SNS COLLEGE OF TECHNOLOGY

Coimbatore-18
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT II – TIME RESPONSE ANALYSIS

TOPIC 7- PI COMPENSATION

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS

- P-CONTROLLER- EXAMPLE

- ROOT LOCUS

- STEADY-STATE ERROR

- GAIN MARGIN & PHASE MARGIN

- ACTIVITY

- PROPORTIONAL AND INTEGRAL(PI) CONTROLLER

- ADVANTAGES AND DISADVANTAGES OF PI CONTROLLERS.

- EXAMPLE

- THE RESPONSE OF THE SYSTEM

- SUMMARY

P-CONTROLLER- EXAMPLE

Proportional Controller (P-controller) with a unique example. With this example reader's knowledge about 'Stability' and 'Steady State Error' will also enhance. Consider the feedback control system shown in Figure

It is Transfer Function of Plant
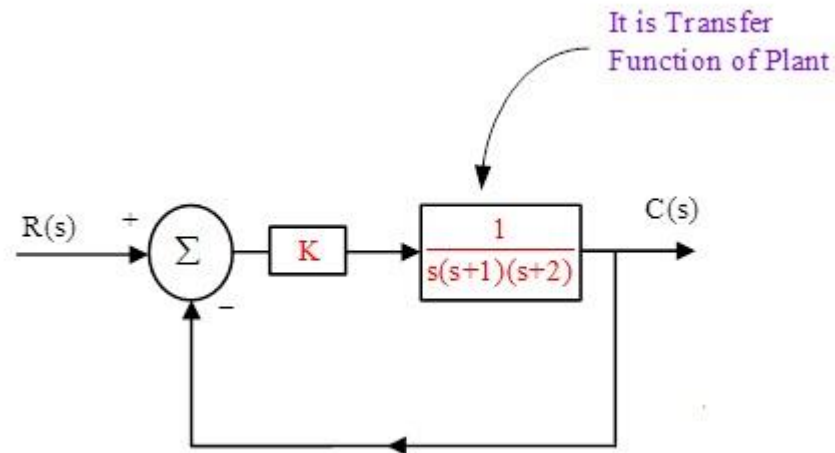
ROOT LOCUS

'K' is called a proportional controller (also called error amplifier).

Characteristics equation of this control system can be written as:

$$s^3+3s^2+2s+K=0$$

If the Routh-Hurwitz is applied in this characteristics equation, then the range of 'K' for the stability can be found as 0<K<6.
(It implies that for the values K>6 system will be unstable; for the value of K=0, the system will be marginally stable).

Root locus of the system shown in Figure. Root Locus provides an idea that what should be the value of 'K'
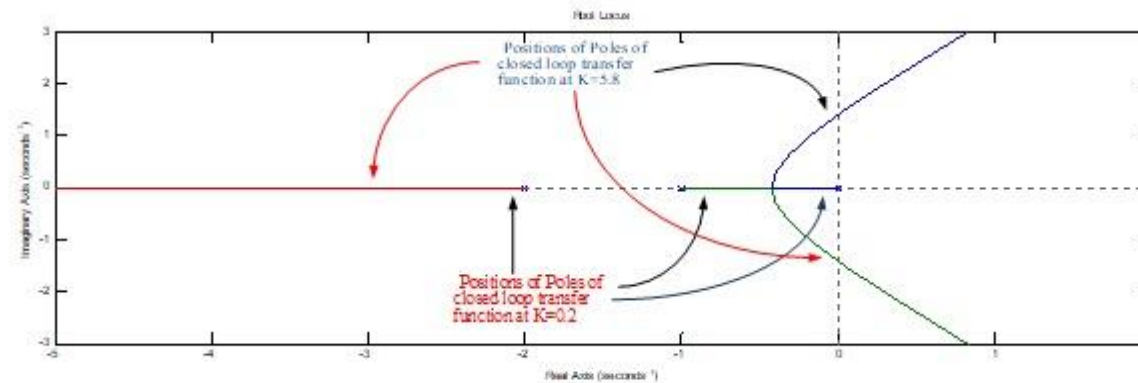
ROOT LOCUS

(You can understand that root locus is drawn for the open-loop transfer function (G(s)H(s), but it gives an idea about the poles of the closed-loop transfer function,

 i.e. Roots = zeros: In characteristics equation

 The Root locus is helpful in designing the value of 'K', i.e. gain of the proportional controller).

So, the system is stable for values such as K= 0.2, 1, 5.8 etc.; but what value we should select.

To analyze each value and show you the results.

high value of 'K' (i.e., for example, K=5.8) will reduce the stability (it is a disadvantage) but improves the steady-state performance (i.e. reduce the steady-state error, which will be an advantage).

