SNS COLLEGE OF TECHNOLOGY

Coimbatore-27
An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT 2 – TIME RESPONSE ANALYSIS

TOPIC 2- IMPULSE AND STEP RESPONSE ANALYSIS OF FIRST ORDER SYSTEMS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- RELATION BETWEEN STANDARD TEST SIGNALS
- LAPLACE TRANSFORM OF TEST SIGNALS
- TIME RESPONSE OF CONTROL SYSTEMS
- INTRODUCTION- FIRST ORDER SYSTEM
- IMPULSE RESPONSE OF 1^{ST} ORDER SYSTEM
- ACTIVITY
- STEP RESPONSE OF 1^{ST} ORDER SYSTEM
- RELATION BETWEEN STEP AND IMPULSE RESPONSE
- ANALYSIS OF SIMPLE RC CIRCUIT
- EXAMPLE 1
- SUMMARY

RELATION BETWEEN STANDARD TEST SIGNALS

- Impulse
- Step
- Ramp
- Parabolic

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$\int \downarrow \qquad \uparrow \frac{d}{dt}$

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\int \downarrow \qquad \uparrow \frac{d}{dt}$

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\int \downarrow \qquad \uparrow \frac{d}{dt}$

$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

LAPLACE TRANSFORM OF TEST SIGNALS

- Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$L\{\delta(t)\} = \delta(s) = A$$

- Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{u(t)\} = U(s) = \frac{A}{S}$$

LAPLACE TRANSFORM OF TEST SIGNALS

- Ramp

$$r(t) = \begin{cases} At & t \ge 0 \\ 0 & t < 0 \end{cases}$$

$$L\{r(t)\} = R(s) = \frac{A}{s^2}$$

- Parabolic

$$p(t) = \begin{cases} \frac{At^2}{2} & t \ge 0 \\ 0 & t < 0 \end{cases}$$

$$L\{p(t)\} = P(s) = \frac{A}{S^3}$$

TIME RESPONSE OF CONTROL SYSTEMS

- **Transient response depends** → system poles only & not on the type of input → To analyze the transient response using a step input.

- **The steady-state response depends** → system dynamics & the input quantity → To examine using different test signals by final value theorem.

INTRODUCTION- FIRST ORDER SYSTEM

- The first order system has only one pole.

$$\frac{C(s)}{R(s)} = \frac{K}{Ts+1}$$

- Where ***K*** is the D.C gain and ***T*** is the time constant of the system.

- Time constant is a measure of how quickly a 1st order system responds to a unit step input.

- D.C Gain of the system is ratio between the input signal and the steady state value of output.

INTRODUCTION- FIRST ORDER SYSTEM

- The first order system given below.

$$G(s) = \frac{10}{3s+1}$$

  - D.C gain is 10 and time constant is 3 seconds.

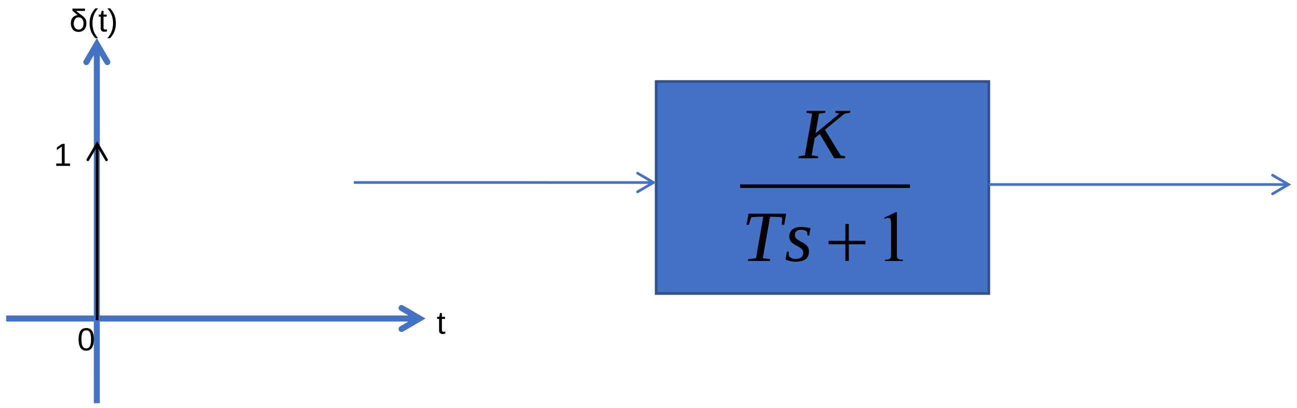
- For the following system $G(s) = \frac{3}{s+5} = \frac{3/5}{1/5s+1}$

  - D.C Gain of the system is 3/5

  - time constant is 1/5 seconds.

