SNS COLLEGE OF TECHNOLOGY

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Coimbatore-35

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT203 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 2 – IIR FILTER DESIGN

TOPIC – CHEBYSHEV FILTER

COMPARISON OF DIGITAL & ANALOG FILTERS

S.No.	Digital Filter	Analog Filter
1	Operates on digital samples of the signal	Operates on analog signals
2	It is governed by linear difference equation	It is governed by linear differential equation
3	It consists of adders, multipliers and delays implemented in digital logic	It consists of electrical components like resistors, capacitors and inductors
4	The filter coefficients are designed to satisfy the desired frequency response	The approximation problem is solved to satisfy the desired frequency response

DESIGN OF LOWPASS DIGITAL CHEBYSHEV FILTER

- For designing a Chebyshev IIR digital filter, analog filter is designed using the given specifications
- Then the analog filter transfer function is transformed to digital filter transfer function by using either Impulse Invariant or Bilinear Transformation
- The analog chebyshev filter is designed by approximating the ideal frequency response using an error function
- The approximation function is selected such that the error is minimized over a band of frequencies

TYPES OF CHEBYSHEV APPROXIMATION

- There are two types of Chebyshev approximation:

1. Type-1 Chebyshev Approximation
2. Type-2 Chebyshev Approximation

- In type - 1 approximation, the error function is selected such that magnitude response is equiripple in the passband and monotonic in the stopband
- In type - 2 approximation, the error function is selected such that magnitude response is monotonic in the passband and equiripple in the stopband
- The type – 2 magnitude response is also called Inverse chebyshev response

ANALOG CHEBYSHEV FILTER

- The magnitude response of Type-1 low pass filter is given by

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2 C_N^2\left(\frac{\Omega}{\Omega_P}\right)}$$

- ϵ – Attenuation Constant

$C_N\left(\frac{\Omega}{\Omega_p}\right)$ = Chebyshev polynomial of order N

ANALOG CHEBYSHEV FILTER

- Attenuation Constant is given by

$$\epsilon = \left[\frac{1}{A_P^2} - 1\right]^{\frac{1}{2}}$$

- $\mathbf{A_p}$ - is the gain or magnitude at pass band edge frequency $\mathbf{\Omega_p}$
- For small values of N the chebyshev polynomial is given by

$$C_N(x) = \begin{cases} \cos(N \cos^{-1} x) & ; \text{ for } |x| \le 1 \\ \cosh(N \cosh^{-1} x) & ; \text{ for } |x| > 1 \end{cases}$$

ANALOG CHEBYSHEV FILTER

- The transfer function of the analog system can be obtained from the magnitude response of Type – 1 low pass filter by substituting **Ω by s/j**

$$H(s)\,H(-s) = \frac{1}{1 + \epsilon^2 C_N^2\left(\frac{s/j}{\Omega_c}\right)}$$

- For the normalized transfer function, let us replace s/ **Ω** **$_c$** **by s$_n$**

$$H(s_n)\,H(-s_n) = \frac{1}{1 + \epsilon^2 C_N^2(-js_n)}$$

PROPERTIES OF CHEBYSHEV FILTERS (TYPE -1)

- The magnitude $|H(j\Omega)|$ oscillates between 1 and $1/\sqrt{1+\epsilon^2}$ within the pass band and so the filter is called equiripple I the pass band
- The normalized magnitude response has a value of $1/\sqrt{1+\epsilon^2}$ at cutoff frequency Ω_c
- The magnitude is monotonic outside the pass band
- The Chebyshev Type – 1 Filters are all pole designs
- With large values of N, the transition from pass band to stop band becomes more sharp and approaches ideal characteristics.

