# SNS COLLEGE OF TECHNOLOGY

**An Autonomous Institution**

**Coimbatore-35**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

# DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

## 19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

## UNIT 1 – DISCRETE FOURIER TRANSFORM

TOPIC – Introduction to DFT

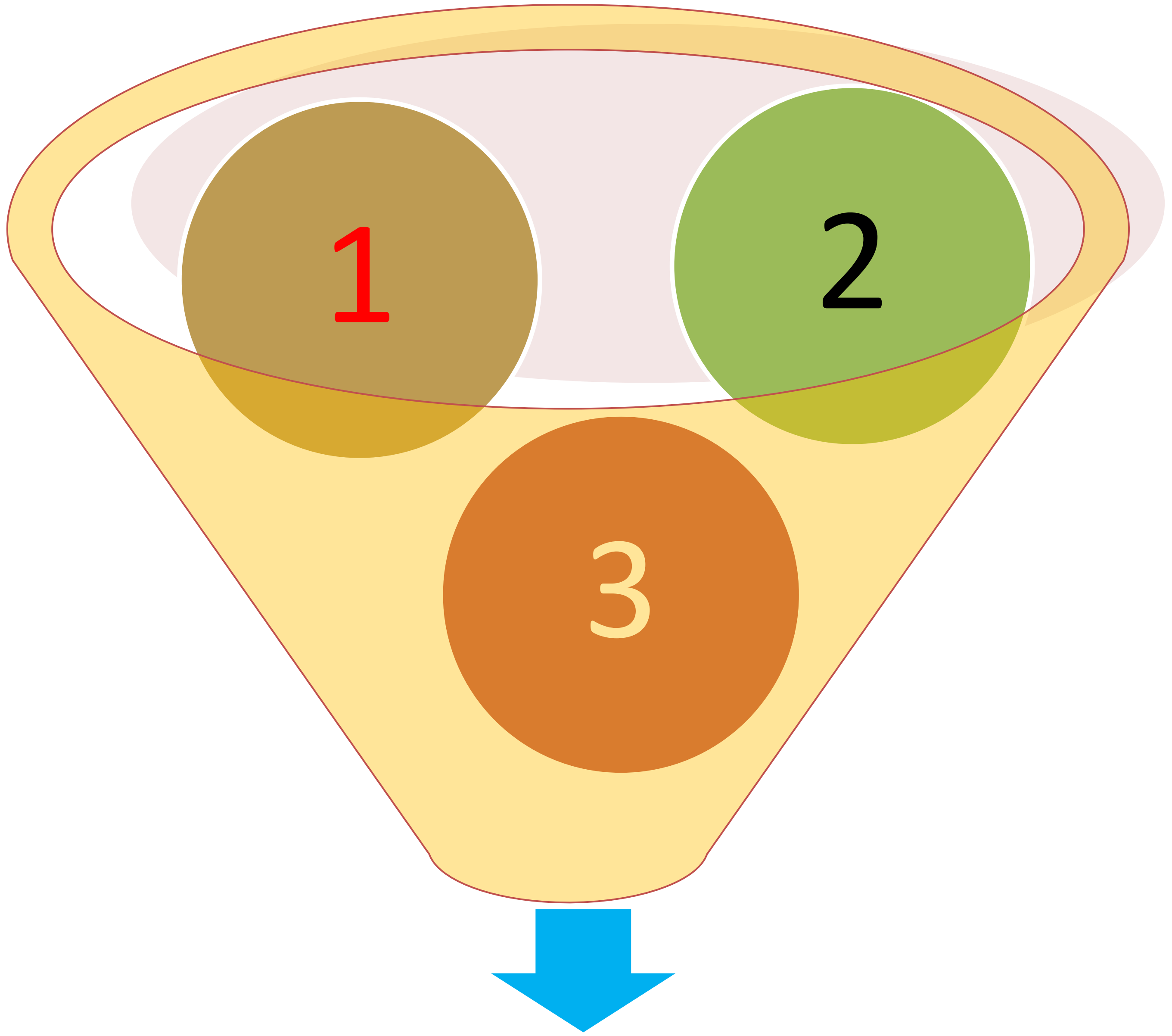
# EMPATHY

1
- Defects in signals is to identified

2
- Conversion from Time domain to frequency domain takes longer time

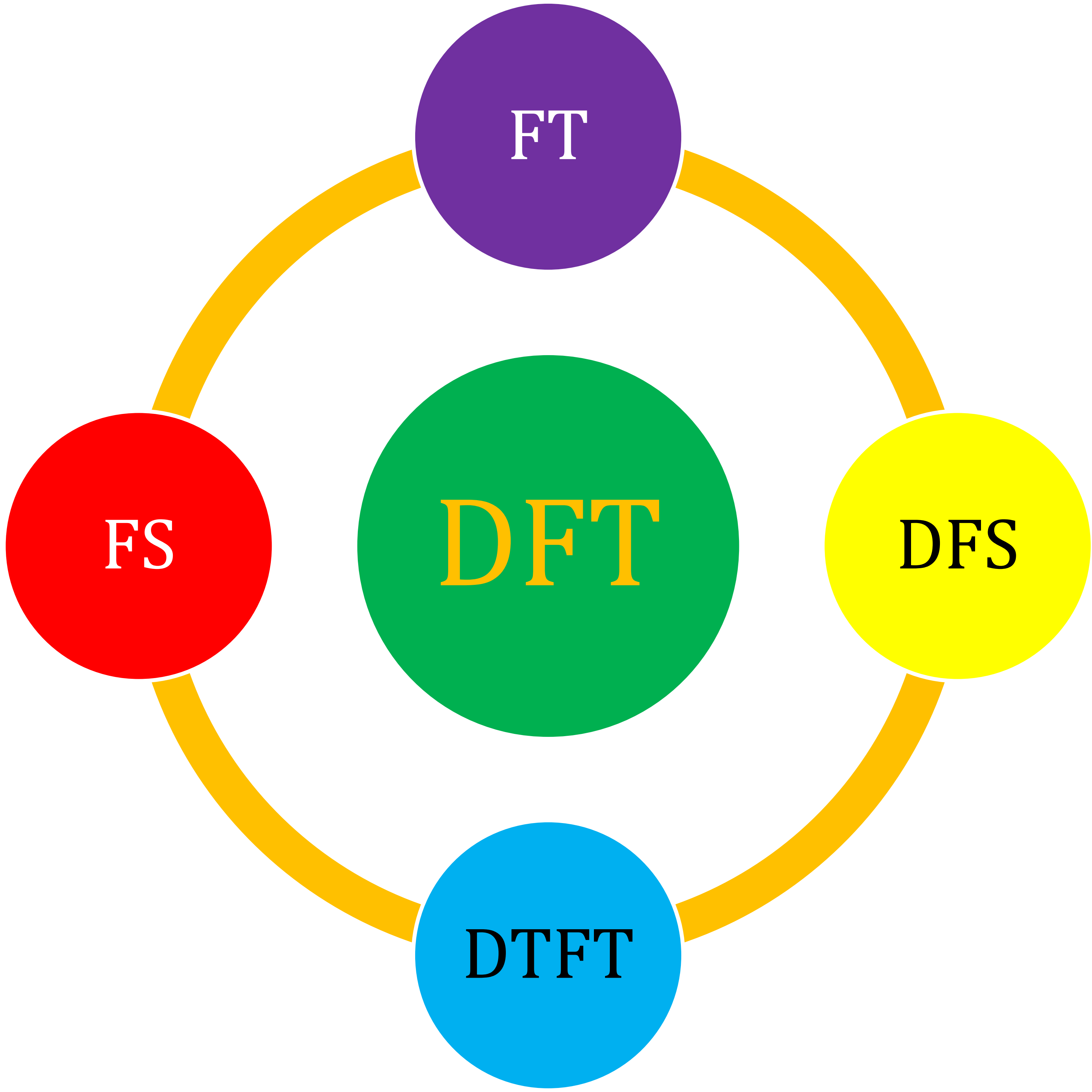
3
- Frequency domain Information must be extracted