IMPULSE RESPONSE OF 1^{ST} ORDER SYSTEM

- Consider the following 1^{st} order system



$$R(s) = \delta(s) = 1$$

$$C(s) = \frac{K}{Ts+1}$$

IMPULSE RESPONSE OF 1ST ORDER SYSTEM

$$C(s) = \frac{K}{Ts+1}$$

- Re-arrange following equation as

$$C(s) = \frac{K/T}{s+1/T}$$

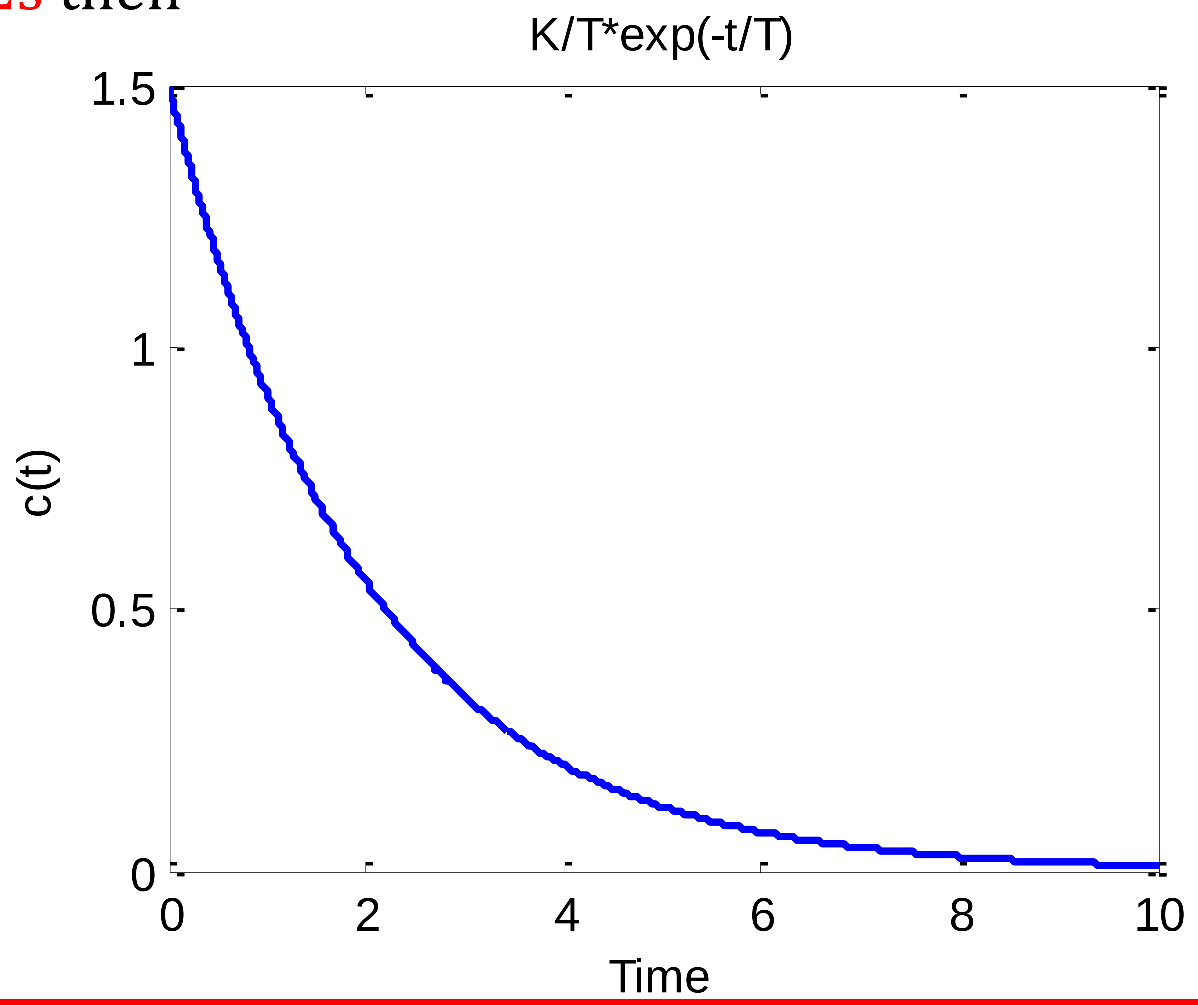
- In order to compute the response of the system in time domain we need to compute inverse Laplace transform of the above equation.