TRANSFER FUNCTION OF ANALOG CHEBYSHEV LOW PASS FILTER

- For a stable and causal filter the poles should lie on the left half of s –plane. Hence the desired filter transfer function is obtained by selecting N number of left half poles
- When N is even all the poles are complex and exist as conjugate pair
- When N is odd, one of the pole is real and all other poles are complex and exist as conjugate pair
- Therefore the transfer function of chebyshev filters will be a product f second order factors

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- N be the order of the filter
- $H(s_n)$ be the normalized Chebyshev lowpass filter function
- When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

- When N is odd,

$$H(s_n) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

$$\text{where, } b_k = 2\, y_N \sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_k = y_N^2 + \cos^2\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{-\frac{1}{N}}\right\}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- N be the order of the filter
- For even values of N parameter B_k are evaluated

$$H(s_n)\Big|_{s_n = 0} = \frac{1}{(1 + \epsilon^2)^{\frac{1}{2}}}$$

- For odd values of N the parameter B_k are evaluated

$$H(s_n)\Big|_{s_n = 0} = 1$$

- While evaluating B_k to take $B_0 = B_1 = B_2 = \ldots.. = B_k$

UNNORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- The unnormalized transfer function is obtained by letting $s_n \rightarrow s/\Omega_c$ in the normalized transfer function, Where Ω_c is the cutoff frequency of the lowpass filter
- H(s) be the normalized Chebyshev low pass filter transfer function
- When N is even, H(s) is obtained by letting $s_n \rightarrow s/\Omega_c$ in normalized Chebyshev low pass filter function

$$\therefore H(s) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}\Bigg|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

UNNORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- H(s) be the normalized Chebyshev low pass filter transfer function
- N be the order of the filter
- Ω_c is the cutoff frequency of the lowpass filter
- When N is odd, H(s) is obtained by letting $s_n \rightarrow s/\Omega_c$ in normalized Chebyshev low pass filter function

$$\therefore H(s) = \frac{B_0}{s_n + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k} \Bigg|_{s_n = \frac{s}{\Omega_c}} = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

ORDER OF ANALOG LOWPASS CHEBYSHEV FILTER

- In Chebyshev filters the frequency response of the filter depends on the order N. The specifications of the filter are given in terms of gain at a passband and stopband frequency
- $\mathbf{A_p}$ - Gain or Magnitude at pass band edge frequency $\mathbf{\Omega_p}$
- $\mathbf{A_s}$ - Gain or Magnitude at Stop band edge frequency $\mathbf{\Omega_s}$