# DEFINE

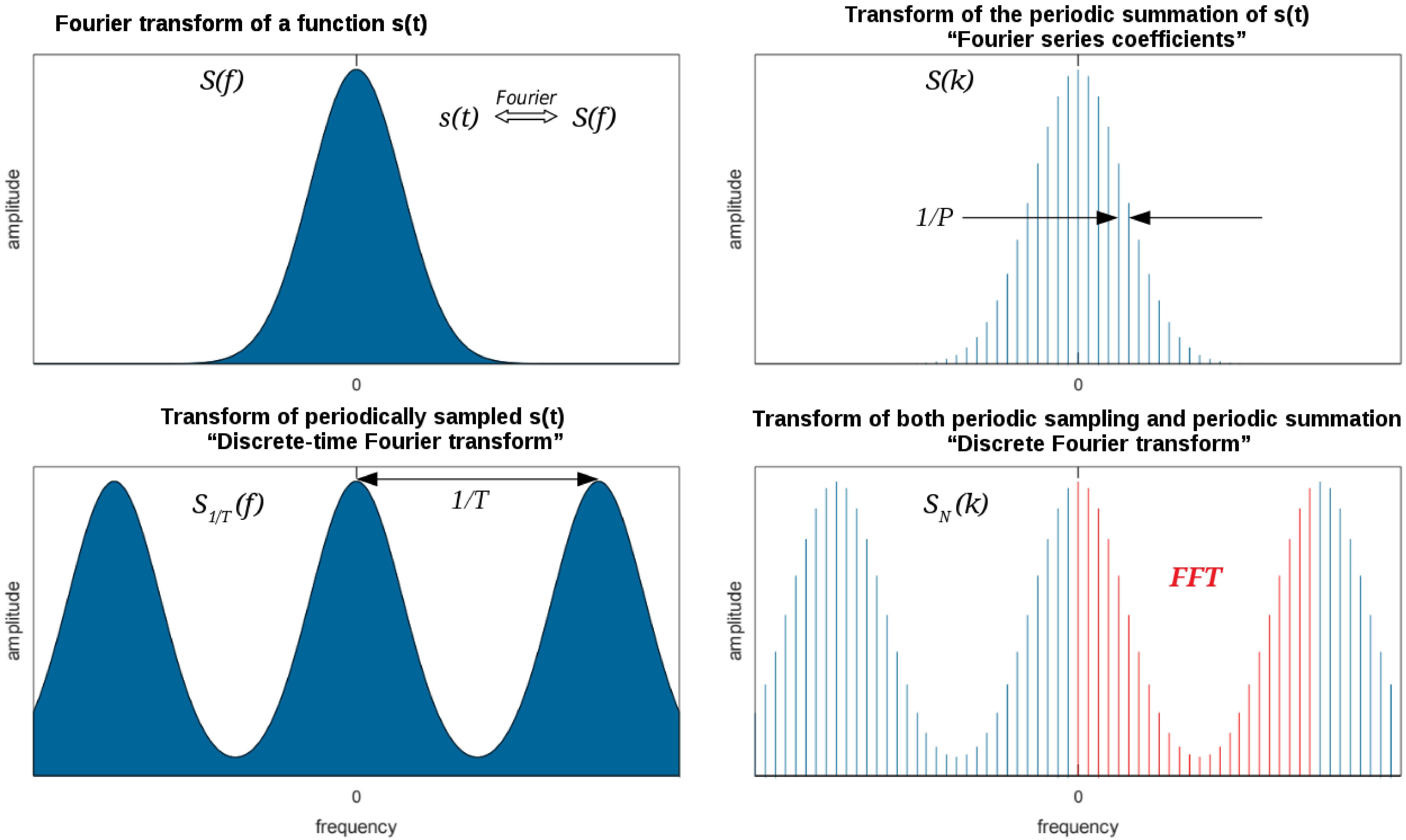



Time to frequency domain conversion and process faster

# IDEATE



# FOURIER COEFFICIENTS REPRESENTATION



# DISCRETE FOURIER TRANSFORM

For a discrete time sequence we define two classes of Fourier Transforms:

- The *DTFT (Discrete Time FT) for sequences having **infinite** duration,*
- The *DFT (Discrete FT) for sequences having **finite** duration.*

