UNIT - 2
STRESSES IN BEAMS

Syllabus.
Types of beams - supports and Loads - Shear force and BM in beams - Cantilever and SSB - overhanging beams - Bending stresses in beams - Applications

Beam - The beam is a horizontal member used in any structure to transfer the load to the column (or) wall

Primary purpose - To resist the externally load applied to it and distribute it to the foundation through the column.

Horizontal beam - carries only Transverse load
sloping beams - carries both Transverse and axial load

The main difference b/w "beam & column" is the beam resists the transverse load and the column transfers the compression load to the Foundation

Types of Beams:
Based on the design requirements, there are many types of beams in construction. It can be classified

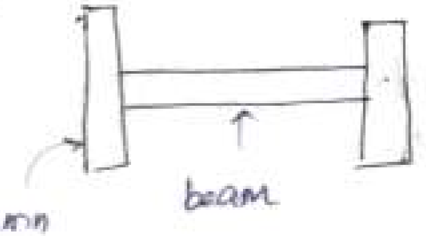
1. Based on Loads & supports
2. Based on Geometry
3. Based on shapes of cross section
4. Based on Equilibrium condition
5. Based on Material
6. Based on construction methods

Based on "Loads & supports":
1. simply supported beam 2. Fixed beam 3. overhanging beam
4. Double overhanging beam 5. continuous beam
6. cantilever beam 7. Trussed beam

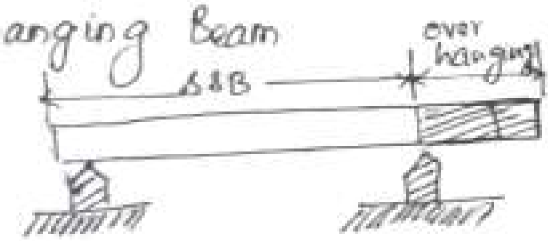
Simply supported Beam:
It is the SSB type in structure. It contains one end roller support and another end provides Pinned support

so The simple beam contains support at both ends, but it is free to rotate.

Fixed Beam. The beam has support at both ends and it resists the motion. Both the ends q beam resists on either wall or column.



Overhanging Beam



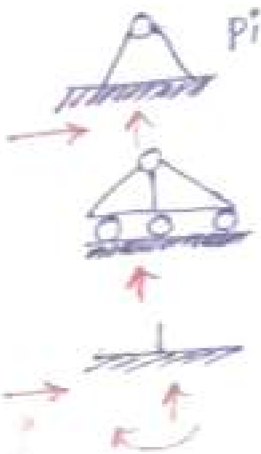
If the end portion q beam is extended

Continuous Beam.

A beam which is provided more than 2 supports

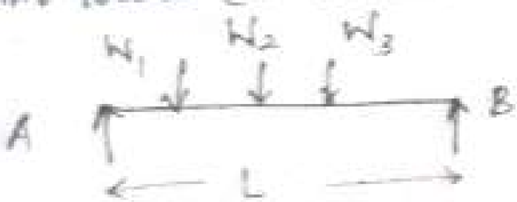
Types q support

supports resist the movements due to the external loads.

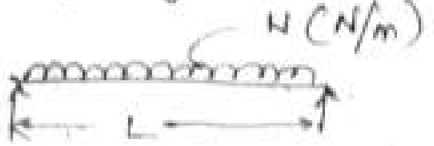


Types q loads

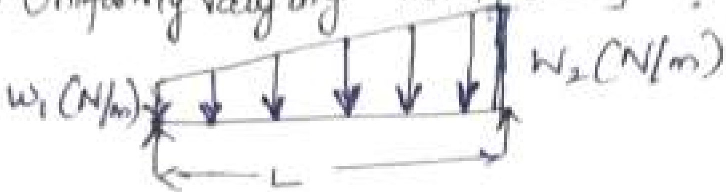
i point loads (or) concenterated loads

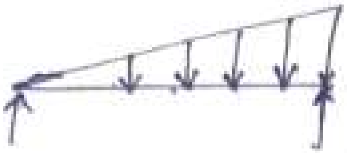


(ii) Uniformly distributed load [UDL]



(iii) Uniformly Varying Load [UVL]

