23MCT205 MECHANICS OF MACHINES

UNIT – III

GEARS AND GEAR TRAINS

Classification of Toothed Wheels

The gears or toothed wheels may be classified as follows:
1. *According to the position of axes of the shafts*. The axes of the two shafts between which the motion is to be transmitted, may be
(*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. These gears are called ***spur gears*** and the arrangement is known as ***spur gearing***. These gears have teeth parallel to the axis of the wheel. Another name given to the spur gearing is ***helical gearing***, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 1 (*a*) and (*b*) respectively. The double helical gears are known as ***herringbone gears***. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.

The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 1 (*c*). These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing***. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as ***helical bevel gears***.

The two non-intersecting and non-parallel *i.e.* non-coplanar shaft connected by gears is shown in Fig. 1(*d*). These gears are called ***skew bevel gears*** or ***spiral gears*** and the arrangement is known as ***skew bevel gearing*** or ***spiral gearing***. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as ***hyperboloids***.

Notes: (*a*) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as ***mitres***.

(*b*) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.
(*c*) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.

2. *According to the peripheral velocity of the gears*. The gears, according to the peripheral velocity of the gears may be classified as:
(*a*) Low velocity, **(*b*)** Medium velocity, and **(*c*)** High velocity.
The gears having velocity less than 3 m/s are termed as ***low velocity*** gears and gears having velocity between 3 and 15 m/s are known as ***medium velocity gears***. If the velocity of gears is more than 15 m/s, then these are called ***high speed gears***.

3. *According to the type of gearing*. The gears, according to the type of gearing may be classified as:
(*a*) External gearing, **(*b*)** Internal gearing, and **(*c*)** Rack and pinion.

In ***external gearing***, the gears of the two shafts mesh externally with each other as shown in Fig. 2 (*a*). The larger of these two wheels is called ***spur wheel*** and the smaller wheel is called **pinion.** In an external gearing, the motion of the two wheels is always ***unlike***, as shown in Fig. 2 (*a*).

In ***internal gearing***, the gears of the two shafts mesh ***internally*** with each other as shown in Fig. 2. (*b*). The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***. In an internal gearing, the motion of the two wheels is always ***like***, as shown in Fig. 2 (*b*).

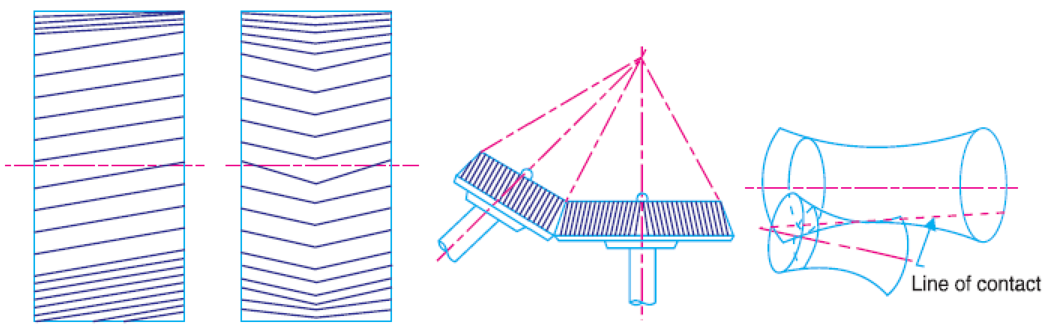
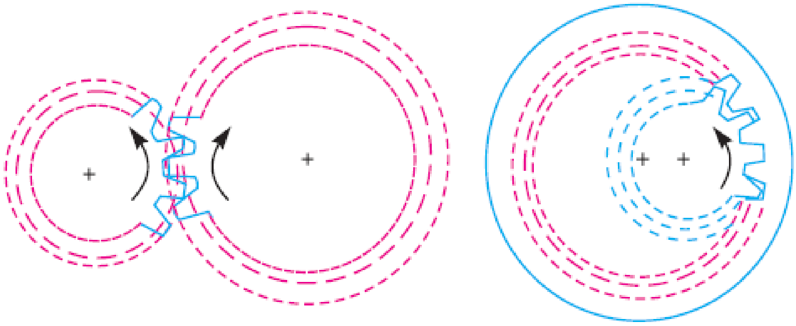


Fig. 1 (*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.



