DISCRETE FOURIER TRANSFORM

The DFT of $x(n)$ is obtained by sampling one period $(0 \leq \omega \leq 2\pi)$ of the discrete time Fourier transform $X(e^{j\omega})$ at a finite number (N) of freq points.

The DFT is defined along with no of samples and is called N-point DFT.

$$X(k) = X(e^{j\omega}) \Big|_{\omega = \frac{2\pi k}{N}} \; ; \; k = 0, 1, 2 \dots N-1$$

Let $x(n) \rightarrow$ Discrete time signal of length L

$X(k) \rightarrow$ DFT of $x(n)$

N-point DFT of $x(n)$, where $N \geq L$ is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}} \qquad k = 0, 1, 2 \dots N-1$$

Inverse Discrete Fourier Transform :-

The Inverse DFT of the sequence $X(k)$ of length N is defined as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi kn}{N}} \quad , n = 0, 1, 2 \dots N-1$$

The relation between $x(n)$ and $X(k)$ is expressed as

$$x(n) \xrightarrow{\text{DFT}} X(k)$$

$$X(k) \xrightarrow{\text{IDFT}} x(n)$$

Problems :-

1) Find the DFT of a given sequence $x(n) = \{1, 2, 3, 4\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$N = 4$ $\quad x(0) = 1, \quad x(1) = 2, \quad x(2) = 3, \quad x(3) = 4$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi nk}{4}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi nk}{2}}$$

$k = 0$

$$X(0) = x(0)\, e^0 + x(1)\, e^0 + x(2)\, e^0 + x(3)\, e^0$$

$$= (1)(1) + (2)(1) + (3)(1) + 4(1)$$

$$X(0) = 10$$

$k = 1,$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n}{2}}$$

$$= x(0)\, e^{\frac{-j\pi(0)}{2}} + x(1)\, e^{-j\pi/2} + x(2)\, e^{\frac{-j\pi(\cancel{2})}{\cancel{2}}} + x(3)\, e^{-j\pi 3/2}$$

$$= (1)(1) + (2)\, e^{-j\pi/2} + 3\, e^{-j\pi} + 4\, e^{-j3\pi/2}$$

$$= 1 + 2\left[\cos\pi/2 - j\sin\pi/2\right] + 3\left[\cos\pi - j\sin\pi\right] + 4\left[\cos 3\pi/2 - j\sin 3\pi/2\right]$$

$$= 1 + 2\left[0 - j\right] + 3(-1) + 4\left[0 - j(-1)\right]$$

$$= 1 - 2j - 3 + 4j = -2 + 2j$$

$$X(1) = -2 + 2j$$

$k = 2,$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n 2}{2}}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi} + x(2)\, e^{-j2\pi} + x(3)\, e^{-j3\pi}$$

$$= 1 + 2\left[\cos\pi - j\sin\pi\right] + 3\left[\cos 2\pi - j\sin 2\pi\right] + 4\left[\cos 3\pi - j\sin 3\pi\right]$$

$$= 1 + 2\left[-1 - j(0)\right] + 3\left[1 - j(0)\right] + 4\left[(-1) - j(0)\right]$$

$$= 1 - 2 + 3 - 4 = -2. \quad X(2) = -2$$

$k = 3,$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n(3)}{2}}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j3\pi/2} + x(2)\, e^{-j3\pi} + x(3)\, e^{-j9\pi/2}$$

$$= (1)(1) + 2\left[\cos 3\pi/2 - j\sin 3\pi/2\right] + 3\left[\cos 3\pi - j\sin 3\pi\right] + 4\left[\cos 9\pi/2 - j\sin 9\pi/2\right]$$

$$= 1 + 2\left[0 - j(-1)\right] + 3\left[(-1) - j(0)\right] + 4\left[0 - j(1)\right]$$

$$= 1 + 2j - 3 - 4j \Rightarrow -2 - 2j$$

$$X(3) = -2 - 2j$$

$$X(k) = \{10, -2+2j, -2, -2-2j\}$$

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② Find the DFT of sequence $x(n) = \{\underset{\uparrow}{1}, 0, 1, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}}$$