$$L^{-1}\left(\frac{C}{s+a}\right) = Ce^{-at} \qquad c(t) = \frac{K}{T}e^{-t/T}$$

IMPULSE RESPONSE OF 1ST ORDER SYSTEM

- If K=3 and T=2s then

$$c(t) = \frac{K}{T} e^{-t/T}$$



ACTIVITY

1.Tsunamis are not caused by
(a) Hurricanes
(b) Earthquakes
(c) Undersea landslides
(d) Volcanic eruptions

4.Where was the electricity supply first introduced in India
(a) Mumbai
(b) Dehradun
(c) Darjeeling
(d) Chennai

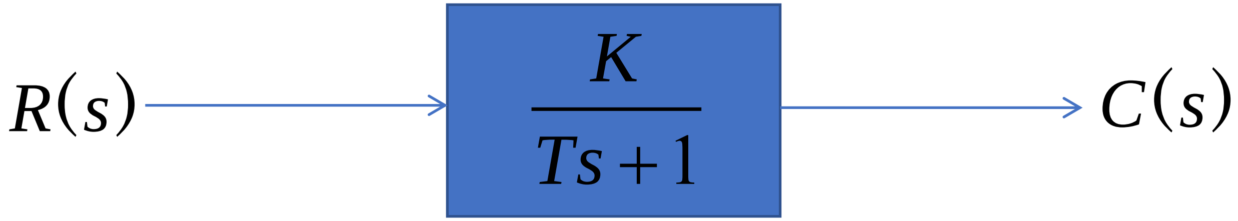
2.Professor Amartya Sen received the Nobel Prize in this field.
a) Literature
b) Electronics
c) Economics
d) Geology

5.According to Swachh Survekshan 2017 ranking of the following city in India?
1) Mysuru
2) Bhopal
3) Indore
4) Visakhapatnam (Vizag)

3.First human heart transplant operation conducted by Dr. Christian Bernard on Louis Washkansky, was conducted in
A.1958
B.1922
C.1967
D.1968

STEP RESPONSE OF 1ST ORDER SYSTEM

- Consider the following 1st order system



$$R(s) = U(s) = \frac{1}{s} \qquad C(s) = \frac{K}{s(Ts+1)}$$

- In order to find out the inverse Laplace of the above equation, we need to break it into partial fraction expansion.