(*a*) External gearing. (*b*) Internal gearing.

Fig. 2

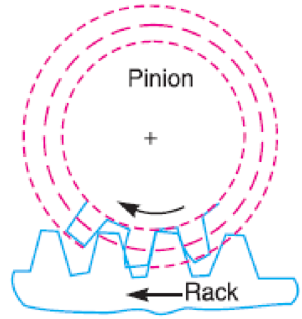


Fig. 3. Rack and pinion.

Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line, as shown in Fig. 3. Such type of gear is called ***rack and pinion***. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and ***vice-versa*** as shown in Fig.3.

4. ***According to position of teeth on the gear surface***. The teeth on the gear surface may be **(*a*)** straight, **(*b*)** inclined, and **(*c*)** curved.
The spur gears have straight teeth whereas helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

Terms Used in Gears

1. ***Pitch circle***. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

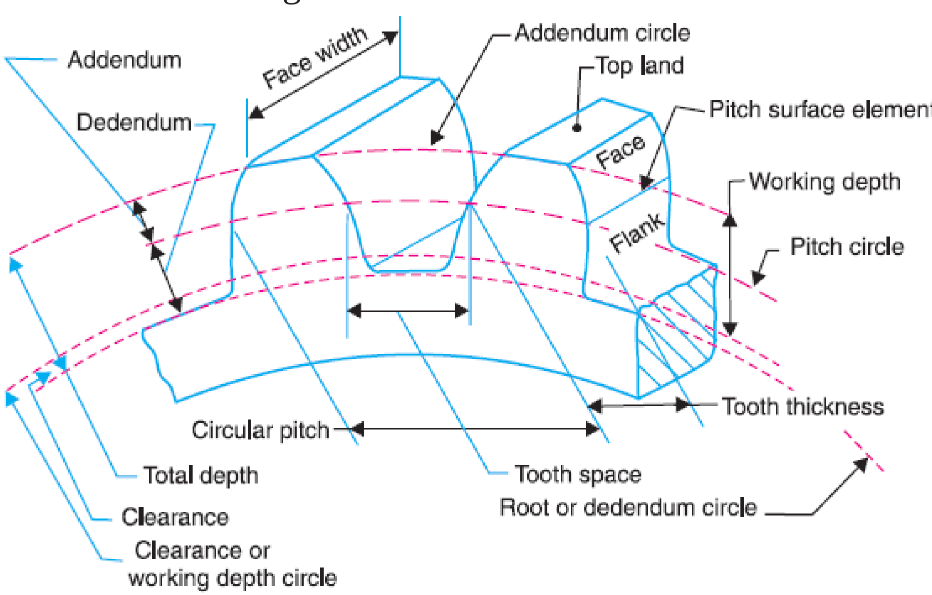


Fig. 4. Terms used in gears.

2. ***Pitch circle diameter***. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.
3. ***Pitch point***. It is a common point of contact between two pitch circles.
4. ***Pitch surface***. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.
5. ***Pressure angle or angle of obliquity***. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are 14.5^o & 20^o
6. ***Addendum***. It is the radial distance of a tooth from the pitch circle to the top of the tooth.
7. ***Dedendum***. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
8. ***Addendum circle***. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.
9. ***Dedendum circle***. It is the circle drawn through the bottom of the teeth. It is also called root circle.
10. ***Circular pitch***. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,
Circular pitch, $p_c = \pi D/T$
where D = Diameter of the pitch circle, and
T = Number of teeth on the wheel.
11. ***Diametral pitch.*** It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d . Mathematically,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Diametral pitch, $$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because\ p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. ***Module.*** It is the ratio of the pitch circle diameter in millimeters to the number of teeth.It is usually denoted by m. Mathematically, Module, $m = D/T$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. ***Clearance***. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

14. ***Total depth***. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

Length of the path of contact. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

Arc of contact. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e*.

