SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT203 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

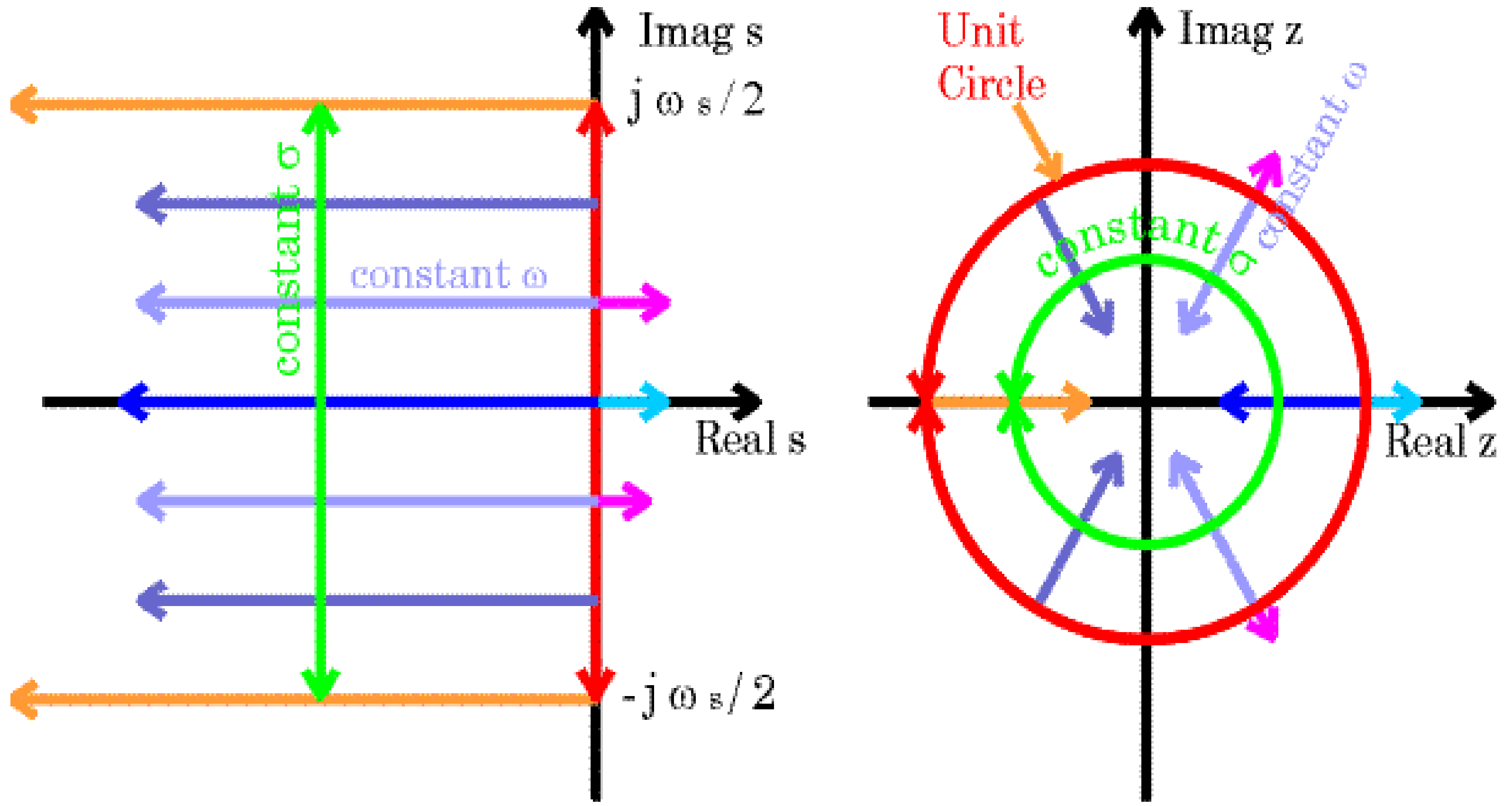
UNIT 2 – IIR FILTER DESIGN

TOPIC – REALIZATION STRUCTURES FOR IIR FILTERS

Z TRANSFORM

- Z transform is used for the analysis of discrete time signals.
- It is more broad compared to Discrete Time Fourier Transform
- It is very much useful in discrete time signals as well as system analysis
- x(n) and X(Z) is called Z transform pair

x(n) ⟺ **X(Z)**



LTI DT SYSTEM

- **System Transfer Function:** Ratio of the output to the input.

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

- **Frequency Response:**

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

DIFFERENCE EQUATION

- **Difference Equation:** It is an efficient way to implement discrete time systems
- The convolution of input sequence x(n) and unit sample response h(n) gives the output y(n)

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

- Two types of systems depending upon the length of unit sample response h(k)

LTI DISCRETE TIME SYSTEMS

- **Finite Impulse Response (FIR) Systems:** Unit sample response (or) Impulse response h(n) has finite no. of terms

$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

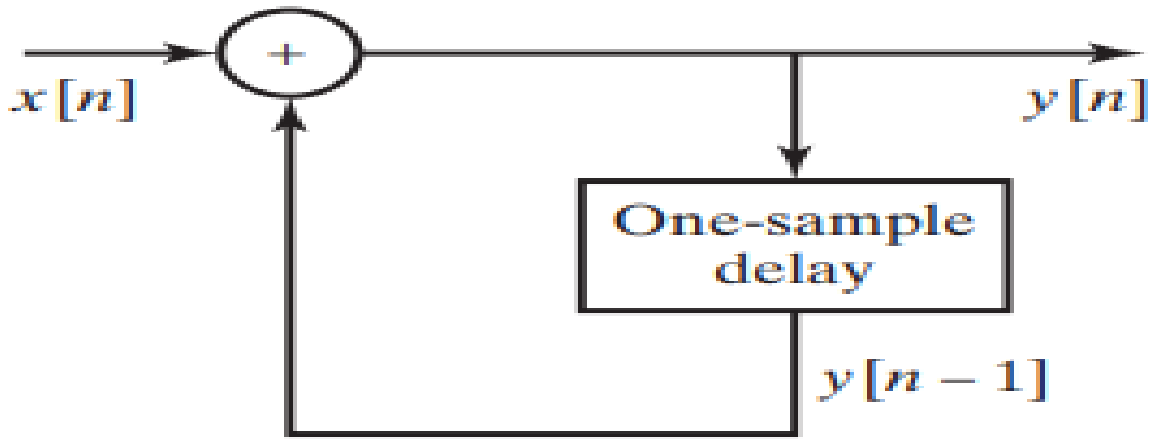
- **Infinite Impulse Response (IIR) Systems:** Length of Unit sample response (or) Impulse response h(n) is infinite

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

- **Recursive Systems:** Output y(n) depends on present and past inputs as well as past output

$$y(n) = \sum_{k=0}^{n} x(k)$$

- **Non Recursive Systems:** Output y(n) depends on present and past input.



