SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF MECHANICAL ENGINEERING

23MET205 & STRENGTH OF MATERIALS

II YEAR / IV SEM

UNIT – 1

STRESS, STRAIN & DEFORMATION OF SOLIDS

ANALYSIS OF BARS OF COMPOSITE SECTIONS

A bar, made up two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compressive when subjected to an axial tensile or compressive loads, is called a composite bar.

For the composite bar the following two points are important:

1. The extension or compression in each bar is equal. Hence determination per unit length i.e. strain in each bar is equal.
2. The total external load on the composite bar is equal to the sum of the loads carried by each different material.

A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter of 4cm. The composite bar is ten subjected to an axial pull of 45000 N. If the length of each bar is equal to 15cm, determine. (i) The stresses in the rod and tube, and (ii) Load carried by each bar Take E for steel = 2.1 x 105 N/mm2 and for copper = 1.1 x 105 N/mm2

Sol Given:

Dia of steel rod = 3cm = 30mm

∴ Area of steel rod,

$$A_e = \frac{\pi}{4}(30)^2 = 706.86\text{mm}^2$$

External dia. of copper tube
= 5cm = 50mm

Internal dia. of copper tube
= 4cm = 40mm

∴ Area of copper tube,

$$A_e = \frac{\pi}{4}(50^2\text{-}40^2)\text{mm}^2 = 706.86\text{mm}^2$$

Axial pull on composite bar, P = 45000 N
Length of each bar L = 15cm
Young's modulus for steel, $E_S = 2.1 \times 10^5$ N/mm^2
Young's modulus for copper $E_c = 1.1 \times 10^5$ N/mm^2
(i) The stress in the rod and tube
Let σ_S = Stress in steel
P_S = Load carried by steel rod
σ_c = Stress in copper, and
P_c = Load carried by copper tube.

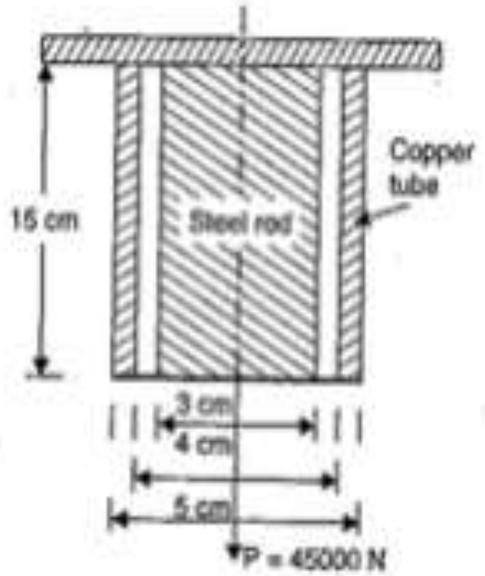

Fig. 1.9 Composite bar

Now strain in steel = Strain in copper $\left(\because \frac{\sigma}{E} = \text{Strain}\right)$

or $$\frac{\sigma_S}{E_S} = \frac{\sigma_c}{E_c}$$

$$\therefore \sigma_S = \frac{E_S}{E_c} \times \sigma_c = \frac{2.1 \times 10^6}{11 \times 106} \times \sigma_c = 1.900\ \sigma_c$$

Now Stress $= \frac{\text{Load}}{\text{Area}}$, $\therefore$ Load = Stress x Area

Load on steel + load on copper = Total load

$\sigma_S \times A_S + \sigma_c \times A_c = P$ $(\because$ Total Load = P)

or $1.909\ \sigma_c \times 706.86 + 706.86 = 45000$

or $\sigma_c\ (1.909 \times 706.86 + 706.86) = 45000$

or $2056.25\ \sigma_c = 45000$

$$\therefore \sigma_c \frac{45000}{2056.25} = 21.88 \text{N/mm}^2 \text{Ans}$$

Substituting the value of σ_c in equation (i), we get

$\sigma_c = 1.909 \times 21.88$ N/mm^2

$= 41.77$ N/mm^2. Ans

(ii) Load carried by each bar

As Load = Stress x Area

$\therefore$ Load carried by steel rod

$P_s = \sigma_S \times A_S$

$=$ $41.77 \times 706.86 = 29525.5$ N. Ans

Load Carried by copper tube,

$P_c = 45000 - 29525.5$

$=$ 15474.5 N. Ans

A compound tube consists of a steel tube 140mm internal diameter and 160mm external diameter and an out brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900 kN. Find the stresses and the load carried by each tube and the amount if 21 shortens. Length of each tube is 140mm. Take E for Steel as 2 x 105 N/mm2 and for brass as 1 x 105 N/mm2 .

Sol Given:

Internal dia. of steel tube = 140mm

External dia. of steel tube = 160mm

$\therefore$ Area of steel tube, $A_a = \frac{\pi}{4}(160^2 - 140^2) = 4712.4 mm^2$

Internal dia. of brass tube = 160mm

External dia. of brass tube= 180mm

$\therefore$ Area of steel tube, $A_b = \frac{\pi}{4}(180^2 - 160^2) = 5340.7.4 mm^2$

Axial load carried by compound tube,

$P = 900\ kN = 900 \times 1000 = 900000N$

Length of each tube $L = 140mm$

E for steel $E_a = 2 \times 10^5\ N/mm^2$

E for brass $E_b = 1 \times 10^5\ N/mm^2$

Let σ_a = Stress in steel in N/mm^2 and

σ_b = Stress in brass in N/mm^2

Now strain in steel = Strain in brass $\left(\because \text{Strain} \frac{\text{Stress}}{E}\right)$

or $\frac{\sigma_S}{E_S} = \frac{\sigma_b}{E_b}$

$\therefore \sigma_S = \frac{E_a}{E_b} \times \sigma_b = \frac{2 \times 10^6}{1 \times 10^5} \times \sigma_b = 2\sigma_b$

Now load on steel + Load on brass = Total load

or $\sigma_S \times A_a + \sigma_b \times A_b = 900000$ ($\because$ Load = Stress x Area)

or $2\sigma_b \times 4712.4 + \sigma_b \times 5340.7 = 900000$ ($\because \sigma_S = 2\sigma_b$)

or $147655\ \sigma_b = 900000$

$\therefore \sigma_b = \frac{900000}{14765.5} = 60.95 N/mm^2$ .Ans

Substituting the value of P_b in equation (i), we get

$\sigma_s = 2 \times 60.95 = 121.9\ N/mm^2$. Ans.

Load carried by brass tube

= Stress x Area

$= \sigma_b \times A_b = 60.95 \times 5340.7N$

$= 325515\ N = 325.515\ kN$ Ans.

Load carried by steel tube

$= 900 - 325.515 = 574.485\ kN$. Ans.

Decrease in the length of the compound tube

= Decrease in length of either of the tubes

= Decrease in length of brass tube

= Strain in brass tube x original length

$= \frac{\sigma_b}{E_b} \times L = \frac{60.95}{1 \times 10^5} \times 140 = 0.0853\ mm$ Ans