(*a*) ***Arc of approach***. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) ***Arc of recess***. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Gear Materials

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The nonmetallic materials like wood, raw hide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.

The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important. The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness. The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig.5. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at *K* (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). *MN* is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.

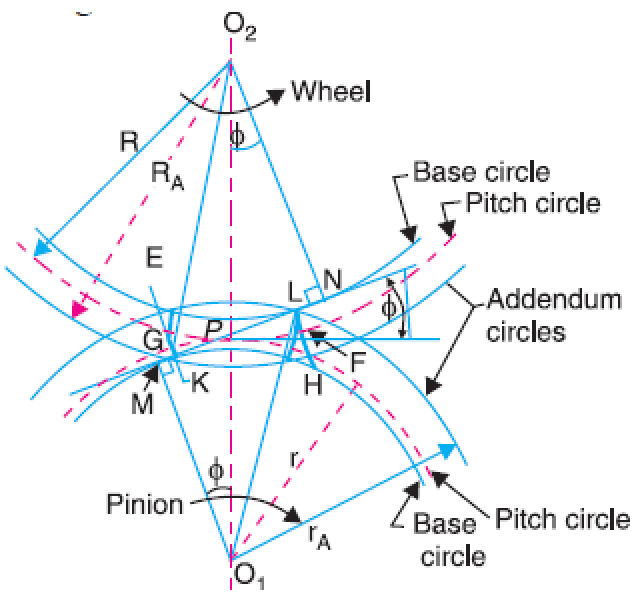


Fig. 5. Length of path of contact.

The length of path of contact is the length of common normal cutoff by the addendum circles of the wheel and the pinion. Thus the length of path of contact is *KL* which is the sum of the parts of the path of contacts *KP* and *PL*. The part of the path of contact *KP* is known as ***path of approach*** and the part of the path of contact *PL* is known as ***path of recess.***

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P =$ Radius of pitch circle of wheel.

From Fig. 12.11, we find that radius of the base circle of pinion,

$$O_1M = O_1P \cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P\cos\phi = R\cos\phi$$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

and

$$PN = O_2P\sin\phi = R\sin\phi$$

∴ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angled triangle O_1ML,

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

and

$$MP = O_1P\sin\phi = r\sin\phi$$

∴ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

We know that the length of the arc of approach (arc GP)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc PH)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Since the length of the arc of contact GPH is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

$$p_c = \text{Circular pitch} = \pi m, \text{ and}$$

$$m = \text{Module.}$$

Problem 1. *A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.*

Solution. Given : $t = 30$; $T = 80$; $\varphi = 20°$; $m = 12$ mm ; Addendum = 10 mm

Length of path of contact

We know that pitch circle radius of pinion,

$r = m.t / 2 = 12 \times 30 / 2 = 180$ mm

and pitch circle radius of gear,

$R = m.T / 2 = 12 \times 80 / 2 = 480$ mm

∴Radius of addendum circle of pinion,

$r_A = r + \text{Addendum} = 180 + 10 = 190$ mm

and radius of addendum circle of gear,

$R_A = R + \text{Addendum} = 480 + 10 = 490$ mm

We know that length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi \quad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(490)^2 - (480)^2 \cos^2 20°} - 480 \sin 20° = 191.5 - 164.2 = 27.3 \text{ mm}$$

and length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(190)^2 - (180)^2 \cos^2 20°} - 180 \sin 20° = 86.6 - 61.6 = 25 \text{ mm}$$

We know that length of path of contact,

$$KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \quad \textbf{Ans.}$$

Length of arc of contact

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{52.3}{\cos 20°} = 55.66 \text{ mm } \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$p_c = \pi.m = \pi \times 12 = 37.7 \text{ mm}$$

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{p_c} = \frac{55.66}{37.7} = 1.5 \text{ say } 2 \textbf{ Ans.}$$

Problem 2. *Two involute gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find :* **1.** *The angle turned through by pinion when one pair of teeth is in mesh ; and* **2.** *The maximum velocity of sliding.*