$$y(n) = \sum_{k=0}^{M} h(k)\, x(n-k)$$

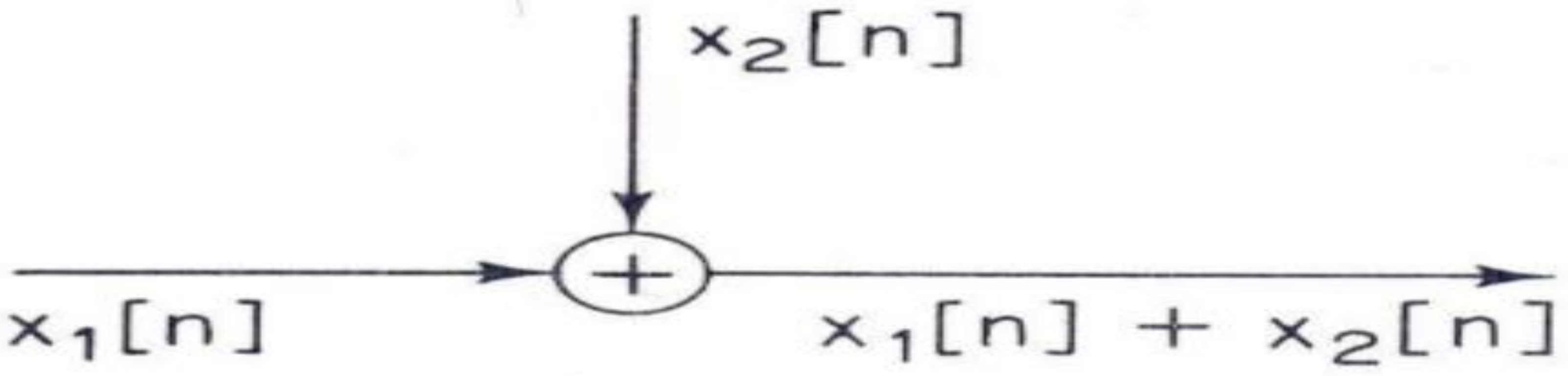
BLOCK DIAGRAM

- The discrete time systems are represented by block diagrams.
- They are also called structures of discrete time systems.
- It can be classified into four types

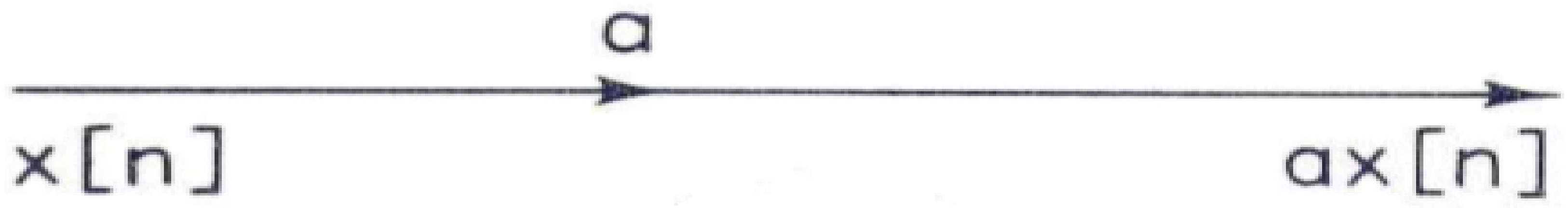
1. Direct Form I
2. Direct Form II
3. Cascade Form and
4. Parallel Form