$x(0) = 1,\ x(1) = 0$

$x(2) = 1,\ x(3) = 0$

$\boxed{N=4}$

$N = 4,$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\cancel{2}\pi nk}{\cancel{4}\,2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi nk}{2}}$$

$k = 0,$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi nk}{2}}$$

$$= x(0)\, e^{0} + x(1)\, e^{0} + x(2)\, e^{0} + x(3)\, e^{0}$$

$$= (1)\, e^{0} + (0)\, e^{0} + (1)\, e^{0} + (0)\, e^{0}$$

$$X(0) = 1 + 0 + 1 + 0 \Rightarrow 2.$$

$X(0) = 2$

$k = 1,$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n(1)}{2}}$$

$$= x(0)\, e^{\frac{-j\pi(0)(1)}{2}} + x(1)\, e^{-j\pi/2} + x(2)\, e^{-j\pi} + x(3)\, e^{-j\pi(3)/2}$$

$$= (1)\, e^{0} + (0)\, e^{-j\pi/2} + (1)\, e^{-j\pi} + (0)\, e^{-j3\pi/2}$$

$$= 1 + 0 + [\cos\pi - j\sin\pi] + 0 = 1 + [-1 - j(0)] \Rightarrow 0$$

$X(1) = 0$

$k = 2,$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n\cancel{2}}{\cancel{2}}}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi} + x(2)\, e^{-j2\pi} + x(3)\, e^{-j3\pi}$$

$$= 1 + 0 + [\cos 2\pi - j\sin 2\pi] + 0 \Rightarrow 1 + 0 + [1 - 0] + 0$$

$X(2) = 2$

$k = 3,$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n 3/2}$$

$$= x(0)\, e^{-j\pi (0) 3/2} + x(1)\, e^{-j\pi \frac{(1)(3)}{2}} + x(2)\, e^{-j\pi \frac{(2)(3)}{2}} + x(3)\, e^{-j\pi \frac{(3)(3)}{2}}$$

$$= (1)\, e^{0} + (0)\, e^{-j\pi 3/2} + (1)\, e^{-j3\pi} + (0)\, e^{-j 9\pi/2}$$

$$= 1 + [\cos 3\pi - j\sin 3\pi] \Rightarrow 1 + [-1 - j(0)] = 0$$

$X(3) = 0$ $\qquad$ $X(k) = \{2, 0, 2, 0\}$

③ compute IDFT of $X(k) = \{2, 0, 2, 0\}$

$N = 4,$ $\qquad$ $X(0) = 2,\ X(1) = 0,\ X(2) = 2,\ X(3) = 0$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi k n}{N}} \quad , n = 0, 1, 2 \ldots N-1$$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k)\, e^{\frac{j\cancel{2}\pi k n}{\cancel{4}\, 2}}$$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k)\, e^{\frac{j\pi n k}{2}}$$

$n = 0$

$$x(0) = \frac{1}{4}\left[X(0)\, e^{0} + X(1)\, e^{\frac{j\pi (0)(1)}{2}} + X(2)\, e^{\frac{j\pi (0)(2)}{2}} + X(3)\, e^{\frac{j\pi (0)(3)}{2}} \right]$$

$$= \frac{1}{4}\left[(2)(1) + (0)\, e^{0} + 2(e^{0}) + (0)\, e^{0} \right]$$

$$= \frac{1}{4}\left[2 + 0 + 2 + 0 \right] = \frac{4}{4} = 1$$

$x(0) = 1$

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n=1,

$$x(1) = \frac{1}{4}\left[x(0)\,e^{0} + x(1)\,e^{\frac{j\pi(1)(1)}{2}} + x(2)\,e^{\frac{j\pi(1)(2)}{2}} + x(3)\,e^{\frac{j\pi(1)(3)}{2}}\right]$$