# DTFT AND INVERSE DTFT

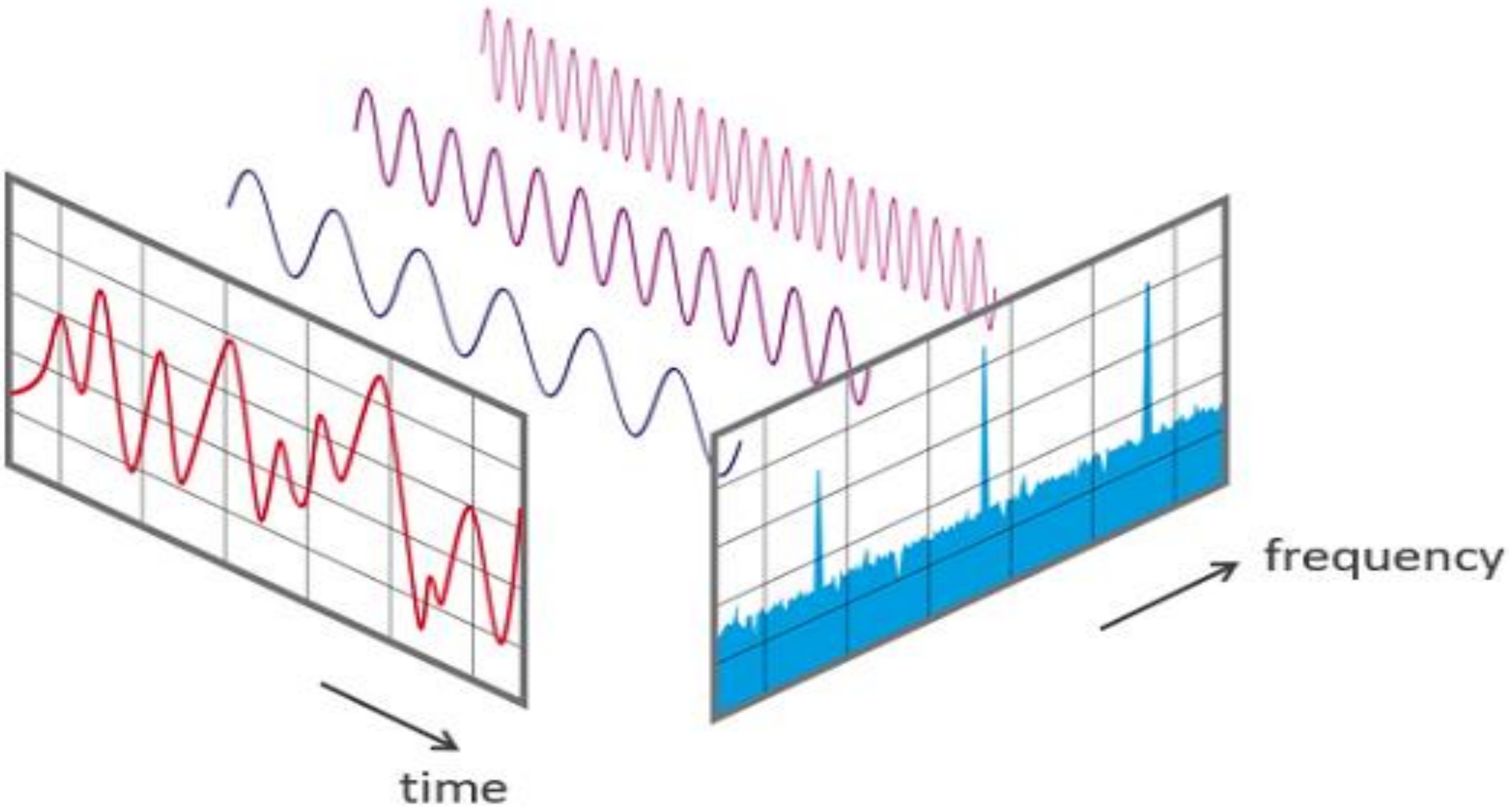
**DTFT**

$$X(\omega) = DTFT\{x(n)\} = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$$