ELEMENTARY BLOCKS

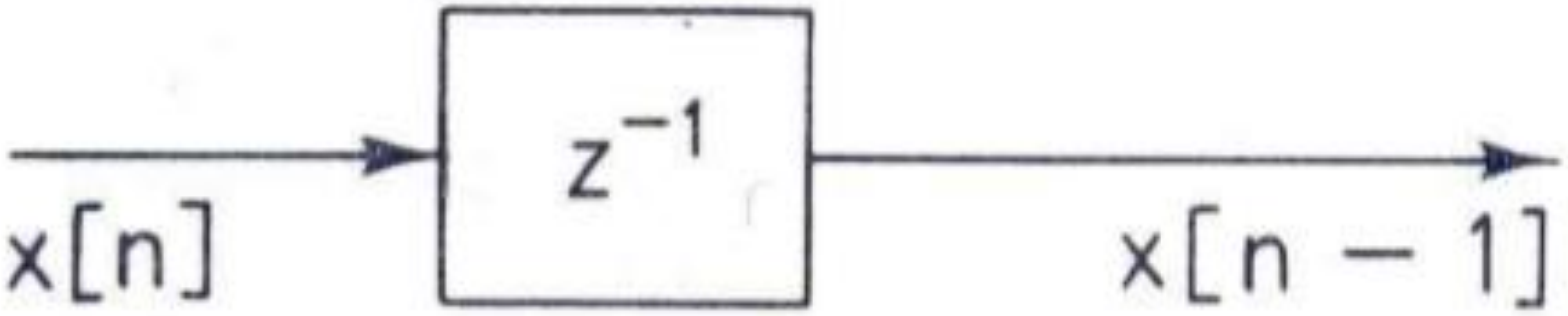
Adder



Constant Multiplier

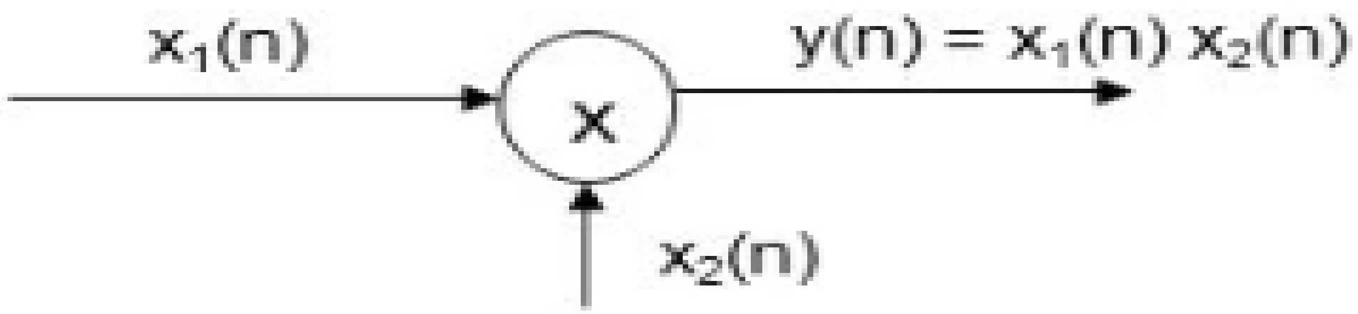


Unit Delay Element

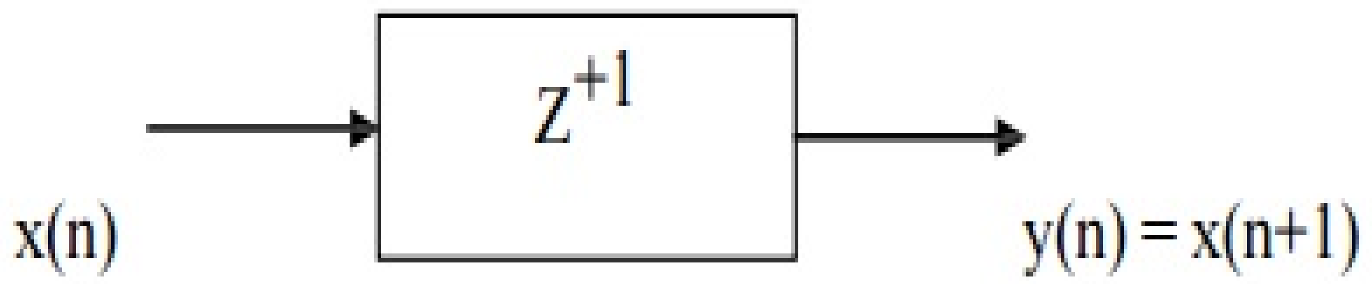


ELEMENTARY BLOCKS

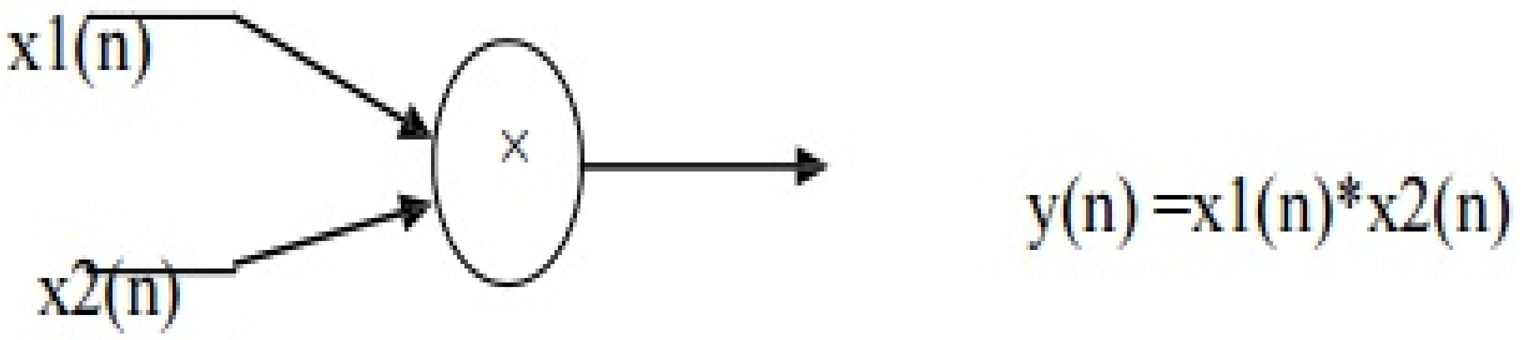
Signal Multiplier



Advancing Element

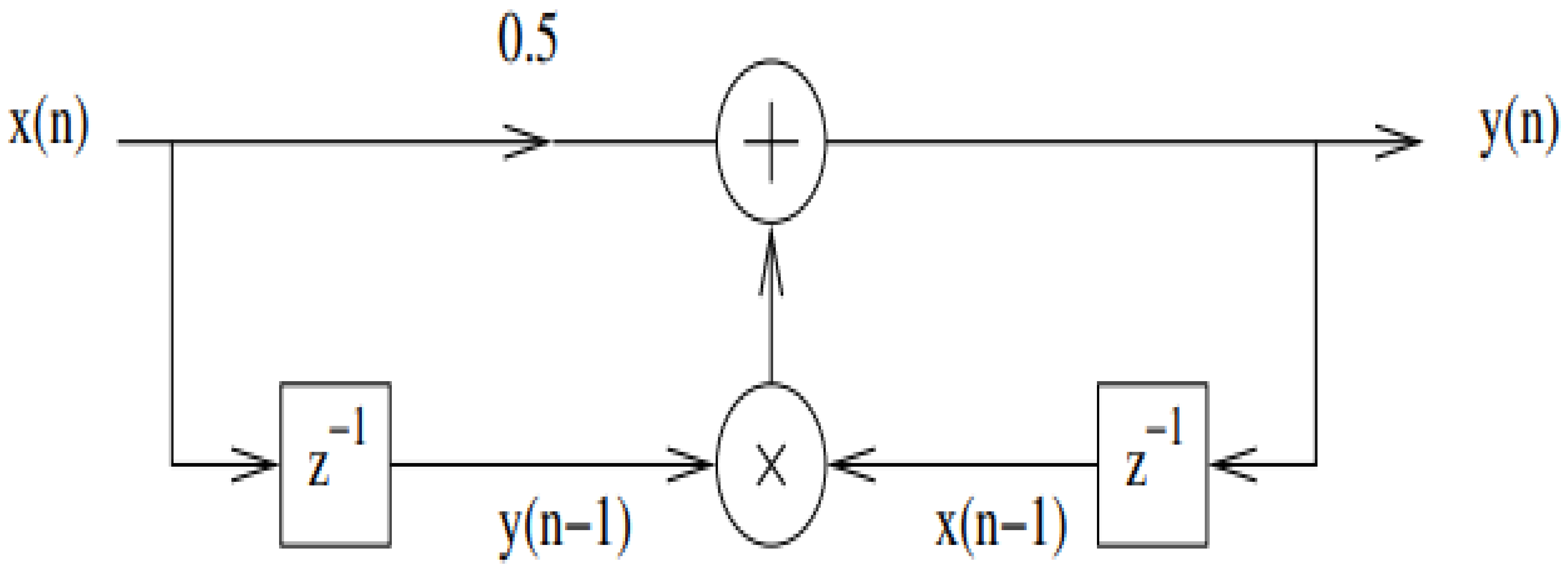


Multiplication

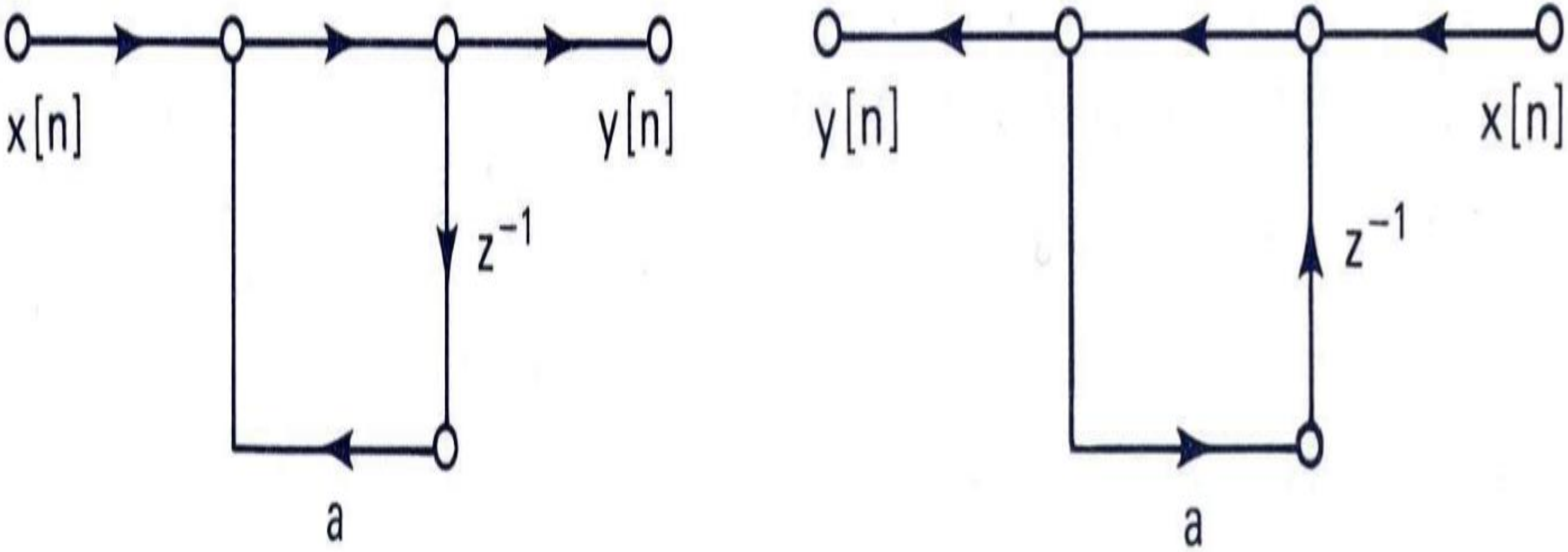