$$C(s) = \frac{K}{s} - \frac{KT}{Ts+1}$$

STEP RESPONSE OF 1ST ORDER SYSTEM

$$C(s) = K\left(\frac{1}{s} - \frac{T}{Ts+1}\right)$$

- Taking Inverse Laplace of above equation

$$c(t) = K\left(u(t) - e^{-t/T}\right)$$

- Where u(t)=1

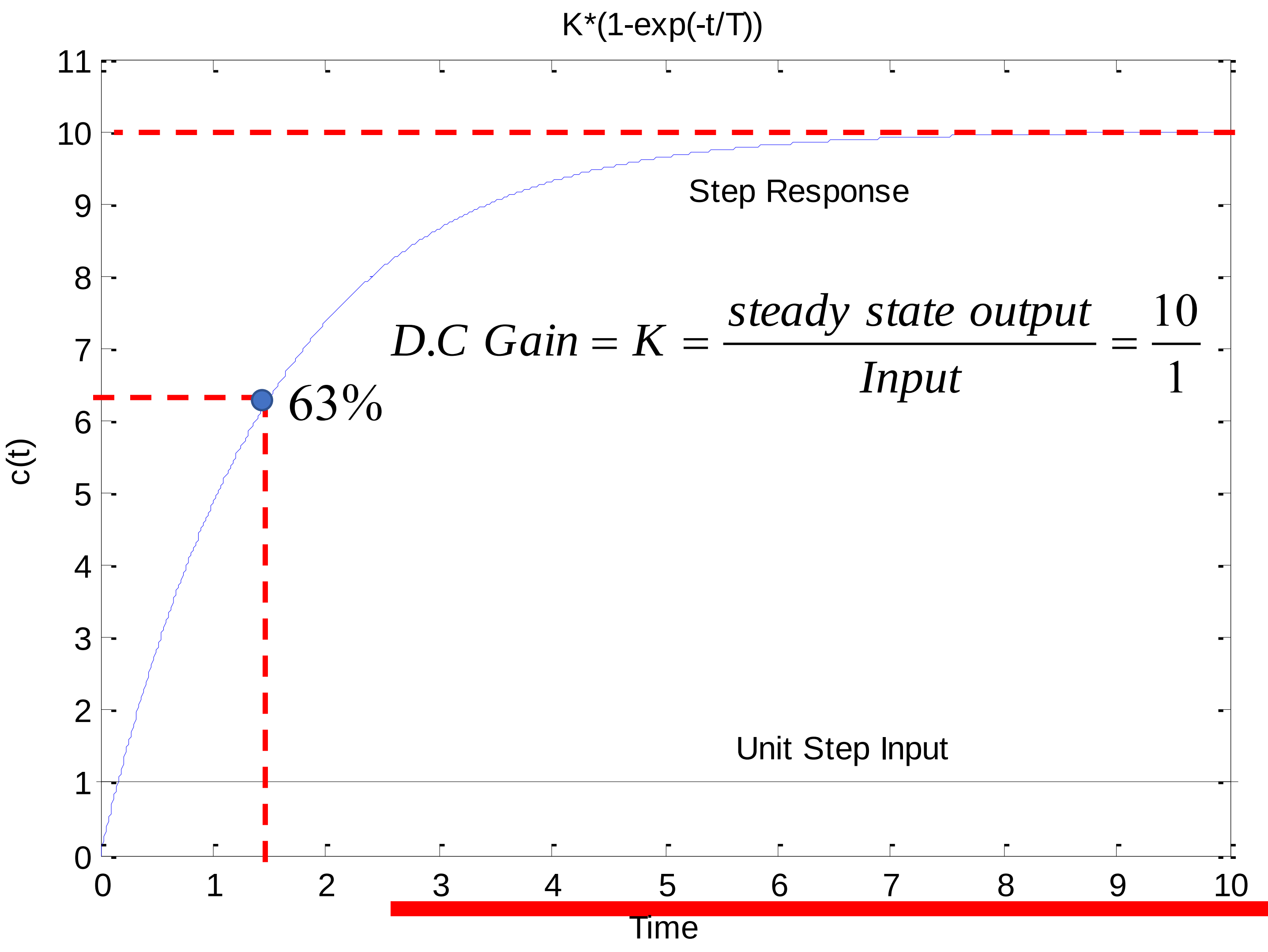
$$c(t) = K\left(1 - e^{-t/T}\right)$$

- When t=T (time constant)