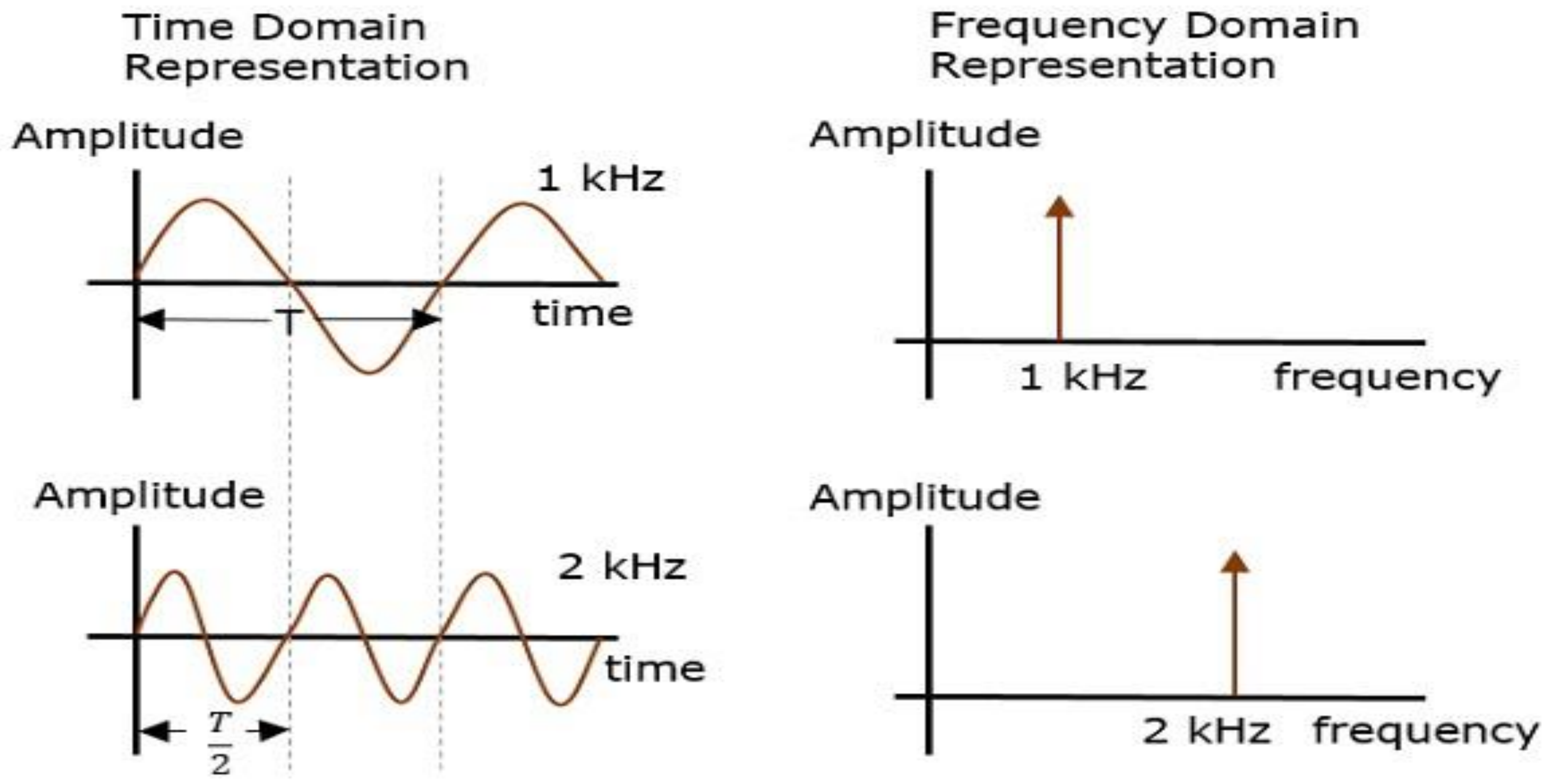
**Inverse DFT**

$$x(n) = IDTFT\{X(\omega)\} = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\omega)e^{j\omega n}d\omega$$

# DISCRETE FOURIER TRANSFORM



# REPRESENTATION OF SIGNALS



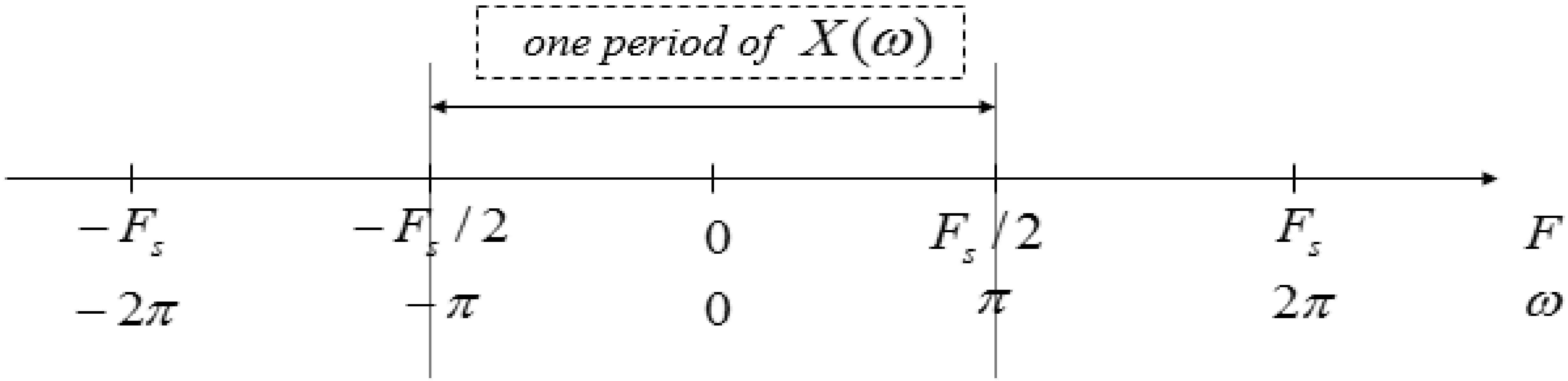