BLOCK DIAGRAM REPRESENTATION

$$y(n) = y(n-1)\,x(n-1) + 0.5\,x(n)$$

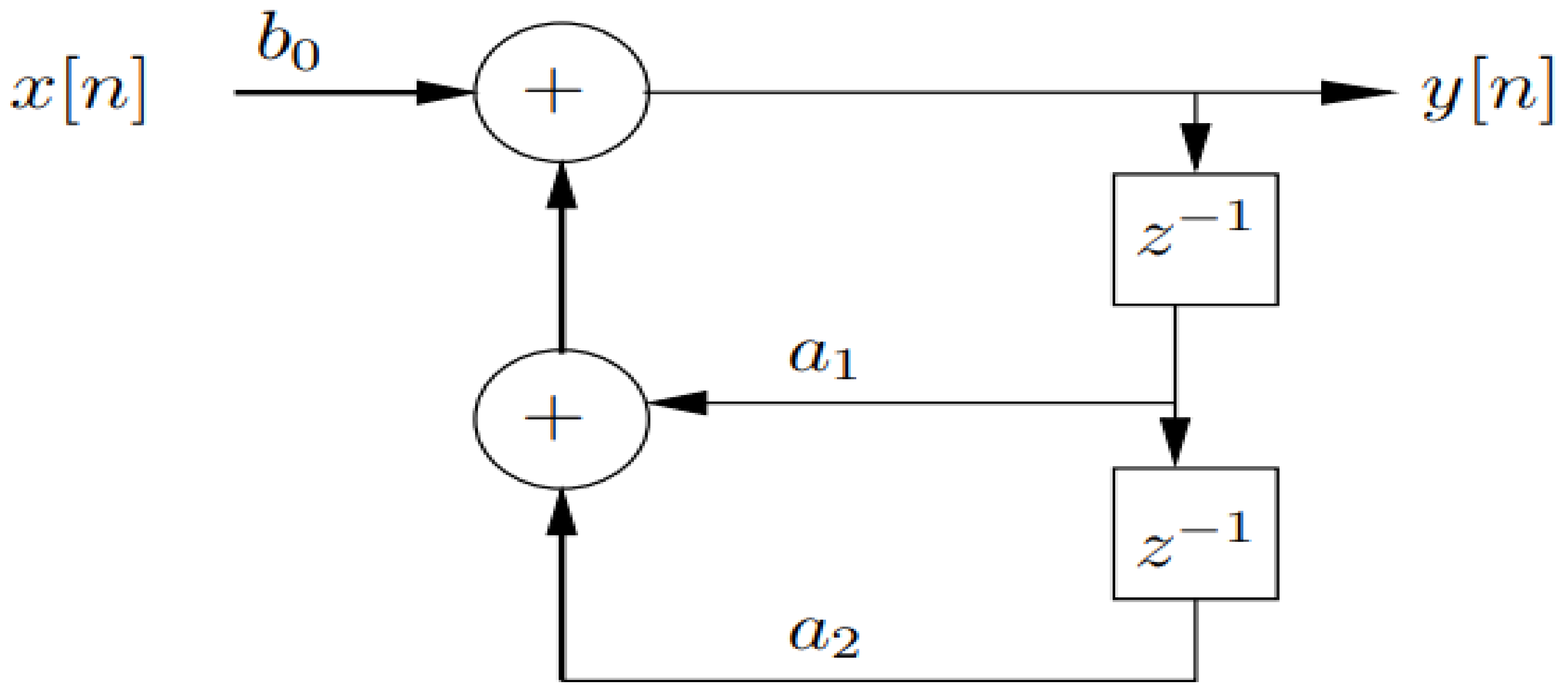


$$H(z) = 1/\ 1 - a\ z^{-1}$$



BLOCK DIAGRAM REPRESENTATION

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n]$$



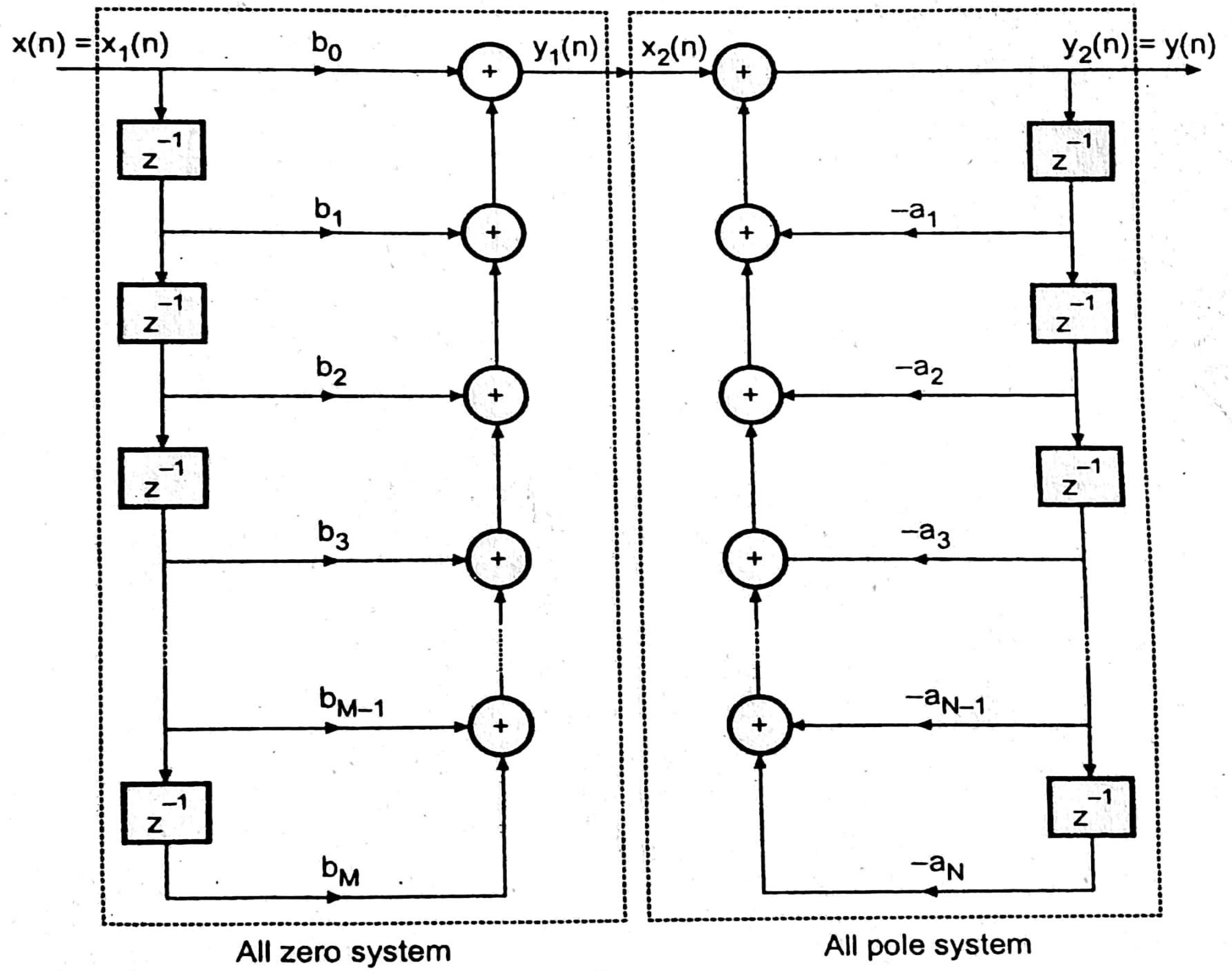
GENERAL DIRECT FORM I

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$\therefore H(z) = H_1(z) \cdot H_2(z).$$



