③ The differential equation of a Physical Phenomenon is given by,

$$\frac{d^2y}{dx^2} + 500x^2 = 0, \quad 0 \le x \le 1$$

Trial function, $y = a_1(x - x^4)$

Boundry conditions are, $y(0) = 0$,

$y(1) = 0$.

Calculate the value of the Parameter a_1 by the following methods

(i) Point collocation ; (ii) subdomain collocation ; (iii) Least square

(iv) Galerkin. $\boxed{a_1 = 41.66}$

④ Find the eigen values of $A = \begin{pmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{pmatrix}$

The characteristic equation is,

$$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0.$$

$a_1 =$ sum of the leading diagonal elements

$= 4 + 10 - 13 = 1$

$a_2 =$ sum of minors of the leading diagonal elements

$$= \begin{vmatrix} 10 & 4 \\ -30 & -13 \end{vmatrix} + \begin{vmatrix} 4 & -10 \\ 6 & -13 \end{vmatrix} + \begin{vmatrix} 4 & -20 \\ -2 & 10 \end{vmatrix}$$

$= -130 + 120 - 52 + 60 + 40 - 40$

$\boxed{\begin{matrix} 122 \\ 536 \\ 121 \end{matrix}}$

$a_2 = -2$.

$$a_3 = |A| = \begin{vmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{vmatrix}$$

$a_3 = 0$.

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$$\lambda^3 - \lambda^2 - 2\lambda = 0.$$

To find eigen values

$$\lambda^3 - \lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda^2 - \lambda - 2) = 0$$

when $\lambda = 0$, $\quad \lambda^2 - \lambda - 2 = 0$

$$\lambda = \frac{1 \pm \sqrt{1 - (4 \times 1 \times -2)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{1+8}}{2}$$

$$= \frac{1 \pm 3}{2} = 2, -1$$

Eigen values : $0, -1, 2$

Rayleigh Ritz method :-

Rayleigh - Ritz method is an integral approach method which is useful for solving complex structural problems, encountered in finite element analysis.

For continuous system,

$$\text{Potential energy, } \pi = \int_{x_1}^{x_2} f(y, y', y'')\, dx$$

Total potential energy of the structure is,

$$\pi = \left\{ \begin{array}{c} \text{Internal} \\ \text{Potential} \\ \text{energy} \end{array} \right\} - \left\{ \begin{array}{c} \text{External} \\ \text{Potential} \\ \text{energy} \end{array} \right\}$$

$$\left\{ U = \frac{EI}{2} \int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \right.$$

$$H = \int_0^l w y \, dx.$$

= Strain energy - Work done by external forces.

The general exact function can be represented as a polynomial or trignometric series with undetermined constants

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

or

$$y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l} + \dots$$

The constants a_0, a_1, a_2 are unknowns known as Ritz parameters of the curve.

The following two conditions must be fulfilled by the approximating function.

1. It should satisfy the geometric boundary conditions
2. The function must have atleast one Ritz parameter.

Formula's used

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$\sin n\pi = 0$

diff

$\sin x = \cos x$ (1)

$\cos x = -\sin x$ (1)

int

$\sin x = -\frac{\cos x}{x}$

$\cos x = \frac{\sin x}{x}$