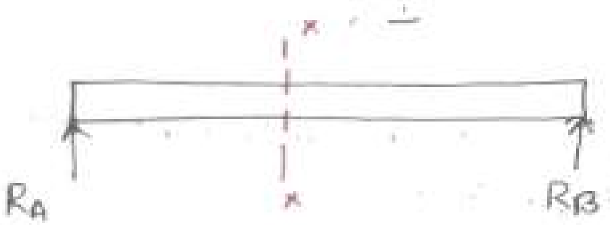


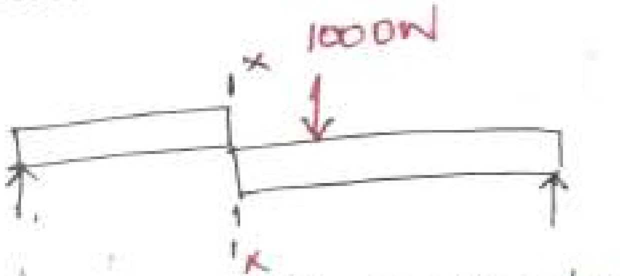


SIGN CONVENTIONS FOR SHEAR FORCE & BENDING MOMENT:

What is shear force diagram?

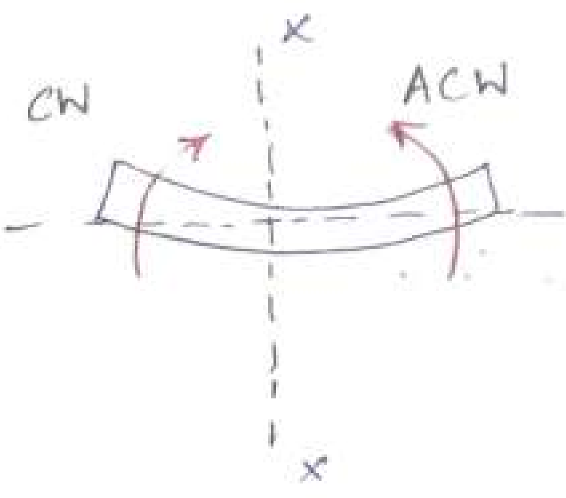
A SFD is the one which shows the variation in shear force along the length of the beam