# DISCRETE FOURIER TRANSFORM

- *The DTFT **X(ω)** is periodic with period **2π***
- *The frequency **ω** is the digital frequency and therefore it is limited to the interval*

$$-\pi < \omega < +\pi$$

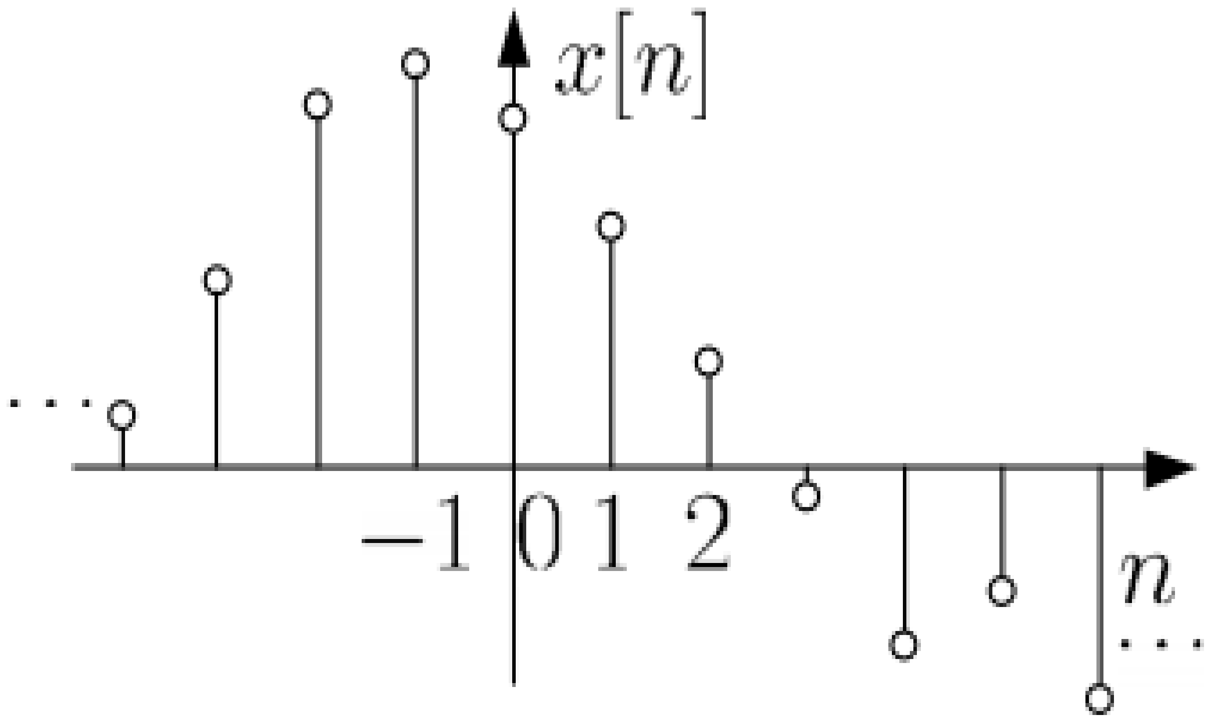
- *The digital frequency **ω** is a normalized frequency relative to the sampling frequency, defined as*

