1) Determine $H(z)$ using impulse Invariance technique for the analog system function :-

$$H_a(S) = \frac{1}{(S+1)(S+2)}$$

$$= \frac{A}{S+1} + \frac{B}{S+2}$$

$$1 = A(S+2) + B(S+1)$$

$S = -2$

$1 = A(0) + B(-1)$

$\boxed{\therefore B = -1}$

$S = -1$

$1 = A(1) + B(0)$

$\boxed{A = 1}$

$$H_a(S) = \frac{1}{S+1} - \frac{1}{S+2}$$

Using Impulse Invariance technique :-

$$H(S) = \sum_{K=1}^{N} \frac{C_K}{S - P_K} \rightarrow H(z) = \sum_{K=1}^{N} \frac{C_K}{1 - e^{P_K T} z^{-1}}$$

$$H(z) = \sum_{k=1}^{N} \frac{1}{1 - e^{P_K \cdot T} z^{-1}}$$

$P_1 = -1, \quad P_2 = -2$

$$H(z) = \frac{1}{1 - e^{(-1)T} z^{-1}} - \frac{1}{1 - e^{(-2)T} z^{-1}}$$

Sampling frequency of 5 samples per second

$$T = \frac{1}{f_S} = \frac{1}{5} = 0.2$$

$$H(z) = \frac{1}{1-e^{-0.2}z^{-1}} - \frac{1}{1-e^{-0.4}z^{-1}}$$

$$= \frac{1}{1-0.8187z^{-1}} - \frac{1}{1-0.670z^{-1}}$$

$$= \frac{1-0.670z^{-1}-1+0.8187z^{-1}}{(1-0.8187z^{-1})(1-0.670z^{-1})}$$

$$= \frac{0.1487z^{-1}}{1-0.670z^{-1}-0.8187z^{-1}+0.548z^{-2}}$$

$$= \frac{0.1487z^{-1}}{1-1.488z^{-1}+0.548z^{-2}}$$

Multiply by z^2 in Num & deno

$$H(z) = \frac{0.1487z}{z^2-1.488z+0.548}$$

2) Determine $H(z)$ using impulse invariance technique for the analog system function :-

$$H_a(s) = \frac{2}{(s+1)(s+2)}$$

$$\frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s+1)$$

Put $S=-2$ Put $S=-1$

$2 = B(-2+1)$ $2 = A(-1+2)$

$\boxed{B=-2}$ $\boxed{A=2}$

$$H_a(S) = \frac{2}{S+1} - \frac{2}{S+2}$$

Using Impulse Invariance technique :-

$$H(S) = \sum_{k=1}^{N} \frac{C_k}{S-P_k} \rightarrow H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_k T} z^{-1}}$$

$$H(z) = \sum_{k=1}^{N} \frac{2}{1-e^{P_k T} z^{-1}}$$

$P_1 = -1$, $P_2 = -2$, $T = 1$

$$H(z) = \frac{2}{1-e^{-1}z^{-1}} - \frac{2}{1-e^{-2}z^{-1}}$$

$$= \frac{2}{1-0.367z^{-1}} - \frac{2}{1-0.135z^{-1}}$$

$$= \frac{2(1-0.135z^{-1}) - 2(1-0.367z^{-1})}{(1-0.367z^{-1})(1-0.135z^{-1})}$$

$$= \frac{\cancel{2} - 0.27z^{-1} - \cancel{2} + 0.72z^{-1}}{1-0.135z^{-1} - 0.367z^{-1} + 0.048z^{-2}}$$

$$= \frac{0.45z^{-1}}{1-0.502z^{-1} + 0.048z^{-2}}$$

Multiply by z^2 in Num & den

$$H(z) = 0.45z / z^2 - 0.502z + 0.048$$

Scanned with CamScanner

(3) $H(s) = \dfrac{10}{s^2+7s+10}$; $T = 0.2$ sec

Find $H(z)$ using Impulse Invariance method.

$$H(S) = \frac{10}{s^2+7s+10} \Rightarrow \frac{10}{(s+5)(s+2)}$$

$$= \frac{A}{s+5} + \frac{B}{s+2}$$

$$10 = A(s+2) + B(s+5)$$

Sub $s=-2$

$$10 = A(0) + B(3)$$

$$\boxed{\therefore B = 10/3}$$

Sub $s=-5$

$$10 = A(-3) + B(0)$$

$$\boxed{A = -10/3}$$

$$H(S) = \frac{-10/3}{s+5} + \frac{10/3}{s+2}$$

Use Impulse Invariance Method :-

$$H(S) = \sum_{k=1}^{N} \frac{c_k}{s-p_k} \Rightarrow H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_kT}z^{-1}}$$