$$c(t) = K\left(1 - e^{-1}\right) = 0.632K$$

STEP RESPONSE OF 1ST ORDER SYSTEM

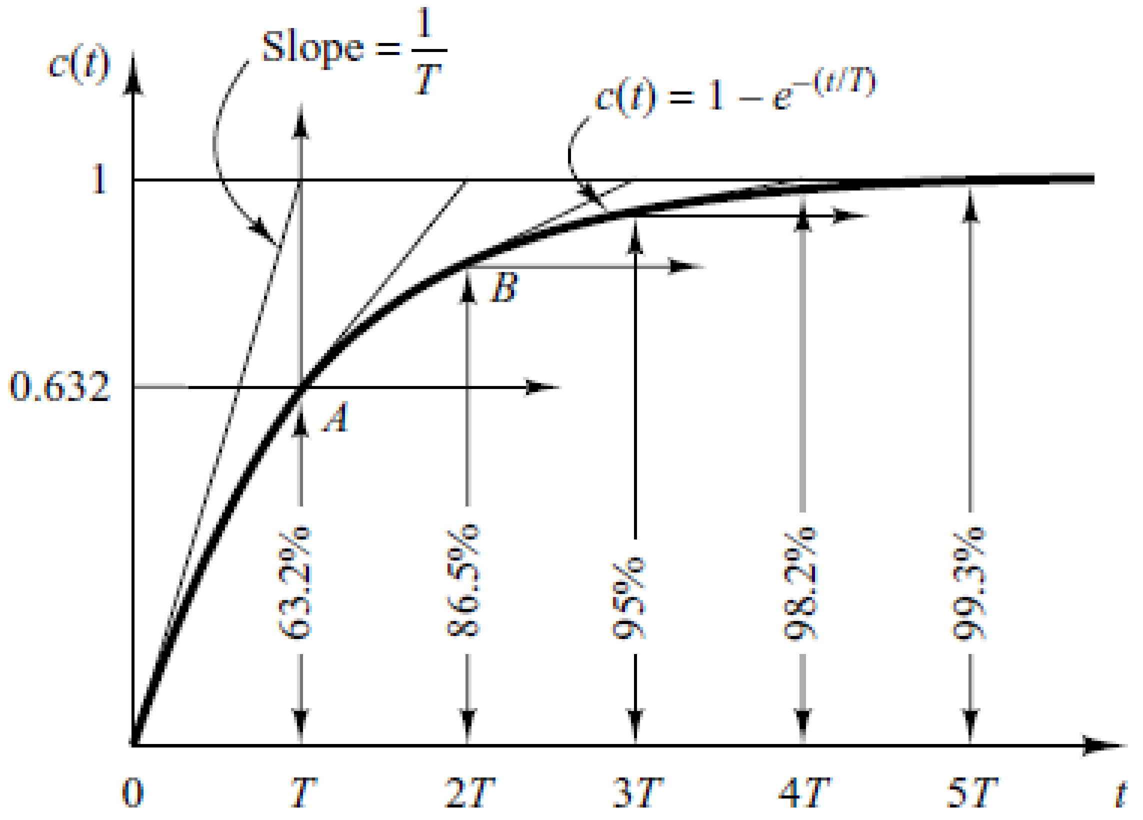
- If K=10 and T=1.5s then $c(t) = K\left(1 - e^{-t/T}\right)$



