4. The following differential equation is available for a physical phenomenon

$$\frac{d^2y}{dx^2} + 50 = 0, \quad 0 \le x \le 10$$

Trial

Trial function is $y = a_1 x(10-x)$.

Boundary conditions are $y(0) = 0$, $y(10) = 0$

find the value of the parameter a_1 by the following methods (1) Point co location metod (2) sub domain colocation method (3) least squares method (4) galerking method.

Sol:-

$$y = a_1 x(10-x)$$

$x = 0$; $y = 0$

$x = 10$; $y = 0$

Hence, it satisfies the boundry conditions.

1. Point co-location method.

$$y = a_1 x(10-x)$$

$$y = 10a_1 x - a_1 x^2$$

$$dy/dx = 10a_1 - 2a_1 x.$$

$$\boxed{\frac{d^2y}{dx^2} = -2a_1}$$

Residual $R = \frac{d^2y}{dx^2} + 50$

$$R = -2a_1 + 50$$

In point co-location method residual $R=0$

$$0 = -2a_1 + 50$$

$$-2a_1 = -50$$

$$\boxed{a_1 = 25}.$$

a_1 value in (1)

$$y = a_1 x (10-x) \Rightarrow 25x(10-x)$$

2) sub domain co-location method.

$$\int_0^{10} R\,dx = 0.$$

$$\int_0^{10} (-2a_1 + 50)\,dx = 0.$$

$$-2a_1 (x)_0^{10} + 50 (x)_0^{10} = 0.$$

$$-2a_1 (10-0) + 50(10-0) = 0$$

$$-20a_1 + 500 = 0$$

$$-20a_1 = 500$$

$$\boxed{a_1 = 25}.$$

a_1 value in (1)

$$y = a_1 x (10-x) \Rightarrow 25x(10-x).$$

(3) Least square's method

$$I = \int_0^{10} R^2\,dx$$

$$\frac{\partial I}{\partial a_1} = \int_0^{10} R \frac{\partial R}{\partial a_1} \, dx.$$

$$R = -2a_1 + 50$$

$$\partial R/\partial a_1 = -2.$$

$$\partial I/\partial a_1 = \int_0^{10} R(-2)\, dx.$$

$$= \int_0^{10} (-2a_1 + 50)(-2)\, dx$$

$$= \int_0^{10} (4a_1 - 100)\, dx.$$

$$= 4a_1 (x)_0^{10} - 100(x)_0^{10}$$

$$\partial I/\partial a_1 = 40a_1 - 1000$$

$$0 = 40a_1 - 1000$$

$$40a_1 = 1000$$

$$\boxed{a_1 = 25}$$

$a_1 = 25$ in (9).

$$y = 25x(10-x)$$

(4). Galerking Method :-

$$\int_0^{10} \omega_i R \, dx = 0$$

$$\omega_i = y = \text{Trial fn}$$

$$\int_0^{10} a_1 x(10-x) \times (-2a_1+50) \times dx = 0$$

$$\int_0^{10} (10a_1 x - a_1 x^2)(-2a_1+50)\, dx = 0$$

$$\int_0^{10} \left[-20a_1^2 x + 500a_1 x + 2a_1^2 x^2 - 50a_1 x^2\right] dx = 0$$

$$-20a_1^2 \left[\frac{x^2}{2}\right]_0^{10} + 500a_1 \left[\frac{x^2}{2}\right]_0^{10} + 2a_1^2 \left[\frac{x^3}{3}\right]_0^{10} - 50a_1 \left[\frac{x^3}{3}\right]_0^{10} = 0$$

$$-20a_1^2 [50] + 500a_1 [50] + 2a_1^2 [333.33] - 50a_1 [333.33] = 0$$

$$-1000a_1^2 + 25000a_1 + 666.66a_1^2 - 16666.66a_1 = 0$$

$$a_1 [25000 - 16666.66] + a_1^2 [-1000 + 666.66] = 0$$

$$8333.34a_1 - 333.34a_1^2 = 0$$

$$8333.34a_1 = 333.34a_1^2$$

$$a_1 = \frac{8333.34}{333.34}$$

$$a_1 = 24.999 \simeq 25$$

$a_1 = 25$ in (y)

$$y = 25x(10-x),$$