STEADY-STATE ERROR

$K_p = \lim_{s \to 0} KG(s)H(s)$, Steady state error $(e_{ss}) = \frac{1}{1+K_p}$ (It is applicable in case of step input)

$K_v = \lim_{s \to 0} sKG(s)H(s)$, Steady state error $(e_{ss}) = \frac{1}{K_v}$ (It is applicable in case of ramp input)

$K_a = \lim_{s \to 0} s^2 KG(s)H(s)$, Steady state error $(e_{ss}) = \frac{1}{K_a}$ (It is applicable in case of parabolic input)

It can be observed that for the high value of 'K', values of Kp, Kv and Ka will be high and steady-state error will below.

Now we will take each case and explain the results

1. At K=0.2

In this case characteristics equation of the system is $s^3 + 3s^2 + 2s + 0.2 = 0$; roots of this equation are -2.088, -0.7909 and -0.1211;

Ignore -2.088 (as it is far away from imaginary axis).

On the basis of the remaining two roots, it can be termed as an over damped system (as both the roots are real & negative, no imaginary parts).

Against step input, its time response is shown in Fig. It can be seen that the response has no oscillations. (if roots are complex then time response exhibits oscillations). The over damped system has damping more than '1'.
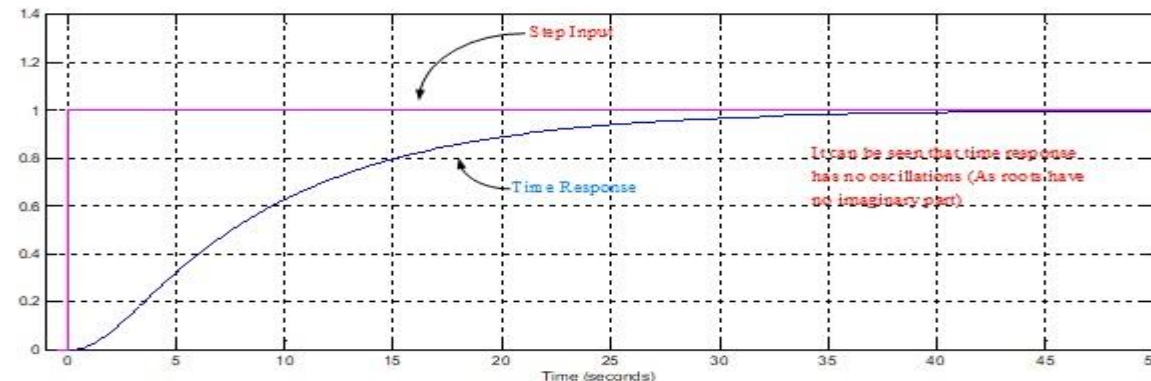
GAIN MARGIN & PHASE MARGIN

In the present case open loop transfer function is
Its Gain Margin (GM)=29.5 dB, Phase Margin (PM)=81.5°

$$G(s)H(s) = \frac{0.2}{s(s+1)(s+2)}$$

It should be noted that in the designing of control systems, overdamped systems are not preferred.

Roots (poles of closed-loop transfer function) should have slight imaginary parts. In the case of over damped, damping is more than '1', while damping around 0.8 is preferred.

https://www.youtube.com/watch?v=eeKn5dkkoj8- Gain Margin

https://www.youtube.com/watch?v=RcGLWrAHh_w- Phase Margin

2. At K=1

In this case characteristics equation of the system is $s^3 + 3s^2 + 2s + 1 = 0$; roots of this equation are -2.3247, -0.3376 ±j0.5623;

We can ignore -2.3247.

On the basis of the remaining two roots, it can be termed as an under damped system (as both the roots are complex having negative real parts).
Against step input, its time response is shown in Fig.
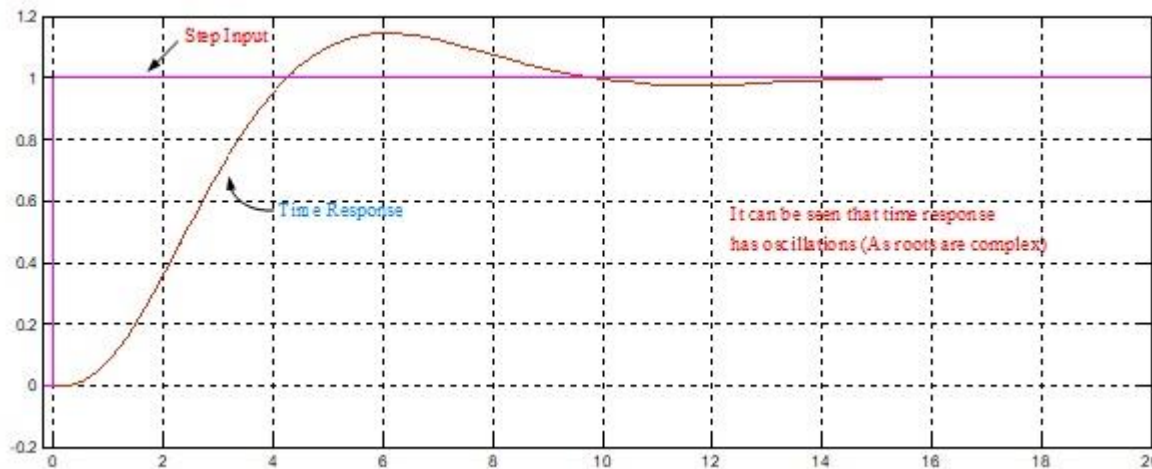
$$G(s)H(s) = \frac{1}{s(s+1)(s+2)}$$