$$D.C\ Gain = K = \frac{steady\ state\ output}{Input} = \frac{10}{1}$$

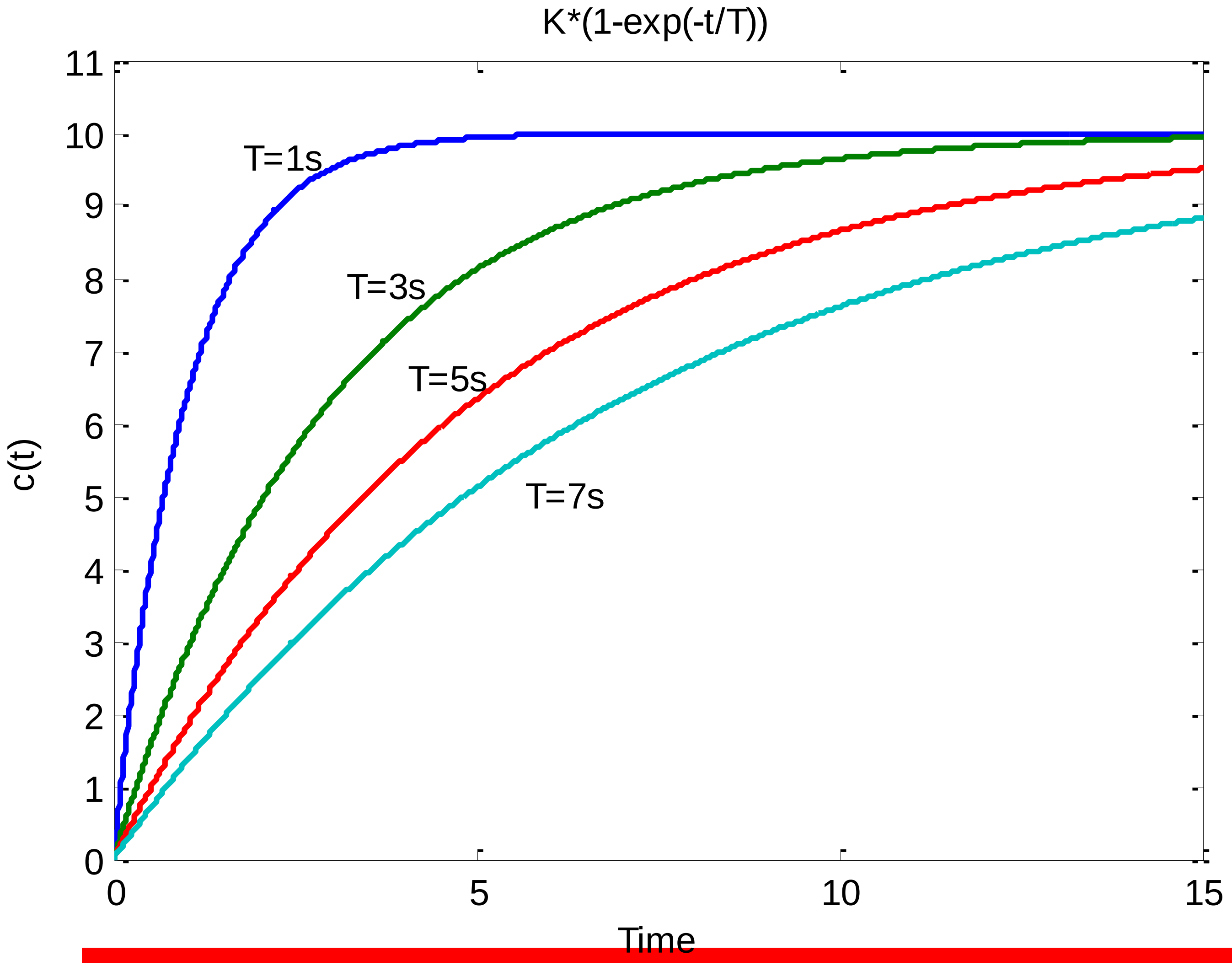
STEP RESPONSE OF 1ST ORDER SYSTEM

- System takes five time constants to reach its final value.