$$\omega = 2\pi \frac{F}{F_s}$$

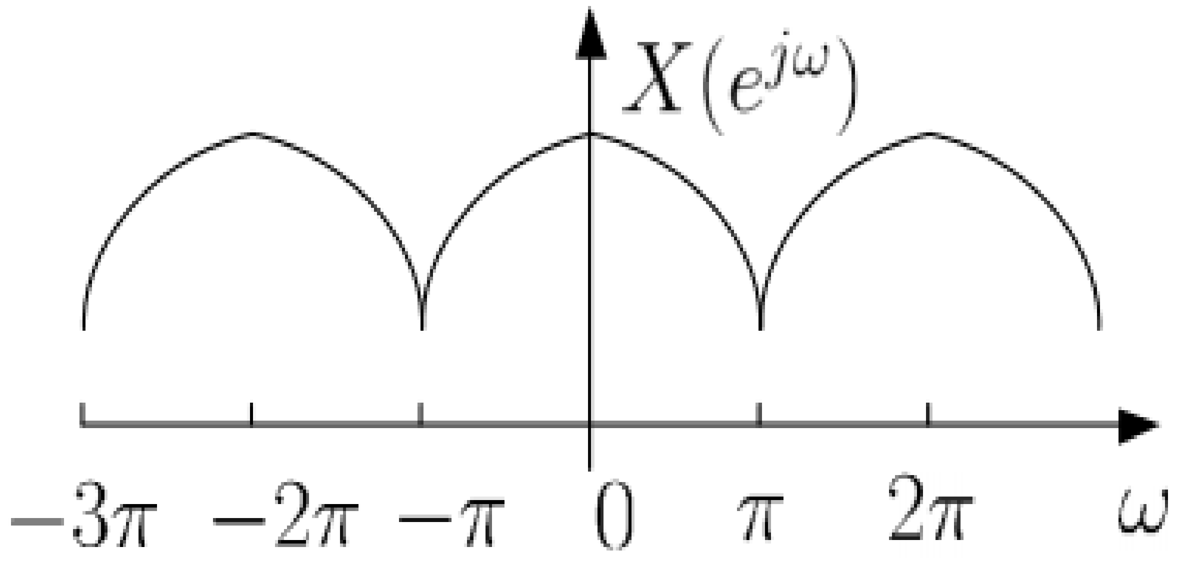


# DFT REPRESENTATION

**Time Domain**




**Freq Domain**



# DISCRETE FOURIER TRANSFORM

*In Discrete Fourier Transform, Given a finite sequence*

$$x = [x(0), x(1), \ldots, x(N-1)]$$

*its Discrete Fourier Transform (DFT) is a finite sequence*

$$X = DFT(x) = [X(0), X(1), \ldots, X(N-1)]$$

***Where***

$$X(k) = \sum_{n=0}^{N-1} x(n) w_N^{kn}, \quad w_N = e^{-j2\pi/N}$$

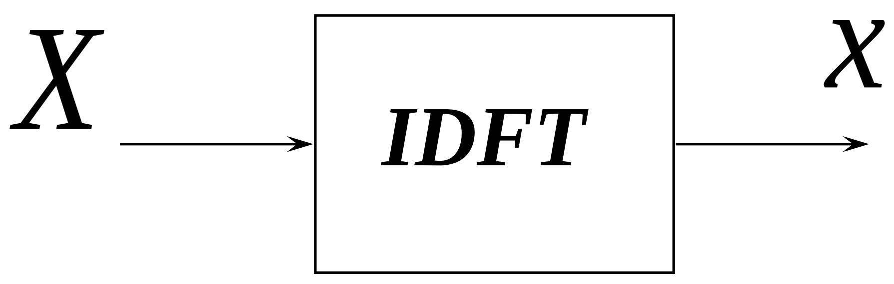
# INVERSE DISCRETE FOURIER TRANSFORM

*In Inverse Discrete Fourier Transform, Given a sequence*

$$X = [X(0), X(1), \ldots, X(N-1)]$$

*its Inverse Discrete Fourier Transform (IDFT) is a finite sequence*

$$x = IDFT(X) = [x(0), x(1), \ldots, x(N-1)]$$

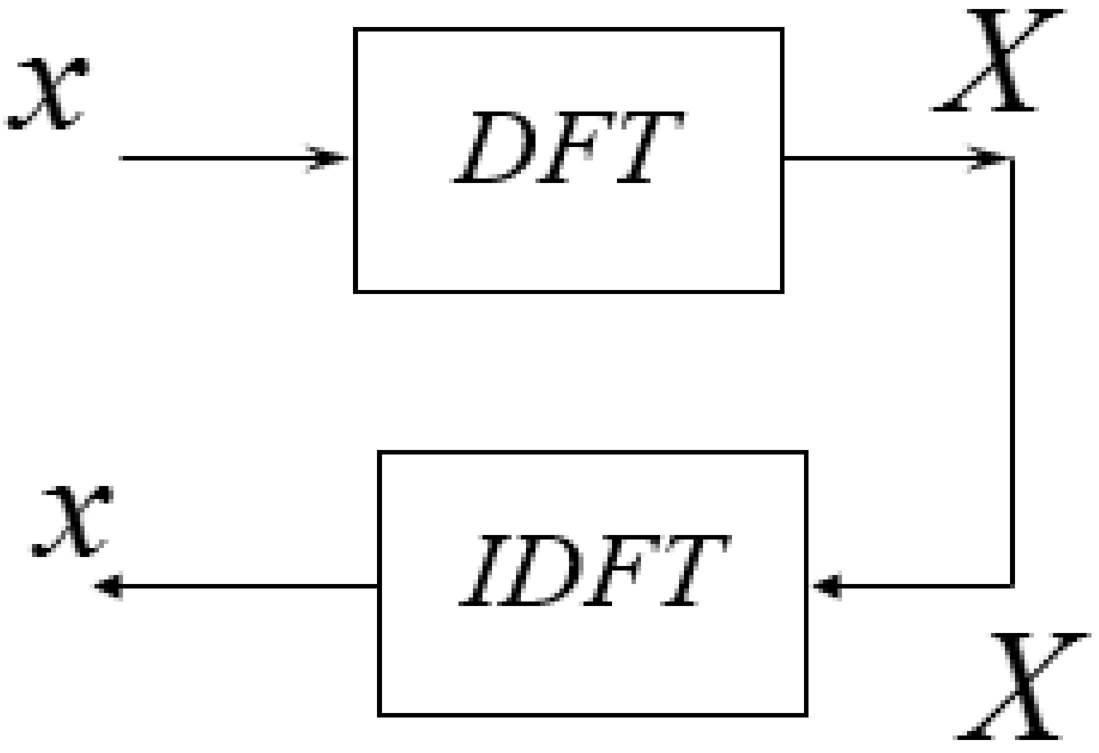



***Where***

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) w_N^{-kn}, \quad w_N = e^{-j2\pi/N}$$