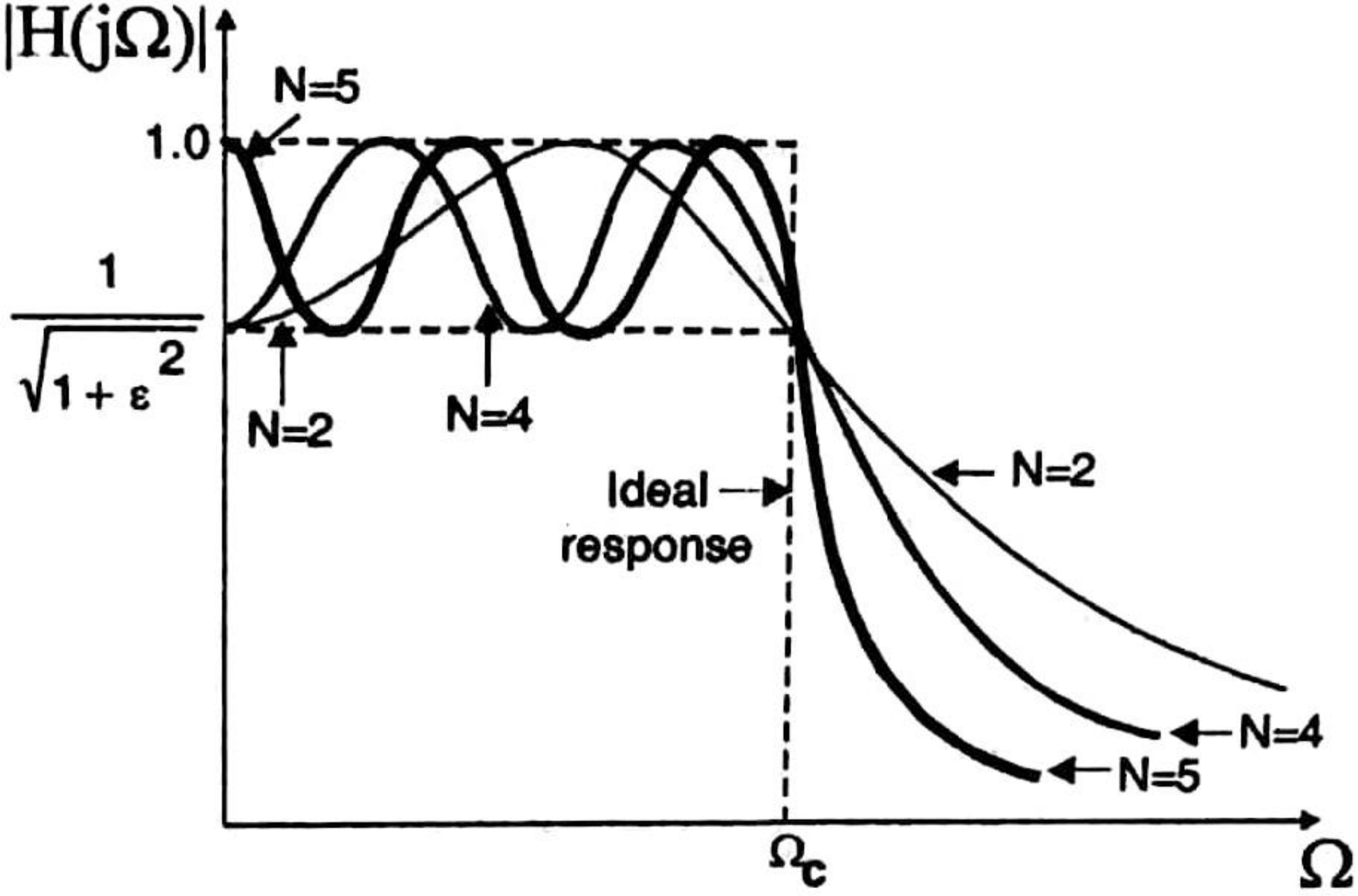
$$N_1 = \frac{\cosh^{-1}\left[\left(\frac{(1/A_s^2) - 1}{(1/A_p^2) - 1}\right)^{\frac{1}{2}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

ORDER OF THE LOWPASS CHEBYSHEV FILTER

- The specifications of the filter are given in terms of dB attenuation at a passband and stopband frequency
- $\alpha_{p,\,dB}$ - dB attenuation at pass band edge frequency Ω_p
- $\alpha_{s,\,dB}$ - dB attenuation at Stop band edge frequency Ω_s