DIRECT FORM II

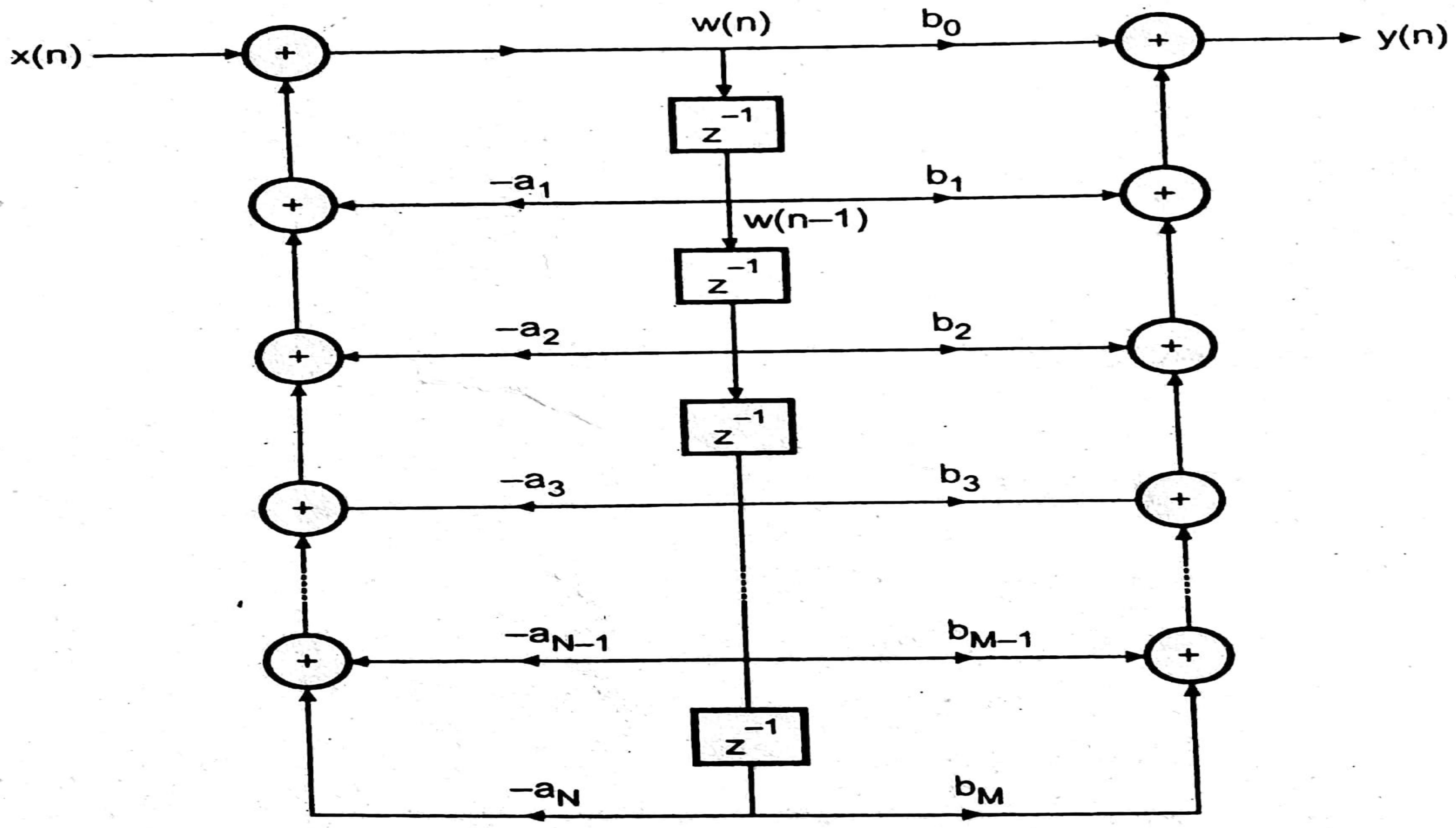
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H(z) = \frac{Y(z)}{X(z)} \Rightarrow \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} \Rightarrow \frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)}$$

$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

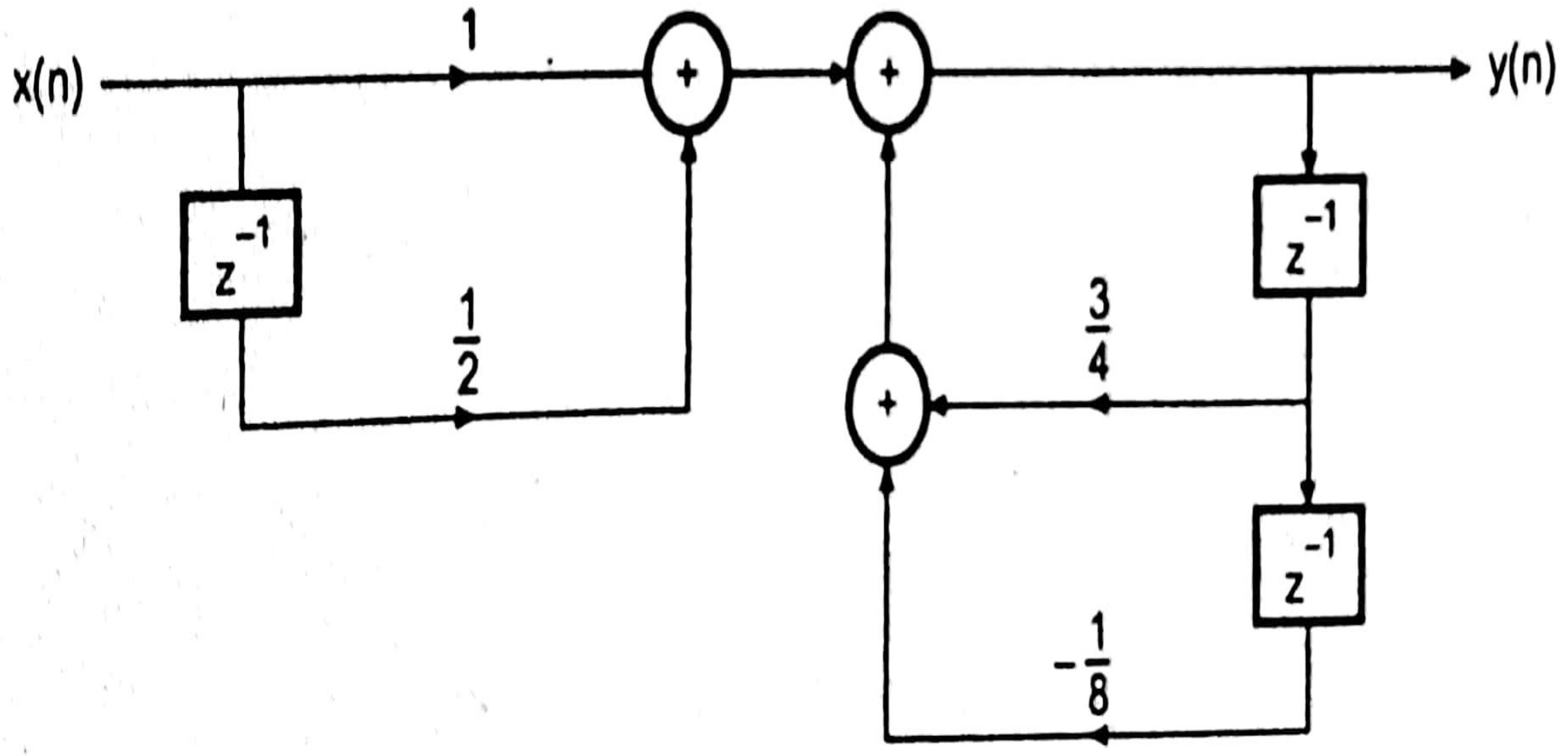
$$H_2(z) = \frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

