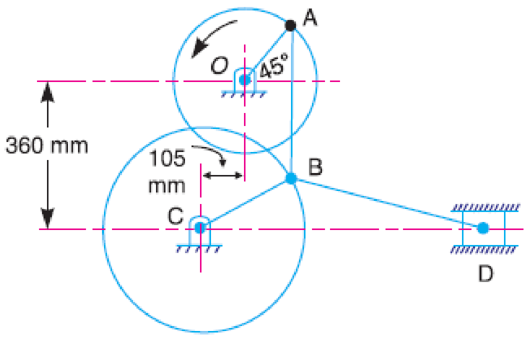
v_B = vector ob = 0.4 m/s **Ans.**
v_D = vector od = 0.24 m/s **Ans.**

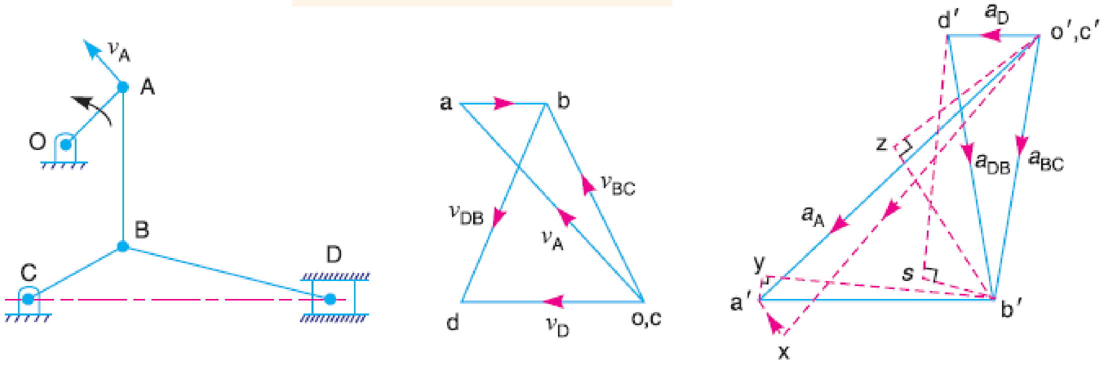
$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s (Anticlockwise).}$$ **Ans.**

a_D = vector $o'd'$ = 0.16 m/s^2 **Ans.**

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{1.28}{0.45} = 2.84 \text{ rad/s}^2 \text{ (Clockwise)}$$ **Ans.**

Problem 4. *In the toggle mechanism shown in Fig., the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a speed of 180 r.p.m. increasing at the rate of 50 rad/s^2. The dimensions of the various links are as follows:*
OA = 180 mm; CB = 240 mm; AB = 360 mm; and BD = 540 mm.
For the given configuration, find **1.** *Velocity of slider D and angular velocity of BD, and* **2.** *Acceleration of slider D and angular acceleration of BD.*





(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.