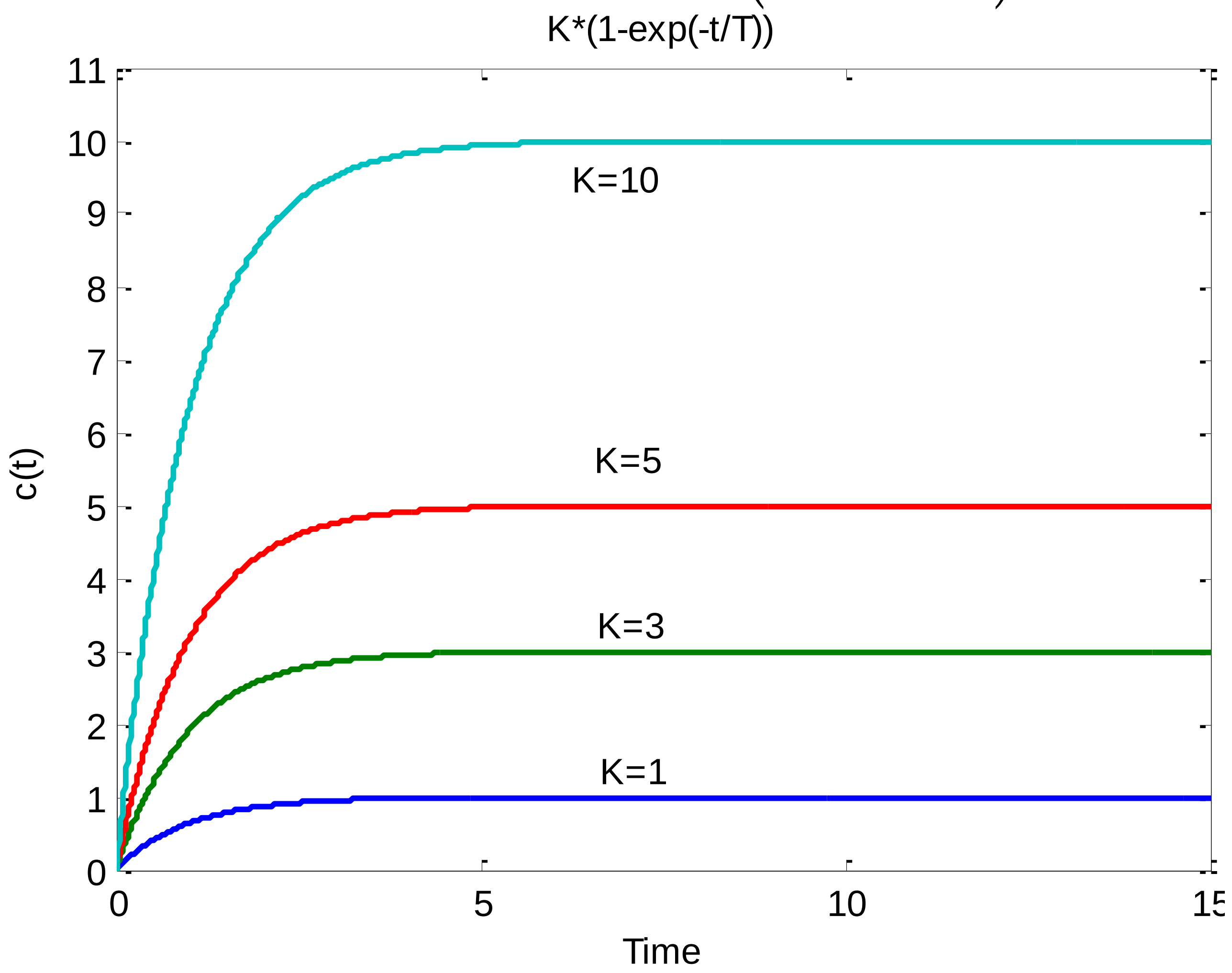
STEP RESPONSE OF 1ST ORDER SYSTEM

- If K=10 and T=1, 3, 5, 7 $c(t) = K\left(1 - e^{-t/T}\right)$



STEP RESPONSE OF 1ST ORDER SYSTEM

- If K=1, 3, 5, 10 and T=1 $c(t) = K\left(1 - e^{-t/T}\right)$



RELATION BETWEEN STEP AND IMPULSE RESPONSE

- The step response of the first order system is

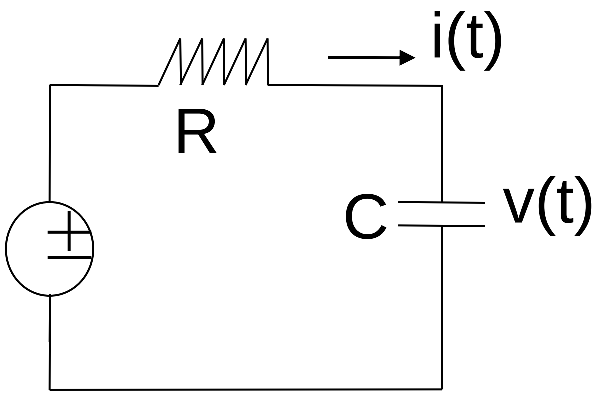
$$c(t) = K\left(1 - e^{-t/T}\right) = K - Ke^{-t/T}$$

- Differentiating c(t) with respect to t yields

$$\frac{dc(t)}{dt} = \frac{d}{dt}\left(K - Ke^{-t/T}\right)$$

$$\frac{dc(t)}{dt} = \frac{K}{T}e^{-t/T}$$

ANALYSIS OF SIMPLE RC CIRCUIT



$$R \cdot i(t) + v(t) = v_T(t)$$

$$i(t) = \frac{d(Cv(t))}{dt} = C\frac{dv(t)}{dt}$$

$$\Rightarrow RC\frac{dv(t)}{dt} + v(t) = v_T(t)$$

state variable: $v(t)$

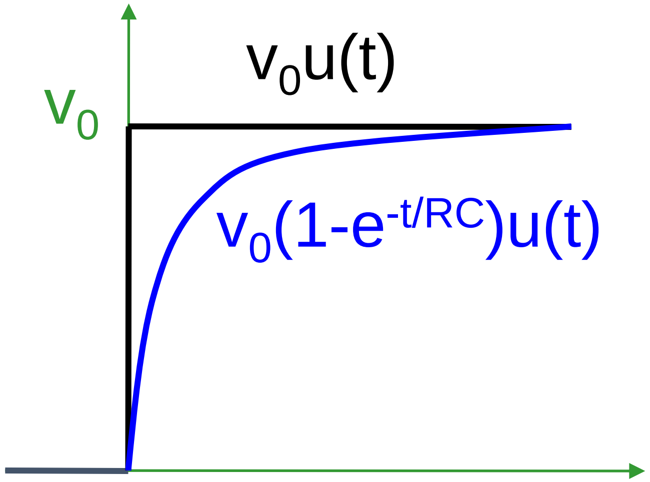
Input waveform: $v_T(t)$

ANALYSIS OF SIMPLE RC CIRCUIT

Step-input response:

$$RC\frac{dv(t)}{dt}+v(t)=v_0u(t)$$

$$v(t)=Ke^{-t/RC}+v_0u(t)$$



match initial state:

$$v(0)=0 \quad \Rightarrow \; K+v_0u(t)=0 \; \Rightarrow \; K+v_0=0$$

output response for step-input:

$$v(t)=v_0(1-e^{-t/RC})u(t)$$

ANALYSIS OF SIMPLE RC CIRCUIT

RC Circuit

- $v(t) = v_0(1 - e^{-t/RC})$ -- waveform

  under step input $v_0u(t)$

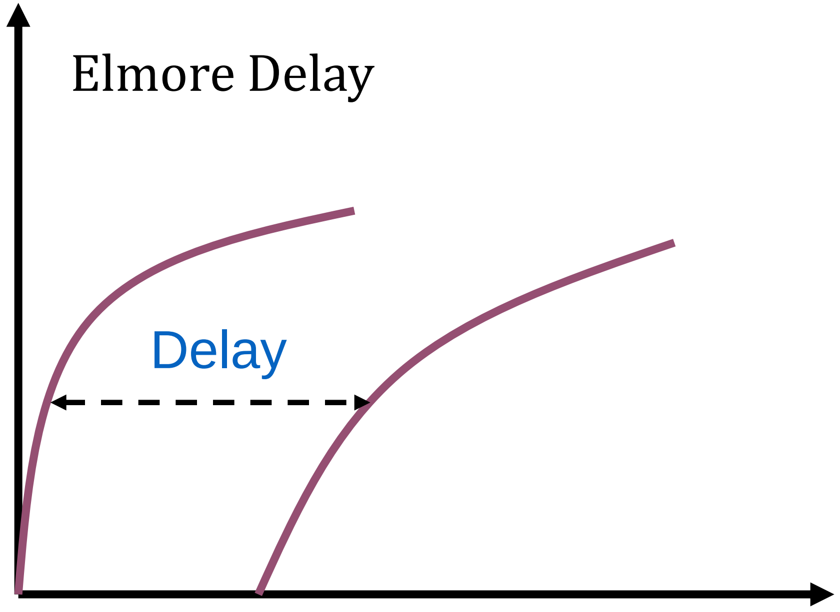
- $v(t)=0.5v_0 \Rightarrow t = 0.69RC$
  - i.e., delay = 0.69RC (50% delay)

  $v(t)=0.1v_0 \Rightarrow t = 0.1RC$

  $v(t)=0.9v_0 \Rightarrow t = 2.3RC$
  - i.e., rise time = 2.2RC (if defined as time from 10% to 90% of Vdd)
- For simplicity, industry uses

  $T_D = RC$ (= Elmore delay)

Elmore Delay



1. 50%-50% point delay
2. Delay=0.69RC

EXAMPLE 1

- **Impulse response of a 1st order system is given below.** $c(t) = 3e^{-0.5t}$
- **Find out: Time constant T, D.C Gain K, Transfer Function ,Step Response**
  - The Laplace Transform of Impulse response of a system is actually the transfer function of the system.
  - Therefore taking Laplace Transform of the impulse response given by following equation.

$$C(s) = \frac{3}{S+0.5} \times 1 = \frac{3}{S+0.5} \times \delta(s)$$

$$\frac{C(s)}{\delta(s)} = \frac{C(s)}{R(s)} = \frac{3}{S+0.5}$$

$$\frac{C(s)}{R(s)} = \frac{6}{2S+1}$$

EXAMPLE 1

- Impulse response of a 1st order system is given below.

$$c(t) = 3e^{-0.5t}$$

- Find out
  - Time constant T=2
  - D.C Gain K=6
  - Transfer Function
  - Step Response

EXAMPLE 1

- For step response integrate impulse response

$$c(t) = 3e^{-0.5t}$$

$$\int c(t)dt = 3\int e^{-0.5t}dt$$

$$c_s(t) = -6e^{-0.5t} + C$$

- We can find out C if initial condition is known e.g. $c_s(0)=0$

$$0 = -6e^{-0.5\times 0} + C$$

$$C = 6$$

$$c_s(t) = 6 - 6e^{-0.5t}$$

EXAMPLE 1

- If initial conditions are not known then partial fraction expansion is a better choice

$$\frac{C(s)}{R(s)} = \frac{6}{2S+1}$$

since $R(s)$ is a stepinput, $R(s) = \frac{1}{s}$

$$C(s) = \frac{6}{s(2S+1)}$$

$$\frac{6}{s(2S+1)} = \frac{A}{s} + \frac{B}{2s+1}$$

$$\frac{6}{s(2S+1)} = \frac{6}{s} - \frac{6}{s+0.5}$$

$$c(t) = 6 - 6e^{-0.5t}$$

SUMMARY