Finite Element Analysis

Objective:

The analysis need certain requirement while designing and assemble of the parts.

TO Find:

1. Displacement of certain point
2. Stress distribution.
3. Natural frequency
4. critical buckling load
5. Vibrations
6. Pressure, velocity and Temperature
7. crack growth, residual strength, fatique life.

METHODS OF ENGINEERING ANALYSIS:

1. Experimental method
2. Analytical method (or) theorical Analysis.
3. Numerical Methods.
   (i) functional approximation
   (ii) finite difference method
   (iii) finite Element (or) Analysis method.

① $A = \begin{bmatrix} -1 & 3 & -2 \\ 2 & -4 & 2 \\ 0 & 4 & 1 \end{bmatrix}$ Find the $|A|$.

Sol:

$$|A| = -1(-4-8) - 3(2-0) - 2(8-0)$$

$$= 12 - 6 - 16$$

$$|A| = -10$$

② $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 4 & 0 \\ 2 & 0 & 3 \end{bmatrix}$. Find cofactors of matrix A.

Sol:

$$a_{11} = \begin{vmatrix} 4 & 0 \\ 0 & 3 \end{vmatrix} = 12 - 0 = 12$$

$$a_{12} = -\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = -(3-0) = -3$$

$$a_{13} = \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} = (0-8) = -8$$

$$a_{21} = -\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = -(3-0) = -3$$

$$a_{22} = +\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = +(9-4) = +5$$

$$a_{23} = -\begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} = -(0-2) = +2$$

$$a_{31} = \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} = (0-8) = -8$$

$$a_{32} = -\begin{vmatrix} 3 & 2 \\ 1 & 0 \end{vmatrix} = -(0-2) = 2$$

$$a_{33} = \begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix} = (12-1) = 11$$

$\therefore C = \begin{bmatrix} 12 & -3 & -8 \\ -3 & 5 & 2 \\ -8 & 2 & 11 \end{bmatrix}$

3) Gauss Elimination method :-

$3x+y-z = 3$

$2x-8y+z = -5$

$x-2y+9z = 8$

Sol:-

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 9 & 8 \\ 2 & -8 & 1 & -5 \\ 3 & 1 & -1 & 3 \end{array}\right] \quad R_3 \leftrightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 9 & 8 \\ 0 & 4 & 17 & 21 \\ 0 & -7 & 28 & 21 \end{array}\right] \quad \begin{array}{l} R_2 \to (R_1 \times 2) - R_2 \\ R_3 \to (R_1 \times 3) - R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 9 & 8 \\ 0 & 1 & 17/4 & 21/4 \\ 0 & -7 & 28 & 21 \end{array}\right] \quad R_2 \to R_2/4$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 9 & 8 \\ 0 & 1 & 17/4 & 21/4 \\ 0 & 0 & 57.75 & 57.75 \\ & & & \cancel{40.25} \end{array}\right] \quad R_3 \to (R_2 \times 7) + R_3$$

By solving,

$$57.75z = 57.75$$

$$z = \frac{57.75}{57.75} = 1$$

$$y + \frac{17}{4}z = \frac{21}{4}$$

$$y = \frac{21}{4} - \frac{17}{4}(1)$$

$$y = 1$$

$x - 2y + 9z = 8$

$x = 1$

RESULT :

$x = 1, y = 1, z = 1.$

$2x_1 + 4x_2 + 2x_3 = 15$, $2x_1 + x_2 + 2x_3 = -5$,

$4x_1 + x_2 - 2x_3 = 0$. By using hauss elemination method.

Soln:

Matrix.

$$\left[\begin{array}{ccc|c} 2 & 4 & 2 & 15 \\ 2 & 1 & 2 & -5 \\ 4 & 1 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & 1 & -2 & 0 \\ 2 & 1 & 2 & -5 \\ 2 & 4 & 2 & 15 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1/4 & -2/4 & 0 \\ 2 & 1 & 2 & -5 \\ 2 & 4 & 2 & 15 \end{array}\right] \quad R_1 \to R_1/4$$

$$\left[\begin{array}{ccc|c} 1 & 1/4 & -2/4 & 0 \\ 0 & 2/4 & 3 & -5 \\ 0 & 14/4 & 3 & 15 \end{array}\right] \quad \begin{array}{l} R_2 \to (R_1 \times -2) + R_2 \\ R_3 \to (R_1 \times -2) + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1/4 & -2/4 & 0 \\ 0 & 1 & 6 & -10 \\ 0 & 0 & -8 & 50 \end{array}\right] \quad \begin{array}{l} R_2 \to (R_2 \times 4/2) \\ R_3 \to (R_2 \times \frac{4}{2} \times \frac{14}{4}) - \end{array}$$

$$-18 x_3 = 50$$

$$x_3 = -2.77$$

$$x_2 + 6x_3 = -10$$

$$x_2 = -10 + 16.662$$

$$x_2 = 6.662$$

$$x_1 + 1/4\, x_2 - 2/4\, x_3 = 0$$

$$x_1 + 1.665 + 1.3885 = 0$$

$$x_1 = -3.0535$$

Ans:-

$$x_1 = -3.0535$$

$$x_2 = 6.662$$

$$x_3 = -2.77.$$

3. $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 2 & 3 \\ 1 & 3 \end{bmatrix}$

Find $A \times B$?

Sol:-

$$A \times B = \begin{bmatrix} 3+6+2 & 5+9+6 \\ 6+8+3 & 10+12+9 \end{bmatrix} = \begin{bmatrix} 11 & 20 \\ 17 & 31 \end{bmatrix}.$$

General steps of finite Element Analysis :-

1. Force method
2. Displacement or stiffness method