DIRECT FORM II



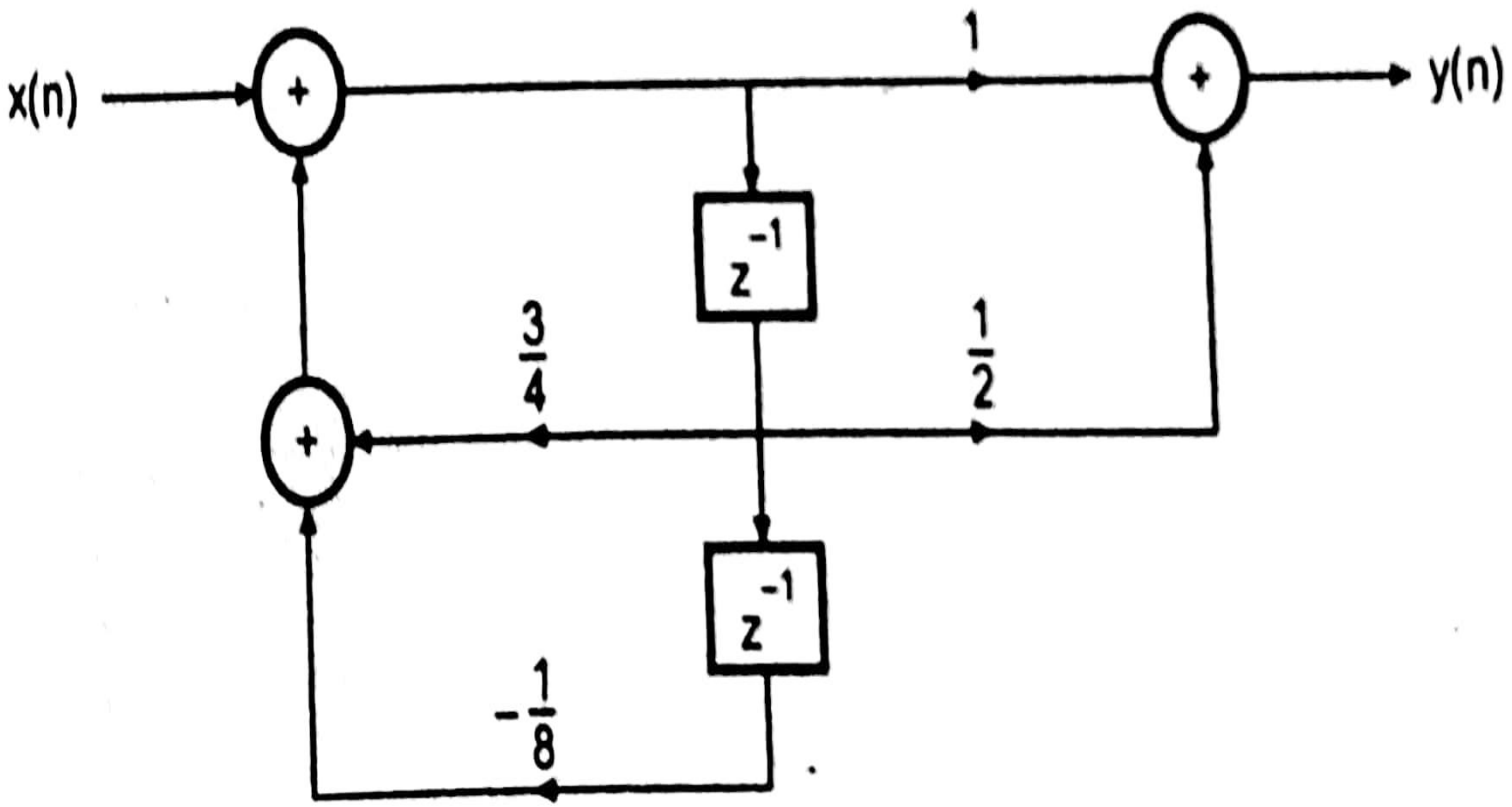
DIRECT FORM I

$$y(n) = 3/4\, y(n-1) - 1/8\, y(n-2) + x(n) + 1/2\, x(n-1)$$



DIRECT FORM II

$$y(n) = 3/4\ y(n-1) - 1/8\ y(n-2) + x(n) + 1/2\ x(n-1)$$



DIRECT FORM I & II

Consider System function : Direct Form I & II

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

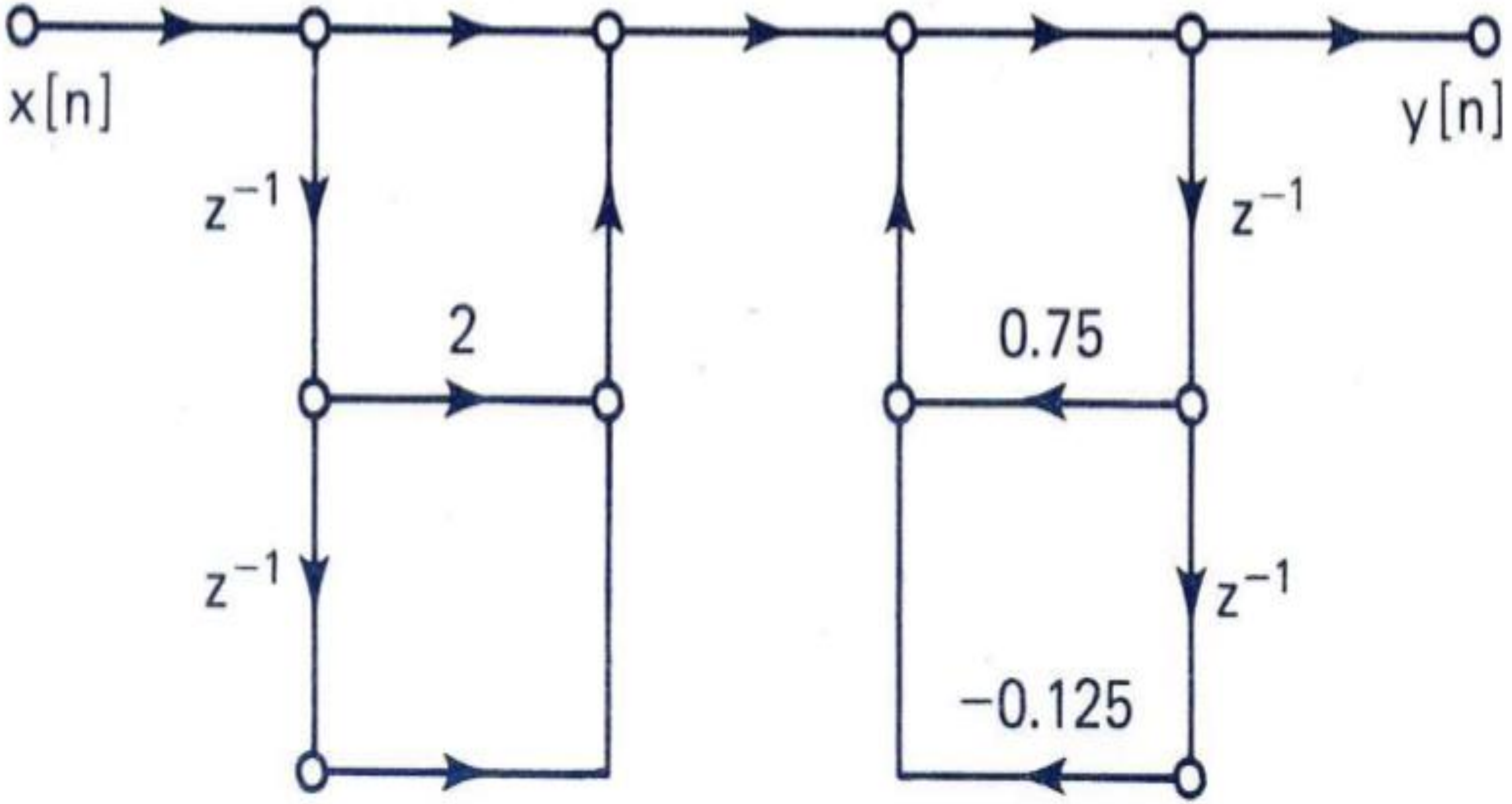
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$X(z) + 2z^{-1}X(z) + z^{-2}X(z) = Y(z) - 0.75z^{-1}Y(z) + 0.125z^{-2}Y(z)$$