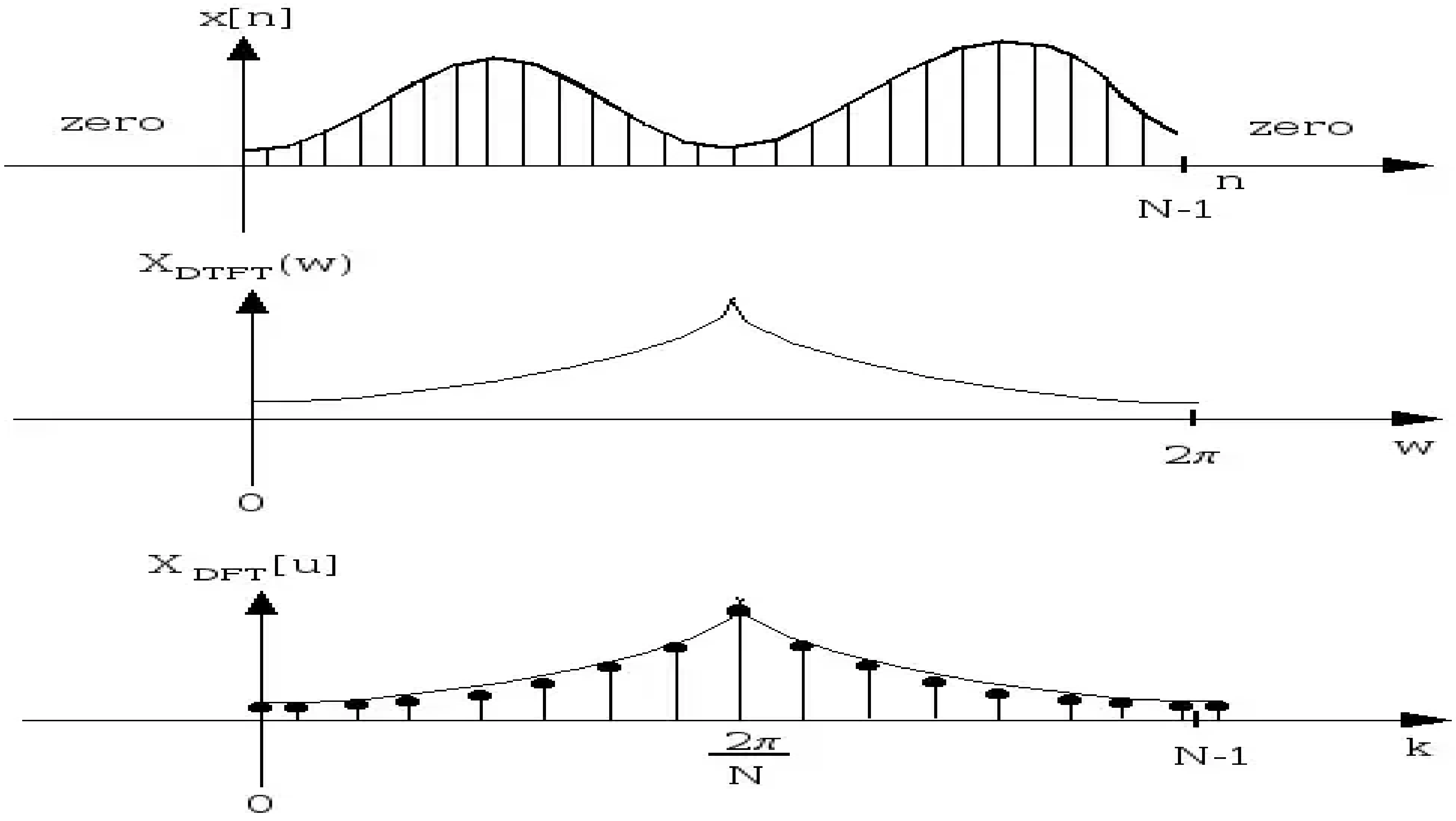
# DISCRETE FOURIER TRANSFORM PAIR

*The DFT and the IDFT form a transform pair.*




*The DFT is a numerical algorithm, and it can be computed by a digital computer.*

# REPRESENTATION OF DTFT & DFT



# PROPERTIES OF DFT

| Property | Time Domain | Frequency Domain |
|---|---|---|
| 1. Linearity | $ax_1[n] + bx_2[n]$ | $aX_1[k] + bX_2[k]$ |
| 2. Time-shifting | $x[n-m]$ | $e^{-j2\pi km}X(k)$ |
| 3. Frequency-shifting (modulation) | $e^{-j2\pi k_0 n/N}x[n]$ | $X(k-k_0)$ |
| 4. Time reversal | $x[-n]$ | $X(-k)$ |
| 5. Conjugation | $x^*[n]$ | $X^*(-k)$ |
| 6. Time-convolution | $x_1[n] \otimes x_2[n]$ | $X_1[k]X_2[k]$ |
| 7. Frequency-convolution | $x_1[n]x_2[n]$ | $\frac{1}{N}X_1[k] \otimes X_2[k]$ |

# APPLICATIONS OF DFT

1. Spectral Analysis
2. Image Processing
3. Signal Processing

**Other Applications:**

1. Sound Filtering
2. Data Compression
3. Partial Differential Equations
4. Multiplication of large integers

# DIFFERENCE B/W DFT & IDFT

| DFT (Analysis transform) | IDFT (Synthesis transform) |
|---|---|
| DFT is finite duration discrete frequency sequence that is obtained by sampling one period of FT. | IDFT is inverse DFT which is used to calculate time domain representation (Discrete time sequence) form of x(k). |
| DFT equations are applicable to causal finite duration sequences. | IDFT is used basically to determine sample response of a filter for which we know only transfer function. |
| Mathematical Equation to calculate DFT is given by $$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\Pi kn/N}$$ | Mathematical Equation to calculate IDFT is given by $$x(n) = 1/N \sum_{n=0}^{N-1} X(k) e^{j2\Pi kn/N}$$ |
| Thus DFT is given by $X(k) = [W_N][x_n]$ | In DFT and IDFT difference is of factor 1/N & sign of exponent of twiddle factor. Thus $x(n) = 1/N\,[W_N]^{-1}[X_K]$ |

# ASSESSMENT

1. Define DFT
2. What is meant by IDFT.
3. Give some applications of Fourier Transform.
4. Define DFT Pair.
5. *The DTFT (Discrete Time FT) for sequences having -------------------- duration*
6. *Determine DFT of $x(n) = \{1,0,1,0\}$*

# THANK YOU