Solution. Given : $\phi = 20°$; $t = 20$; $G = T/t = 2$; $m = 5$ mm ; $v = 1.2$ m/s ; addendum = 1 module = 5 mm

1. *Angle turned through by pinion when one pair of teeth is in mesh*

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 5 \times 20 / 2 = 50 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T / 2 = m.G.t / 2 = 2 \times 20 \times 5 / 2 = 100 \text{ mm} \qquad ...(\because T = G.t)$$

$\therefore$ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum} = 50 + 5 = 55 \text{ mm}$$

and radius of addendum circle of wheel,

$$R_A = R + \text{Addendum} = 100 + 5 = 105 \text{ mm}$$

We know that length of the path of approach (*i.e.* the path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(105)^2 - (100)^2 \cos^2 20°} - 100 \sin 20°$$

$$= 46.85 - 34.2 = 12.65 \text{ mm}$$

and the length of path of recess (*i.e.* the path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2\phi} - r\sin\phi$$

$$= \sqrt{(55)^2 - (50)^2 \cos^2 20°} - 50 \sin 20° = 28.6 - 17.1 = 11.5 \text{ mm}$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = 12.65 + 11.5 = 24.15 \text{ mm}$$

and length of the arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{24.15}{\cos 20^\circ} = 25.7 \text{ mm}$$

We know that angle turned through by pinion

$$= \frac{\text{Length of arc of contact} \times 360^\circ}{\text{Circumference of pinion}} = \frac{25.7 \times 360^\circ}{2\pi \times 50} = 29.45^\circ \quad \textbf{Ans.}$$

2. *Maximum velocity of sliding*

Let ω_1 = Angular speed of pinion, and

ω_2 = Angular speed of wheel.

We know that pitch line speed,

$$v = \omega_1 . r = \omega_2 . R$$

$\therefore$ $\omega_1 = v/r = 120/5 = 24$ rad/s

and $\omega_2 = v/R = 120/10 = 12$ rad/s

$\therefore$ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\, KP \qquad \ldots(\because KP > PL)$$

$$= (24 + 12)\, 12.65 = 455.4 \text{ mm/s} \quad \textbf{Ans.}$$

Problem 3. *A pair of gears, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 r.p.m. Determine the velocity of sliding between the gear teeth faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20° involute form, addendum length is 5 mm and the module is 5 mm.Also find the angle through which the pinion turns while any pairs of teeth are in contact.*

Solution. Given : $T = 40$; $t = 20$; $N_1 = 2000$ r.p.m. ; $\phi = 20^\circ$; addendum = 5 mm ; $m = 5$ mm

We know that angular velocity of the smaller gear,

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 2000}{60} = 209.5 \text{ rad/s}$$

and angular velocity of the larger gear,

$$\omega_2 = \omega_1 \times \frac{t}{T} = 209.5 \times \frac{20}{40} = 104.75 \text{ rad/s} \qquad \ldots\left(\because \frac{\omega_2}{\omega_1} = \frac{t}{T}\right)$$

Pitch circle radius of the smaller gear,

$$r = m.t / 2 = 5 \times 20/2 = 50 \text{ mm}$$

and pitch circle radius of the larger gear,

$$R = m.t / 2 = 5 \times 40/2 = 100 \text{ mm}$$

$\therefore$ Radius of addendum circle of smaller gear,

$$r_A = r + \text{Addendum} = 50 + 5 = 55 \text{ mm}$$

and radius of addendum circle of larger gear,

$$R_A = R + \text{Addendum} = 100 + 5 = 105 \text{ mm}$$

We know that the distance of point of engagement K from the pitch point P or the length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$= \sqrt{(105)^2 - (100)^2 \cos^2 20°} - 100 \sin 20°$$

$$= 46.85 - 34.2 = 12.65 \text{ mm}$$

and the distance of the pitch point P from the point of disengagement L or the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(55)^2 - (50)^2 \cos^2 20°} - 50 \sin 20° = 28.6 - 17.1 = 11.5 \text{ mm}$$