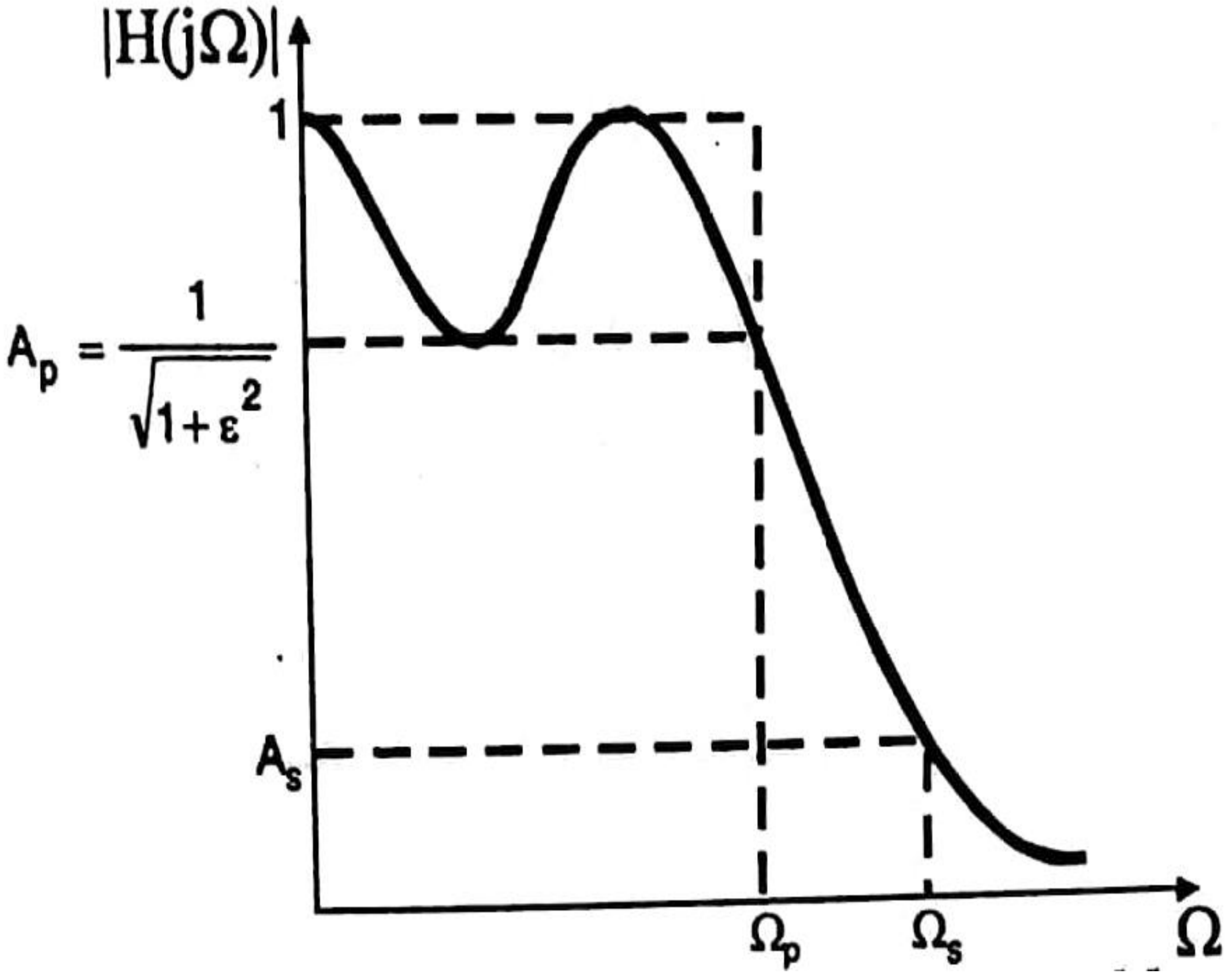
$$N_1 = \frac{\cosh^{-1}\left[\left(\dfrac{10^{0.1\alpha_{s,dB}}-1}{10^{0.1\alpha_{p,dB}}-1}\right)^{\frac{1}{2}}\right]}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 1 LOW PASS FILTER