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_kT}z^{-1}}$$

$$= \frac{-10/3}{1-e^{-1}z^{-1}} + \frac{10/3}{1-e^{-0.4}z^{-1}}$$

$$= \frac{-3.33}{1-0.367z^{-1}} + \frac{3.33}{1-0.670z^{-1}}$$

$$= \frac{-3.33\left(1-0.670z^{-1}\right) + 3.33\left(1-0.367z^{-1}\right)}{\left(1-0.367z^{-1}\right)\left(1-0.670z^{-1}\right)}$$

$$= \frac{-\cancel{3.33} + 2.2311\, z^{-2} + \cancel{3.33} - 1.22\, z^{-2}}{1 - 0.670z^{-1} - 0.367z^{-1} + 0.2458\, z^{-2}}$$

$$= \frac{1.0091\, z^{-2}}{1 - 1.037z^{-1} + 0.245z^{-2}}$$

Multiply by z^2 in Num & den

$$H(z) = \frac{1.0091}{z^2 - 1.037z + 0.2458}$$

(4) For the analog transfer function $H(S) = \frac{4}{S^2+3S+2}$ Determine $H(z)$ using Impulse Invariant transformation.

$T = 0.1$ sec

$$H(S) = \frac{4}{(S+2)(S+1)}$$

$$= \frac{A}{(S+1)} + \frac{B}{(S+2)}$$

$$4 = A(S+2) + B(S+1)$$

$S = -2$

$4 = A(0) + B(-1)$

$\boxed{B = -4}$

$S = -1$

$4 = A(1) + B(0)$

$\boxed{A = 4}$

$$H(S) = \frac{4}{S+1} - \frac{4}{S+2}$$

Use Impulse Invariance Method :-

$$H(s) = \sum_{k=1}^{N} \frac{C_k}{s-P_k} \rightarrow H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_kT}z^{-1}}$$

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_kT}z^{-1}}$$

$$= \frac{4}{1-e^{-0.1}z^{-1}} - \frac{4}{1-e^{-0.2}z^{-1}}$$

$$= \frac{4}{1-0.367z^{-1}} - \frac{4}{1-0.818z^{-1}}$$

$$= \frac{4(1-0.818z^{-1}) - 4(1-0.367z^{-1})}{(1-0.367z^{-1})(1-0.818z^{-1})}$$

$$= \frac{\cancel{4} - 3.272z^{-1} - \cancel{4} + 1.468z^{-1}}{1-0.818z^{-1} - 0.367z^{-1} + 0.300z^{-2}}$$

$$= \frac{-1.804z^{2}}{1-0.451z^{-1} + 0.300z^{-2}}$$

Multiply by z^2 in Num & den

$$H(z) = \frac{-1.804}{z^2 - 0.451z + 0.300}$$

⑤ $H(s) = \frac{2}{(s+1)(s+2)}$; T= 1 sec solve using Bilinear Transformation.

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$$s = \frac{2}{T}\frac{(z-1)}{(z+1)}$$

$$H(z) = \frac{2}{\left[\frac{2}{T}\frac{(z-1)}{(z+1)}+1\right]\left[\frac{2}{T}\frac{(z-1)}{(z+1)}+2\right]}$$

$$H(z) = \frac{2}{\left[2\frac{(z-1)}{(z+1)}+1\right]2\left[\frac{(z-1)}{z+1}+2\right]}$$

$$= \frac{2}{\left[\frac{2(z-1)+(z+1)}{z+1}\right]\left[\frac{2(z-1)+2(z+1)}{z+1}\right]}$$

$$= \frac{2}{\left(\frac{2z-2+z+1}{z+1}\right)\left(\frac{2z-\cancel{2}+2z+\cancel{2}}{z+1}\right)}$$

$$= \frac{2}{\left(\frac{3z-1}{z+1}\right)\left(\frac{4z}{z+1}\right)} \Rightarrow \frac{2(z+1)^2}{12z^2-4z} = \frac{\cancel{2}(z+1)^2}{\cancel{2}(6z^2-2z)}$$

$$= \frac{z^2+2z+1}{6z^2-2z} \quad \text{(or)}$$

$$H(z) = \frac{(z+1)^2}{6z^2-2z}$$

⑥ $H(s) = \frac{1}{(s+1)^2}$; $T = 0.1$ sec

$$H(z) = \frac{1}{\left[\frac{2}{T}\frac{(z-1)}{(z+1)}+1\right]^2}$$

$$= \frac{1}{\left[\frac{2}{0.1}\frac{(z-1)}{(z+1)}+1\right]^2}$$

$$= \frac{1}{\left[\frac{20(z-1)+(z+1)}{z+1}\right]^2}$$

$$= \frac{1}{\left[\frac{20z-20+z+1}{z+1}\right]^2}$$

$$= \frac{1}{\left[\frac{21z-19}{z+1}\right]^2} \Rightarrow \frac{(z+1)^2}{(21z-19)^2}$$