In the present case open loop transfer function is given above
Its Gain Margin (GM)=15.6 dB,
Phase Margin (PM)=53.4°,

Response has oscillations, it is the response of an under damped system

3. At K=5.8

As 5.8 is very close to 6, so you can understand that system is stable, but almost on the border.

You can find the roots of its characteristics equation.

One root can be ignored, the remaining two roots will be very close to the imaginary axis. (Roots of its characteristics equation will be -2.9816, -0.0092±j1.39).

Against step input, its time response is shown in Fig

Figure. Response has oscillations, it is the response of the under damped system (Response in Figureis also belongs to the under damped system)
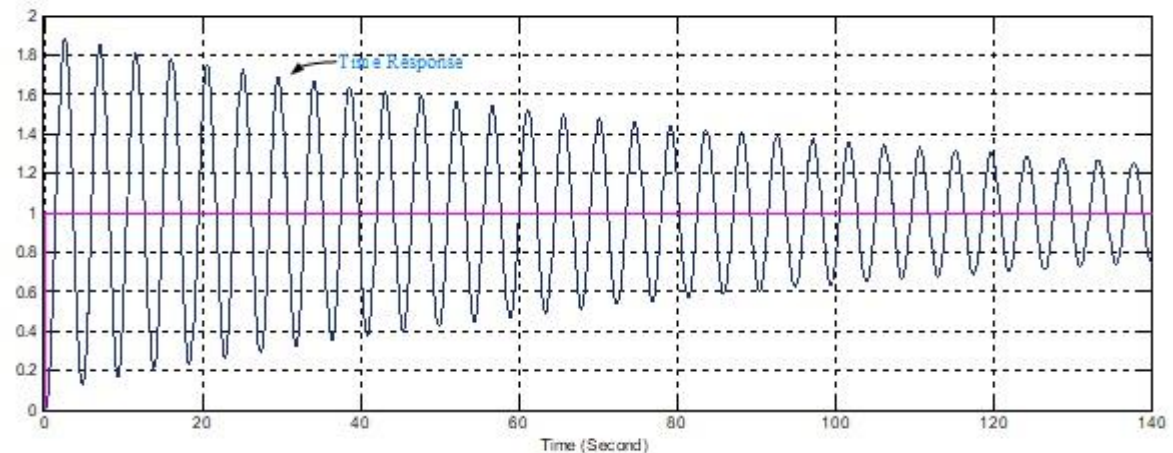
In the present case open loop transfer function is

$$G(s)H(s) = \frac{5.8}{s(s+1)(s+2)}$$

Its Gain Margin=0.294 db, Phase Margin =0.919°

It can be analyzed, as compared to the previous cases, GM & PM are reduced drastically. As the system is very close to instability, hence GM & PM are also very close to zero value.

PROPORTIONAL AND INTEGRAL(PI) CONTROLLER

As the name suggests it is a combination of proportional and an integral controller the output (also called the actuating signal) is equal to the summation of proportional and integral of the error signal.

Now let us analyze proportional and integral controller mathematically.

output is directly proportional to the summation of proportional of error and integration of the error signal, writing this mathematically,

$$A(t) \propto \int_0^t e(t)dt + A(t) \propto e(t)$$

Removing the sign of proportionality

$$A(t) = K_i \int_0^t e(t)dt + K_p e(t)$$

Where, K_i and k_p proportional constant and integral constant respectively.

ADVANTAGES AND DISADVANTAGES OF PI CONTROLLERS.

Through the PI controller, we are adding one pole at origin and one zero somewhere away from the origin (in the left-hand side of complex plane).

As the pole is at the origin, its effect will be more, hence PI controller may reduce the stability;

it reduces steady-state error drastically, due for this reason it is one of the most widely used controllers.

EXAMPLE

The schematic diagram of the PI controller is shown in Fig. 1 Against step input, For the values of K=5.8, K_i=0.2, Its time response, is shown in Fig.2 , At K=5.8 (As a P- controller, it was on the verge of instability, so just by adding the small value of an Integral part, it became unstable.