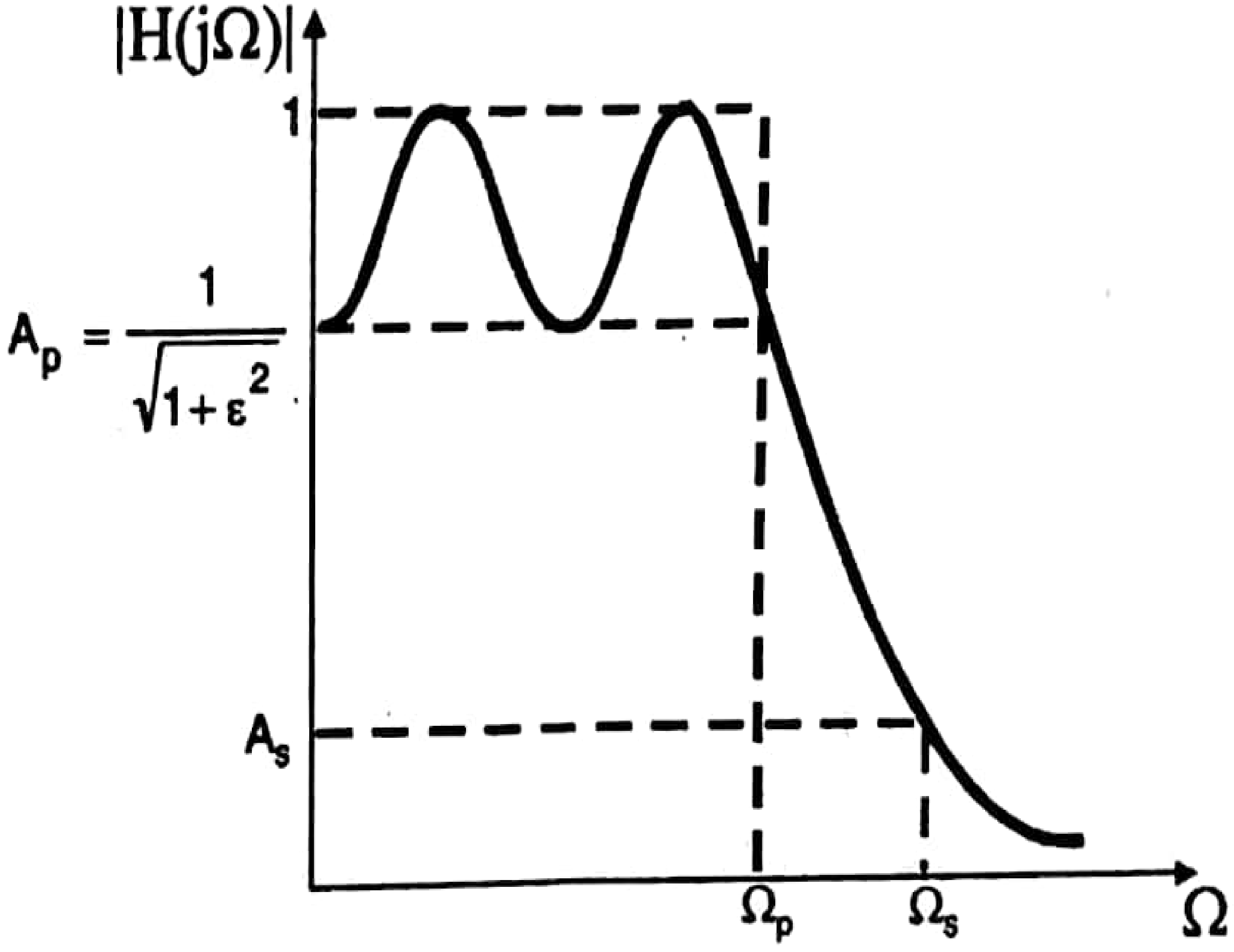


MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 1 FILTERS

N is Odd

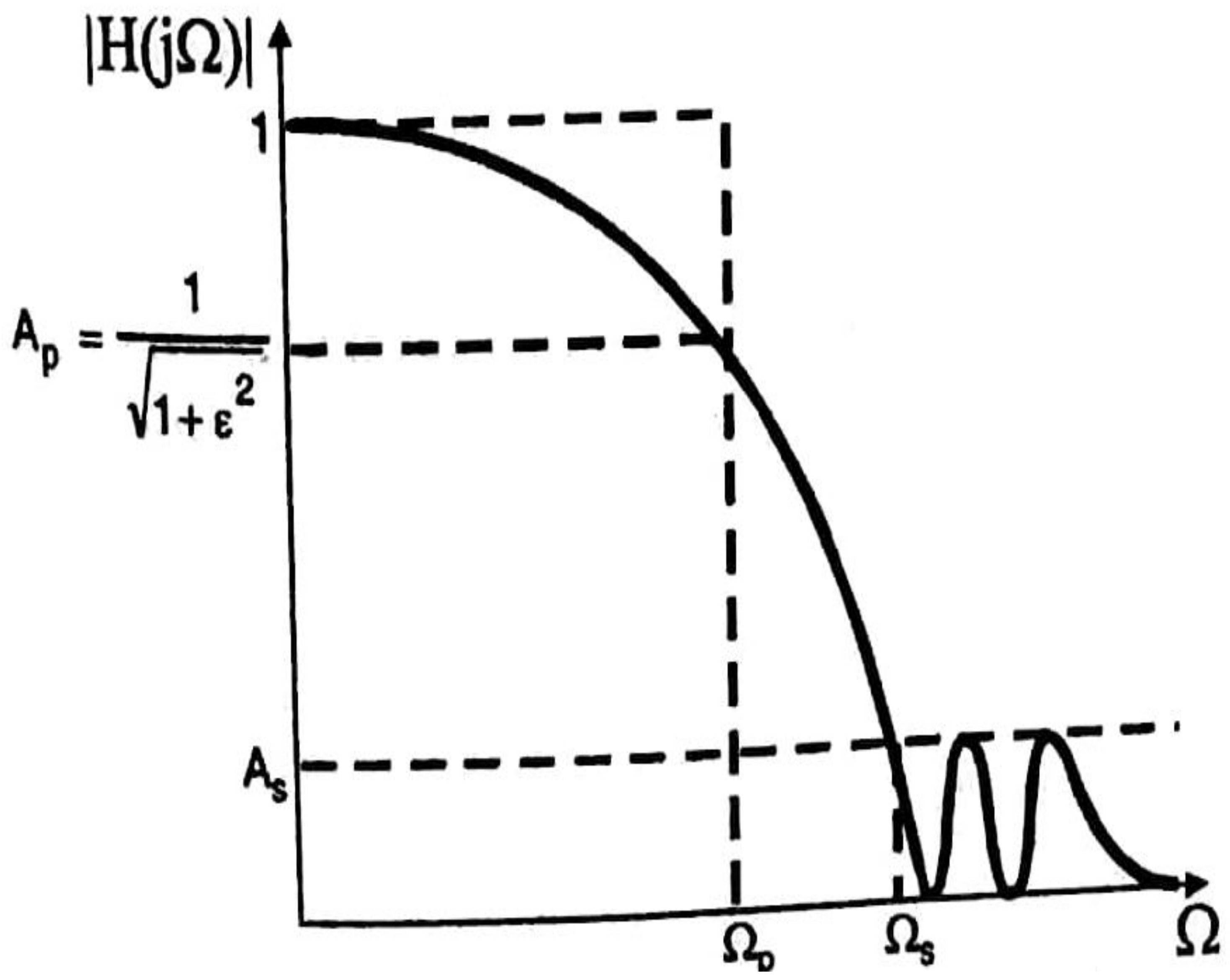


N is Even

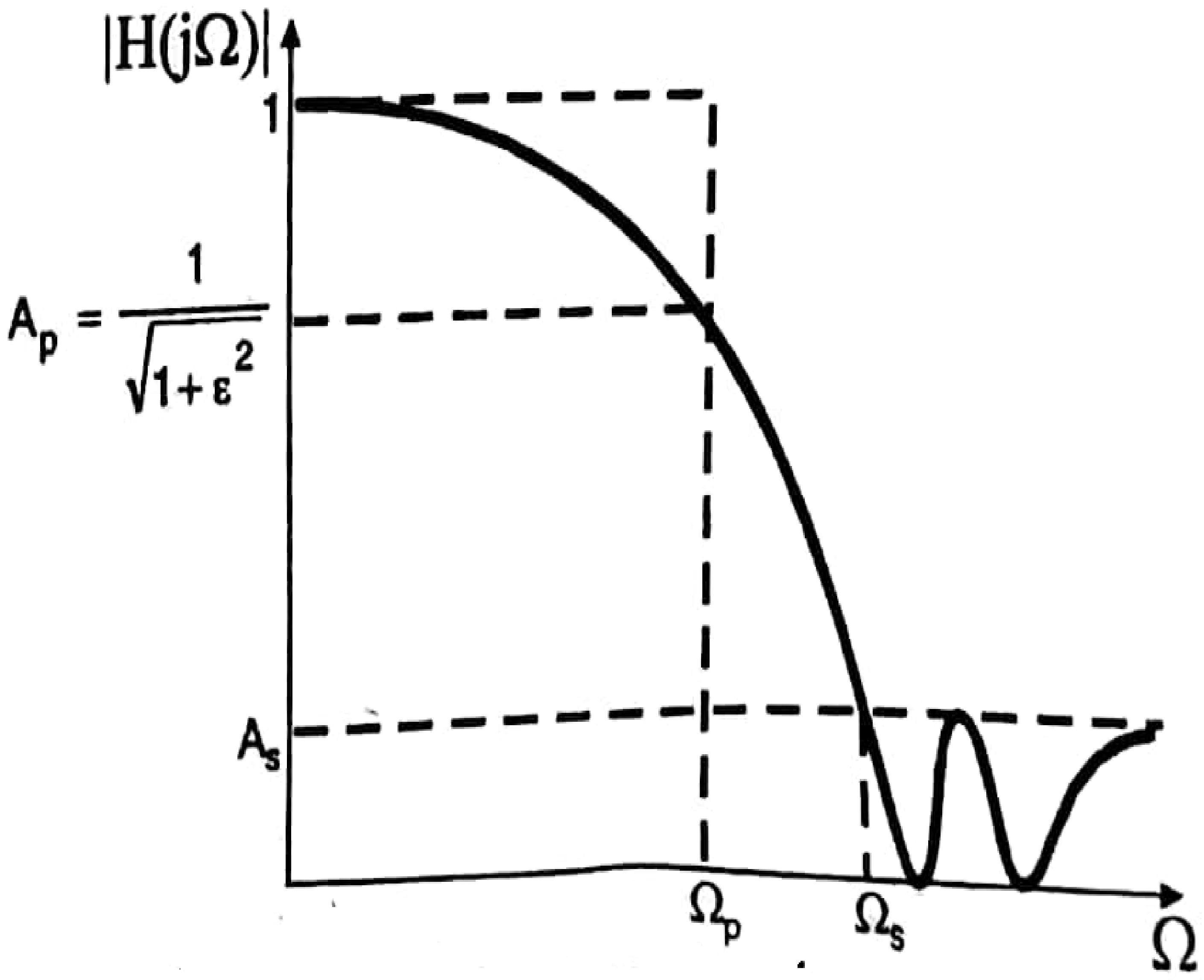


MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 2 FILTERS

N is Odd



N is Even



CUTOFF FREQUENCY OF ANALOG LOWPASS CHEBYSHEV FILTER

- The IIR filters are designed to satisfy a prescribed gain or attenuation at a pass band or stop band frequency. But Practically the cutoff frequency $\mathbf{\Omega_c}$ is used to decide the useful frequency range of the filter
- In chebyshev filter design the passband and stopband specifications are used to estimate the order, N of the filter and $\mathbf{N^{th}}$ order normalized Chebyshev lowpass filter is designed. Then the normalized LPF is unnormalized using the cutoff frequency
- In Chebyshev filters the passband edge frequency, Ω_p is considered as cutoff frequency Ω_c and this cutoff is not equal to 3 dB cutoff frequency Ω_{3dB}

$$\Omega_{3dB} = \Omega_c \cosh\left(\frac{1}{N} \cosh^{-1} \frac{1}{\epsilon}\right)$$