$$= \frac{1}{4}\left[(2)(1) + (0) + (2)\,e^{j\pi} + (0)\right]$$

$$= \frac{1}{4}\left[2 + 2\left(\cos\pi + j\sin\pi\right)\right] \Rightarrow \frac{1}{4}\left[2 + 2\left[-1 + j(0)\right]\right]$$

$$= \frac{1}{4}\left[2 - 2\right] = \frac{1}{4}(0) = 0 \qquad x(1) = 0$$

n=2,

$$x(2) = \frac{1}{4}\left[x(0)\,e^{0} + x(1)\,e^{\frac{j\pi(2)(1)}{2}} + x(2)\,e^{\frac{j\pi(2)(2)}{2}} + x(3)\,e^{\frac{j\pi(2)(3)}{2}}\right]$$

$$= \frac{1}{4}\left[(2) + (0) + 2\,e^{j\pi 2} + 0\right]$$

$$= \frac{1}{4}\left[2 + 2\left(\cos 2\pi + j\sin 2\pi\right)\right] \Rightarrow \frac{1}{4}\left[2 + 2(1+0)\right]$$

$$= \frac{1}{\cancel{4}}\left[\cancel{4}\right] = 1 \qquad x(2) = 1$$

n=3,

$$x(3) = \frac{1}{4}\left[x(0)\,e^{0} + x(1)\,e^{\frac{j\pi(3)(1)}{2}} + x(2)\,e^{\frac{j\pi(3)(2)}{2}} + x(3)\,e^{\frac{j\pi(3)(3)}{2}}\right]$$

$$= \frac{1}{4}\left[(2) + 0 + 2\left[\cos 3\pi + j\sin 3\pi\right] + 0\right]$$

$$= \frac{1}{4}\left[2 + 2(-1+0) + 0\right] \Rightarrow \frac{1}{4}\left[2-2\right] = 0 \qquad x(3) = 0$$

$$x(n) = \{1, 0, 1, 0\}$$

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find the DFT of $x(n) = \{4, 1\}$

$$\boxed{N=2} \qquad x(0) = 4,\ x(1) = 1$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j\cancel{2}\pi nk}{\cancel{2}}}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\pi nk}$$

$k=0$,

$$X(0) = \sum_{n=0}^{1} x(n)\, e^{-j\pi n(0)}$$

$$= x(0)\, e^{0} + x(1)\, e^{0} \Rightarrow (4)\, e^{0} + (1)\, e^{0}$$

$$X(0) = 4+1 \Rightarrow 5 \qquad X(0) = 5$$

$k=1$,

$$X(1) = x(0)\, e^{-j\pi(0)(1)} + x(1)\, e^{-j\pi(1)(1)}$$

$$= (4)(1) + 1\left(e^{-j\pi}\right) \Rightarrow 4 + [\cos\pi - j\sin\pi]$$

$$= 4 - 1 = 3 \qquad X(1) = 3 \qquad X(k) = \{5, 3\}$$

Find the IDFT of $X(k) = \{5, 3\}$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}} \Rightarrow \frac{1}{2} \sum_{k=0}^{1} X(k)\, e^{j\pi nk}$$

$n=0$,

$$x(0) = \frac{1}{2}\left[X(0)\, e^{0} + X(1)\, e^{j\pi(0)(1)}\right]$$

$$= \frac{1}{2}\left[(5)\, e^{0} + (3)\, e^{0}\right] = \frac{1}{\cancel{2}}(\cancel{8}) = 4$$

$$x(0) = 4$$

$n=1$,

$$x(1) = \frac{1}{2}\left[X(0)\, e^{j\pi(1)(0)} + X(1)\, e^{j\pi(1)(1)}\right]$$

$$= \frac{1}{2}\left[(5)(1) + 3\left(e^{j\pi}\right)\right] = \frac{1}{2}\left[5 + 3(\cos\pi + j\sin\pi)\right]$$

$$= \frac{1}{2}[5-3] \Rightarrow \frac{1}{\cancel{2}}(\cancel{2}) = 1 \qquad x(1) = 1$$

$$x(n) = \{4, 1\}$$