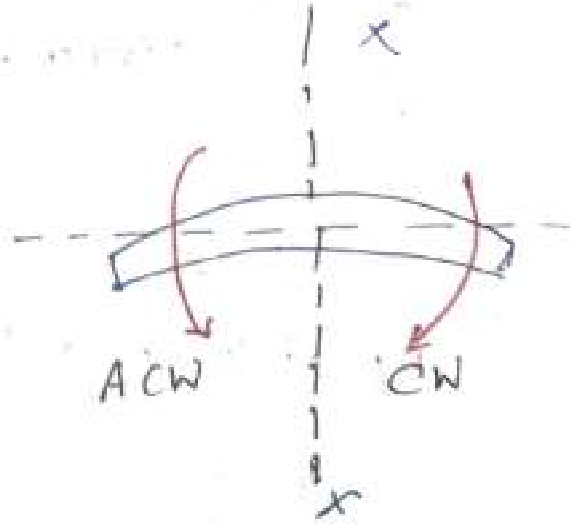




At RHS → After the (x-x) load(↓) [+ve] shear force

LHS → At the left (x-x) load(↑) upwards [-ve] shear force

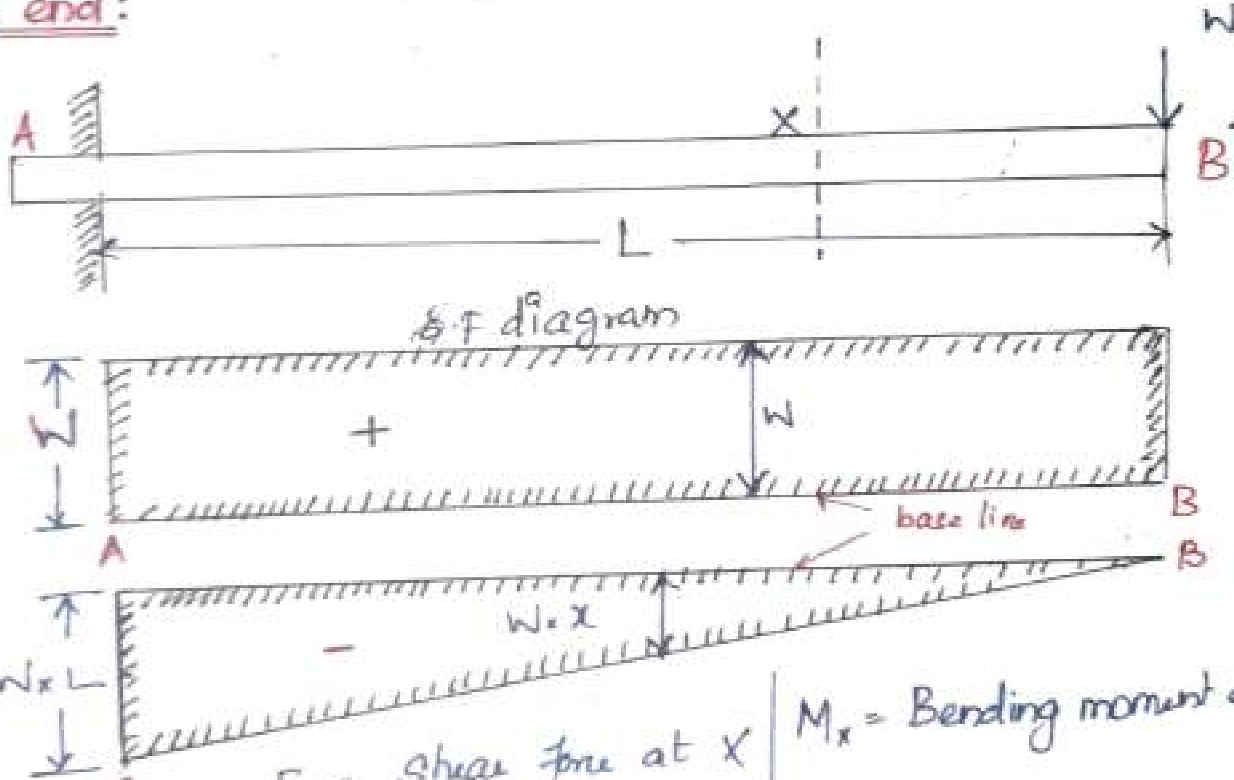




Important points for drawing SFD & BMD:

1. consider the left (or) right portion of section
2. Add the forces [including Reaction] normal to the beam on one of the portion, a force on the right portion acting downwards is '+ve' while force acting upwards is negative.
3. The '+ve' values of shear force and B.M are plotted above the base line and negative values below the base line
4. The SFD will increase (or) decrease suddenly (i.e) by a vertical st. line, where there is vertical point load will be horizontal.
5. The shear force b/w any two vertical loads will be constant and hence the SFD b/w two vertical loads will be horizontal.
6. The BM at the two supports of SSB and at the free end of cantilever ~~wis~~ will be zero.

SFD & BMD for a cantilever with point load at the Free end:



F_x = Shear force at X

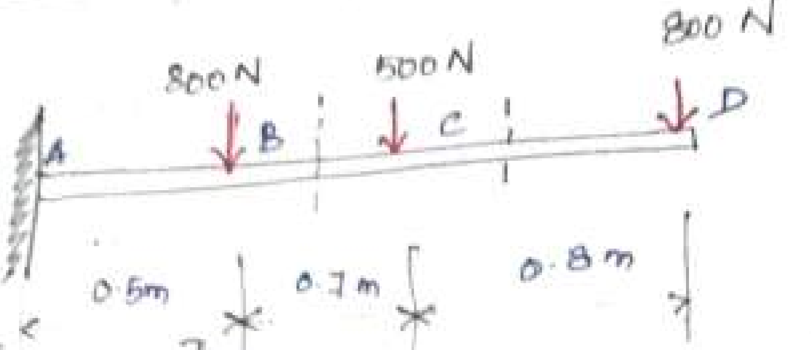
M_x = Bending moment at X

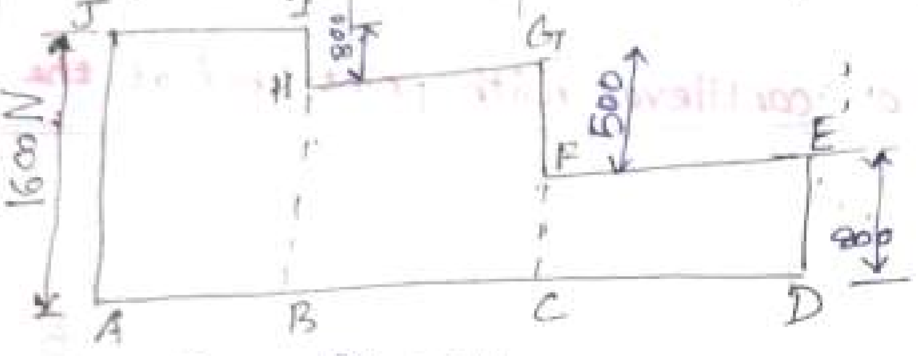
The S.F and B.M.D for several concentrated loads acting on cantilever

PROBLEMS

1. A Cantilever beam of length 2m carries the point loa[ds] as shown in fig. Draw the SFD and BMD for th[e] cantilever beam

<u>Given</u> $l = 2m$;





Shear Force Diagram

<u>Shear Force Diagram</u>

S.F is defined as unbalanced vertical forces to the right (or) left side of section.