Please note the Integral part reduces the stability, which does not mean that system will be always unstable. In the present case, we have added an integral part and the system became unstable).
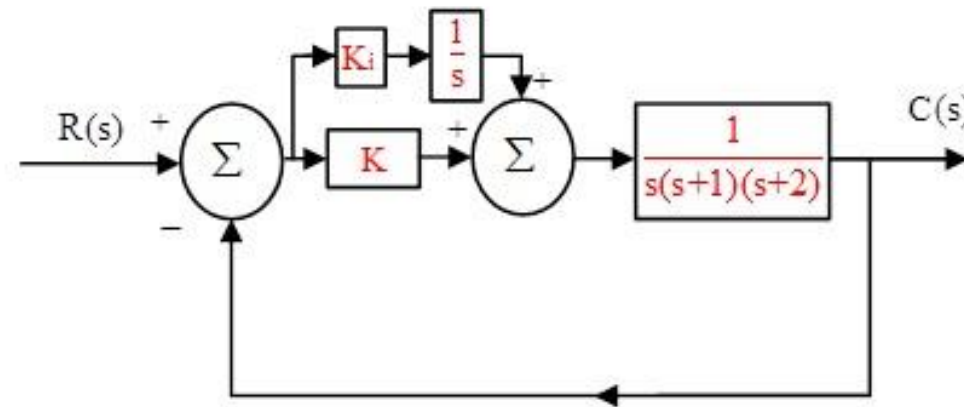

Figure-1: The closed loop control system with PI Controller

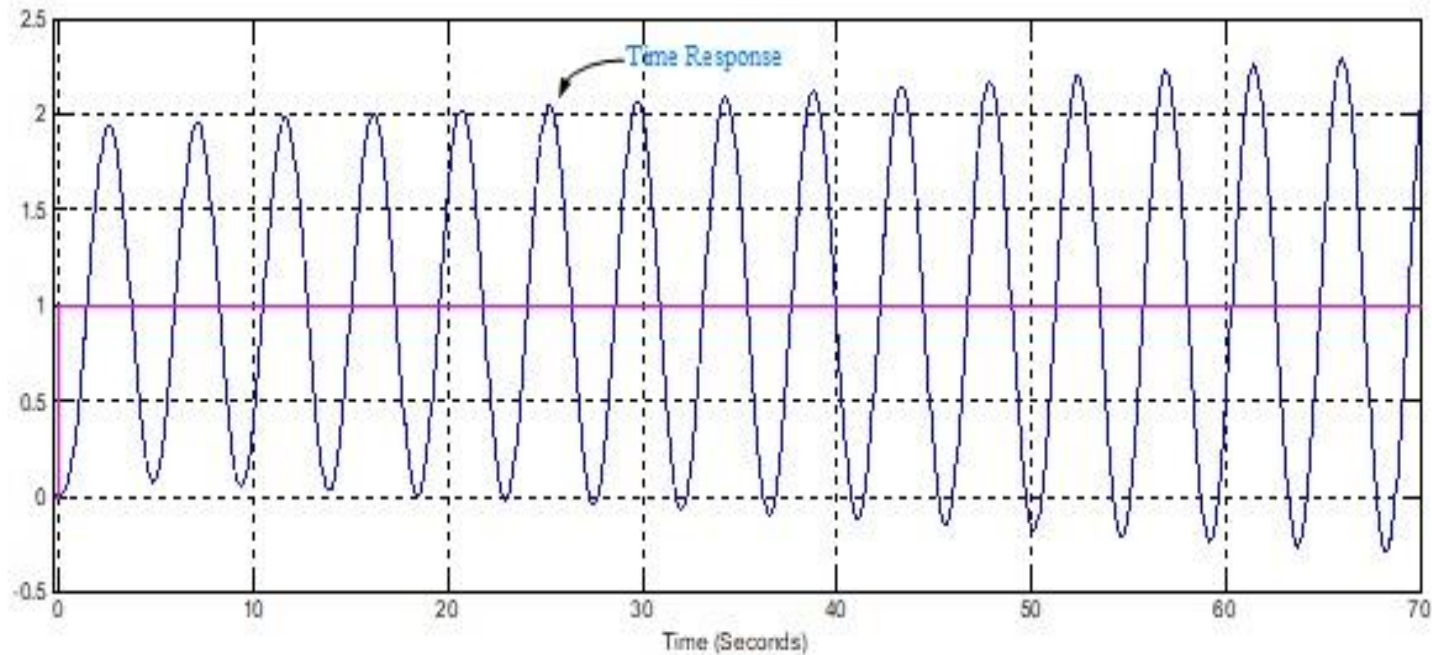# THE RESPONSE OF THE SYSTEM SHOWN IN FIG.-2, WITH K=5.8, KI=0.2

Figure-2

SUMMARY