DESIGN PROCEDURE FOR LOWPASS DIGITAL CHEBYSHEV IIR FILTER

- $\boldsymbol{\omega_p}$ - Pass band edge digital frequency in rad /sample
- $\boldsymbol{\omega_s}$ - Stop band edge digital frequency in rad/sample
- $\mathbf{A_p}$ - Gain at pass band edge frequency $\boldsymbol{\omega_p}$
- $\mathbf{A_s}$ - Gain at Stop band edge frequency $\boldsymbol{\omega_s}$
- $\mathbf{T = 1/ F_s}$ - Sampling time in sec.
- Where F_s = Sampling frequency in Hz
- $\boldsymbol{\Omega_p}$ - Pass band edge analog frequency corresponding to $\boldsymbol{\omega_p}$
- $\boldsymbol{\Omega_s}$ - Stop band edge analog frequency corresponding to $\boldsymbol{\omega_s}$

1. Choose either Bilinear or Impulse Invariant transformation and determine the specifications of equivalent analog filter

- The gain or attenuation of analog filter is same as digital filter
- **Bilinear Transformation:**

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} \qquad \Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2}$$

- **Impulse Invariant Transformation:**

$$\Omega_p = \frac{\omega_p}{T} \qquad \Omega_s = \frac{\omega_s}{T}$$

ORDER OF THE LOWPASS DIGITAL CHEBYSHEV FILTER

2. Decide the order N of the filter. In order to estimate the order N, Calculate the Parameter N_1 using the following equation:

$$N_1 = \frac{\cosh^{-1}\left[\left(\frac{(1/A_s^2)-1}{(1/A_p^2)-1}\right)^{\frac{1}{2}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

- Choose N such that, $N \geq N_1$, Usually N is chosen as nearest integer just greater than N_1

3. Determine the normalized transfer function $H(s_n)$ of the analog lowpass filter function

- When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

- When N is odd,

$$H(s_n) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

where, $b_k = 2\, y_N \sin\left(\frac{(2k-1)\pi}{2N}\right)$

$$\epsilon = \left[\frac{1}{A_P^2} - 1\right]^{1/2}$$

$$c_k = y_N^2 + \cos^2\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{-\frac{1}{N}}\right\}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- N be the order of the filter
- For even values of N parameter B_k are evaluated

$$H(s_n)\big|_{s_n = 0} = \frac{1}{(1 + \epsilon^2)^{\frac{1}{2}}}$$

- For odd values of N the parameter B_k are evaluated

$$H(s_n)\big|_{s_n = 0} = 1$$

- While evaluating B_k to take $B_0 = B_1 = B_2 = ... = B_k$

UNNORMALIZED ANALOG TRANSFER FUNCTION

4. Determine the unnormalized analog transfer function H (s) is obtained by replacing s_n by s/Ω_c in the normalized transfer function of the low pass filter function

- When N is even,

$$\therefore H(s) = \prod_{k=1}^{\frac{N}{2}} \left. \frac{B_k}{s_n^2 + b_k s_n + c_k} \right|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

- When N is odd,

$$\therefore H(s) = \frac{B_0}{s_n + c_0} \prod_{k=1}^{\frac{N-1}{2}} \left. \frac{B_k}{s_n^2 + b_k s_n + c_k} \right|_{s_n = \frac{s}{\Omega_c}} = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

DESIGN PROCEDURE FOR LOWPASS DIGITAL CHEBYSHEV IIR FILTER

5. Determine the transfer function of digital filter H(z). Using the suitable transformation to transform H(s) to H(z). When the Impulse invariant transformation is employed, if $T<1$, then multiply H(z) by T to normalize the magnitude.

6. Realize the digital filter transfer function H(z) by a suitable structure

7. Verify the design by sketching the frequency response $H(e^{j\omega})$

$$\mathbf{H(e^{j\omega}) = H(z) / z = e^{j\omega}}$$

ASSESSMENT

1. What is Chebyshev approximation?
2. How will you choose the order N for a Chebyshev Filter?
3. List the Properties of Chebyshev Filter.
4. Compare Analog filter and Digital filter?
5. Analog filter transfer function is converted to a digital filter transfer function by using either -------------------- (or) -------------------------
6. Define Sampling Time.
7. Attenuation Constant is given by ----------------
8. List the types of Chebyshev filters.

THANK YOU