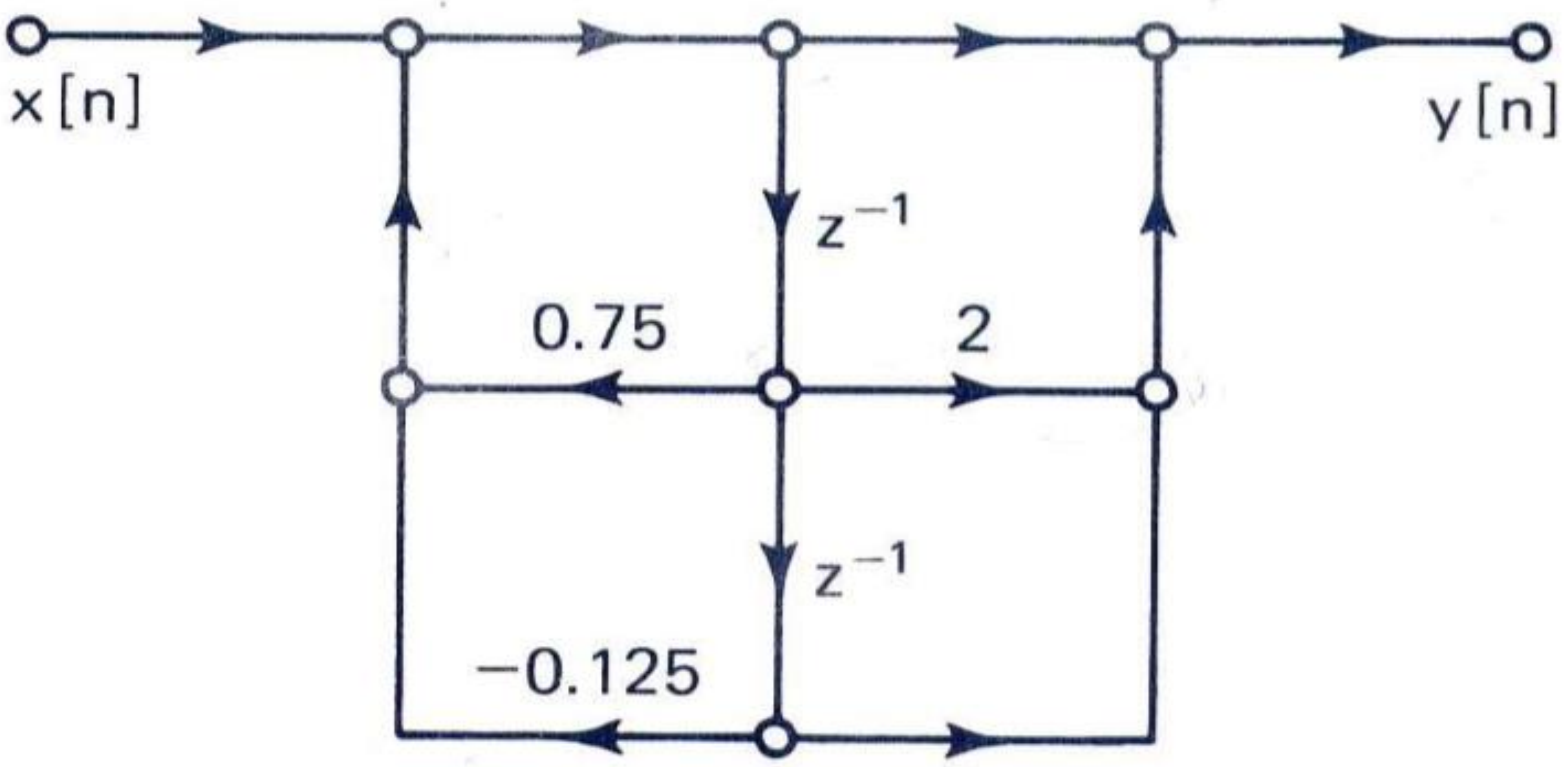
$$x(n) + 2x(n-1) + x(n-2) = y(n) - 0.75y(n-1) + 0.125y(n-2)$$

$$y(n) = x(n) + 2x(n-1) + x(n-2) + 0.75y(n-1) - 0.125y(n-2)$$

DIRECT FORM I



DIRECT FORM II



CASCADE & PARALLEL FORM

Cascade form :-

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$= \frac{(1+z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})}$$