Velocity of sliding at the point of engagement

We know that velocity of sliding at the point of engagement K,

$$v_{SK} = (\omega_1 + \omega_2)\, KP = (209.5 + 104.75)\, 12.65 = 3975 \text{ mm/s}$$ **Ans.**

Velocity of sliding at the pitch point

Since the velocity of sliding is proportional to the distance of the contact point from the pitch point, therefore the velocity of sliding at the pitch point is zero. **Ans.**

Velocity of sliding at the point of disengagement

We know that velocity of sliding at the point of disengagement L,

$$v_{SL} = (\omega_1 + \omega_2)\, PL = (209.5 + 104.75)\, 11.5 = 3614 \text{ mm/s}$$ **Ans.**

Angle through which the pinion turns

We know that length of the path of contact,

$$KL = KP + PL = 12.65 + 11.5 = 24.15 \text{ mm}$$

and length of arc of contact $= \dfrac{KL}{\cos \phi} = \dfrac{24.15}{\cos 20°} = 25.7$ mm

Circumference of the smaller gear or pinion

$$= 2\, \pi\, r = 2\pi \times 50 = 314.2 \text{ mm}$$

$\therefore$ Angle through which the pinion turns

$$= \text{Length of arc of contact} \times \frac{360°}{\text{Circumference of pinion}}$$

$$= 25.7 \times \frac{360°}{314.2} = 29.45°$$ **Ans.**

Problem 4. *Two gear wheels mesh externally and are to give a velocity ratio of 3 to 1. The teeth are of involute form ; module = 6 mm, addendum = one module, pressure angle = 20°. The pinion rotates at 90 r.p.m. Determine :* **1.** *The number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel,* **2.** *The length of path and arc of contact,* **3.***The number of pairs of teeth in contact, and* **4.** *The maximum velocity of sliding.*

Solution. Given : $G = T/t = 3$; $m = 6$ mm ; $A_P = A_W = 1$ module = 6 mm ; $\phi = 20°$; $N_1 = 90$ r.p.m. or $\omega_1 = 2\pi \times 90/60 = 9.43$ rad/s

1. *Number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel*

We know that number of teeth on the pinion to avoid interference,

$$t = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1} = \frac{2 \times 6}{\sqrt{1 + 3(3+2)\sin^2 20°} - 1}$$

$= 18.2$ say 19 **Ans.**

and corresponding number of teeth on the wheel,

$$T = G.t = 3 \times 19 = 57 \text{ Ans.}$$

2. *Length of path and arc of contact*

We know that pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 19/2 = 57 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum on pinion } (A_P) = 57 + 6 = 63 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 57/2 = 171 \text{ mm}$$

∴ Radius of addendum circle of wheel,

$$R_A = R + \text{Addendum on wheel } (A_W) = 171 + 6 = 177 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(177)^2 - (171)^2\cos^2 20°} - 171\sin 20° = 74.2 - 58.5 = 15.7 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2\phi} - r\sin\phi$$

$$= \sqrt{(63)^2 - (57)^2 \cos^2 20^\circ} - 57\sin 20^\circ = 33.17 - 19.5 = 13.67 \text{ mm}$$

∴ Length of path of contact,

$$KL = KP + PL = 15.7 + 13.67 = 29.37 \text{ mm} \quad \textbf{Ans.}$$

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{29.37}{\cos 20^\circ} = 31.25 \text{ mm} \quad \textbf{Ans.}$$

3. ***Number of pairs of teeth in contact***

We know that circular pitch,

$$p_c = \pi \times m = \pi \times 6 = 18.852 \text{ mm}$$

∴ Number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{p_c} = \frac{31.25}{18.852} = 1.66 \text{ say } 2 \quad \textbf{Ans.}$$

4. ***Maximum velocity of sliding***

Let ω_2 = Angular speed of wheel in rad/s.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t}$ or $\omega_2 = \omega_1 \times \frac{t}{T} = 9.43 \times \frac{19}{57} = 3.14$ rad/s

∴ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\, KP \qquad \ldots(\because KP > PL)$$

$$= (9.43 + 3.14)\, 15.7 = 197.35 \text{ mm/s} \quad \textbf{Ans.}$$