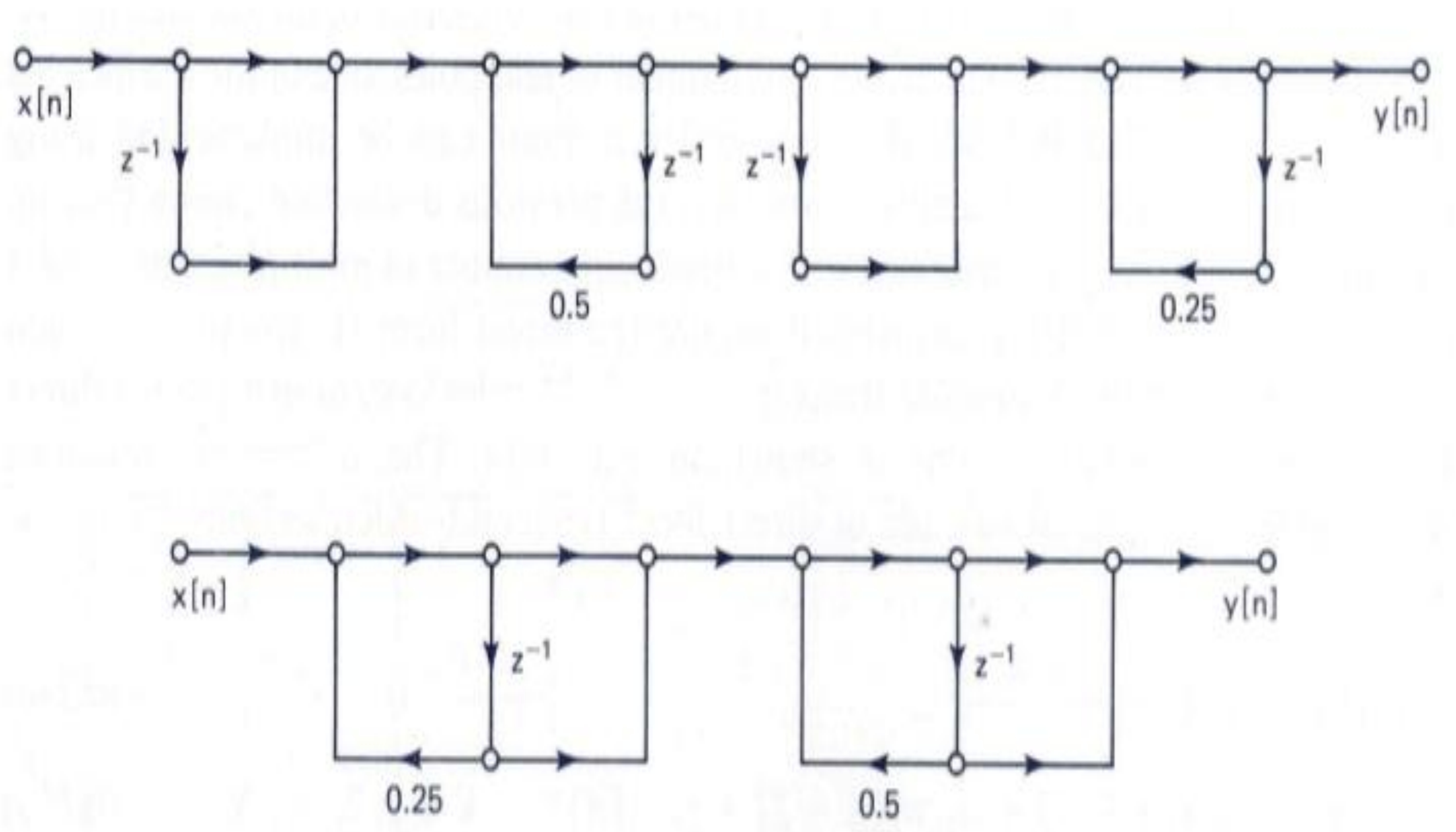
Parallel form :-

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

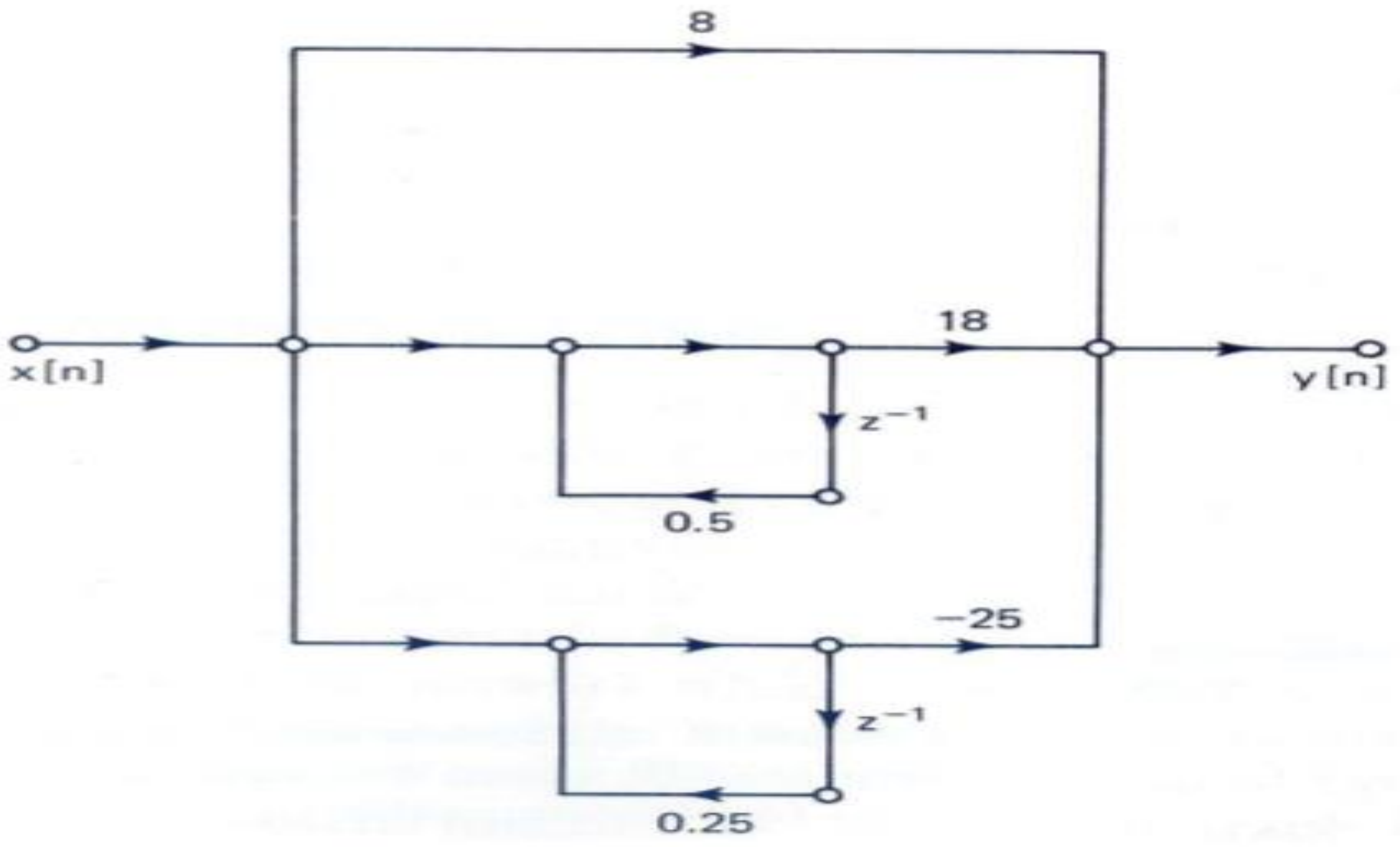
$$= 8 + \frac{-7 + 8z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$= 8 + \frac{18}{1-0.5z^{-1}} - \frac{25}{1-0.25z^{-1}}$$

CASCADE FORM



PARALLEL FORM



ASSESSMENT

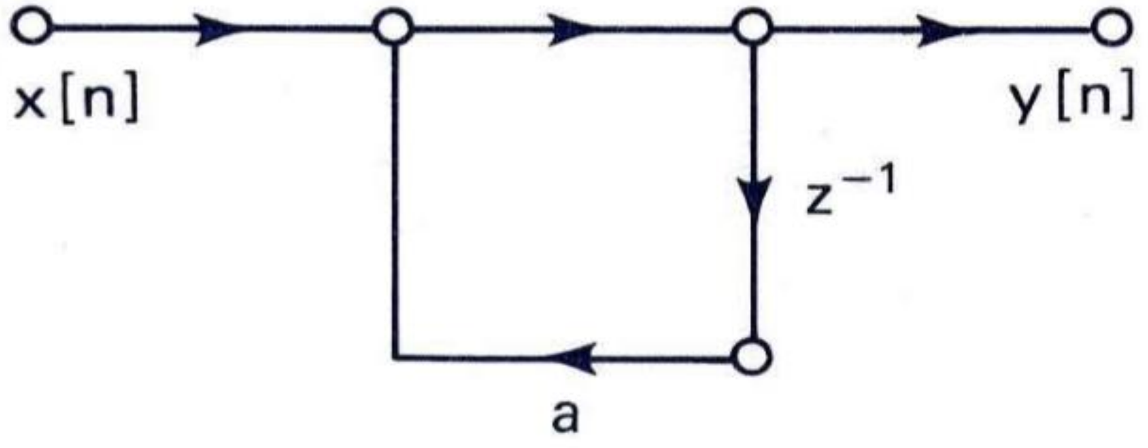
1. Define block diagram.

2. Mention the structures to represent block diagram.

3. The system transfer function of LTI DT system is ----------------

4. List the summary of elementary blocks to represent discrete time systems.

5. Identify the difference equation:



THANK YOU