S.F at D (or) $F_D = +800$ N

S.F at C (or) $F_C = +800 + 500 = +1300$ N

S.F at B (or) $F_B = +800 + 500 + 300 = +1600$ N

S.F at A (or) $F_A = +1600$ N

S.F takes place b/w 'A & B' is <u>constant</u>
'B & C' is <u>constant</u>
'C & D' is <u>constant</u>

<u>Bending Moment diagram</u>

The B.M at D is <u>zero</u>

(1) The B.M at any section b/w C & D at a distance x and D is given by

$$M_x = -800 \times x$$

which follows a straight line law.

At C, the value of $x = 0.8m$

$\therefore$ B.M. at C, $M_C = -800 \times 0.8 = -640$ N-m

ii) The B.M at any section b/w B & C at a distance 'x' from D is given by

$$M_x = -800 \times x$$

[At C, $x = 0.8$ and at B, $x = 0.8 + 0.7 = 1.5m$;] Hence here 'x' varies from 0.8 to 1.5

$$M_x = -800x - 500(x - 0.8)$$

B.M. also varies from B to C by straight line law.

Take B.M at B is obtaining by sub $x = 1.5m$

$$M_B = -800x - 500(x - 0.8) = -1550 \text{ N-m}$$

B.M at A is obtaind by sub $x = 2m$

$$M_A = -800 \times 2 - 500(2 - 0.8) - 800(2 - 1.5)$$

$$\boxed{M_A = -2350 \text{ N-m}}$$

Hence B.M at diff points will be given as

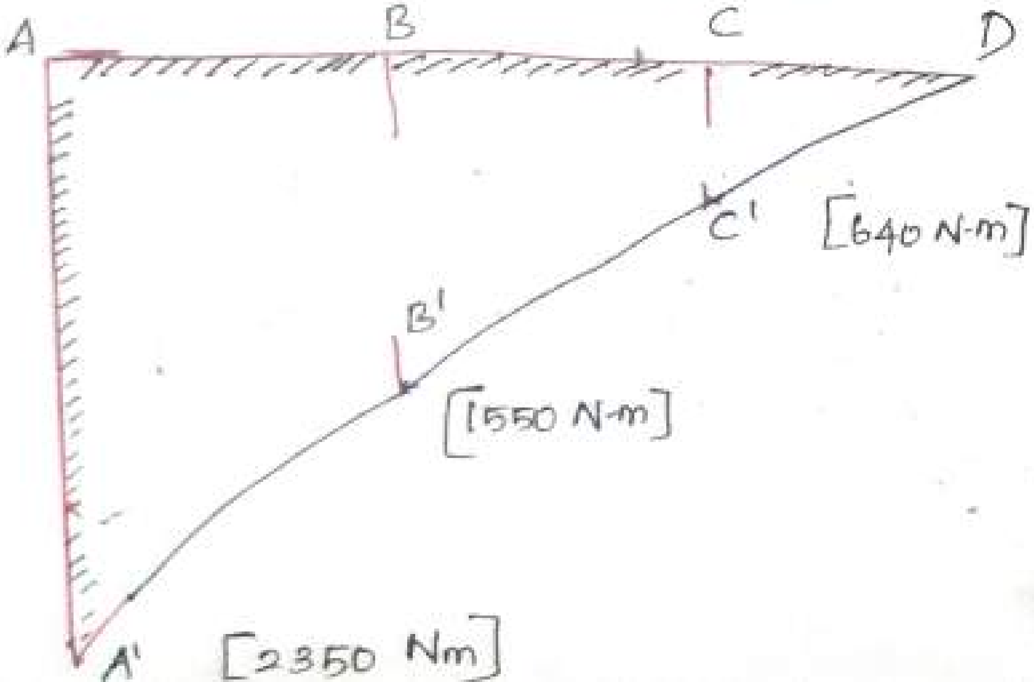
$M_D = 0$

$M_C = -640$ N-m

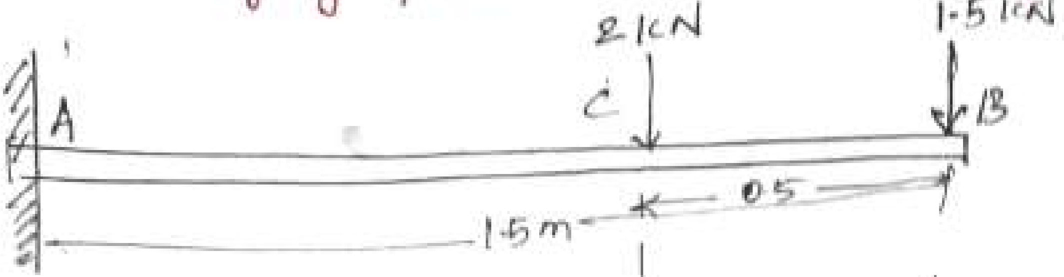
$M_B = -1550$ N-m

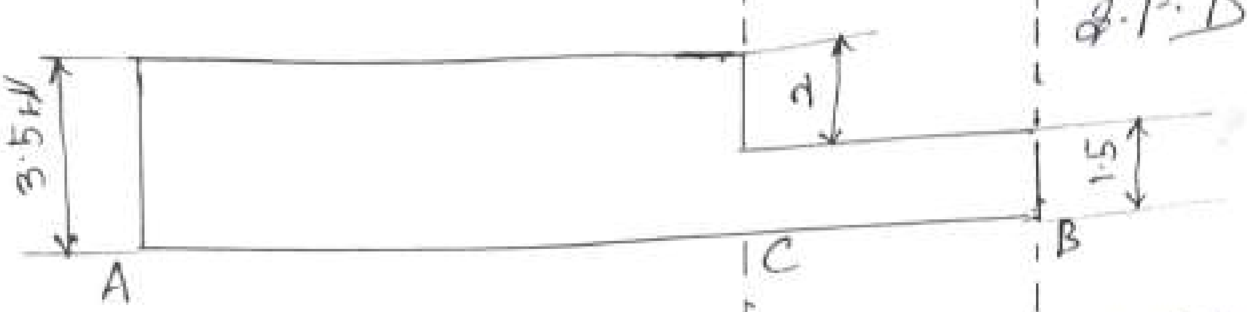
$M_A = -2350$ N-m

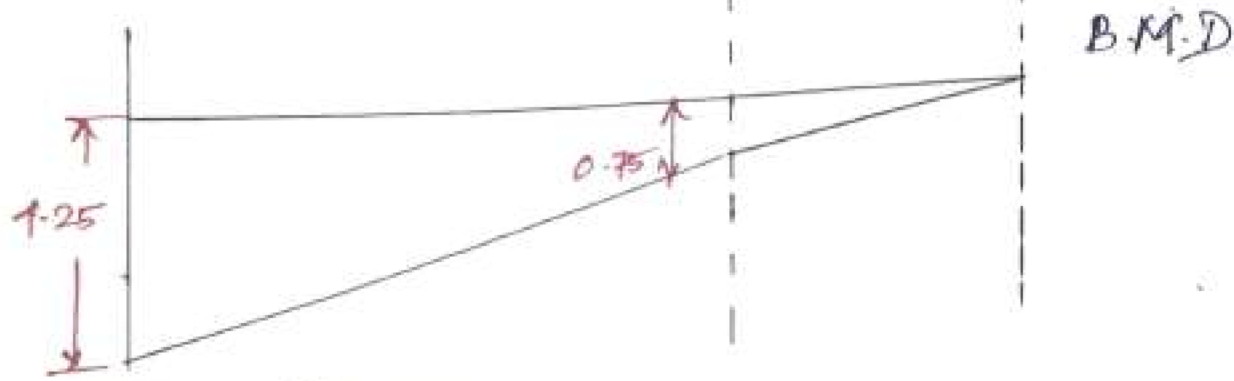
B.M. Diagram



Draw the shear force and B.M for a cantilever beam of span 1.5m. carrying point loads as shown in fig







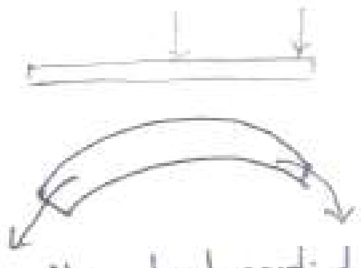
Shear Force Diagram:

$F_B = +W_1 = +1.5\ kN$

$F_C = +(1.5 + W_2) = +(1.5+2) = 3.5\ kN$

$F_A = +3.5\ kN$

B.M. Diagram | $M_B = 0$



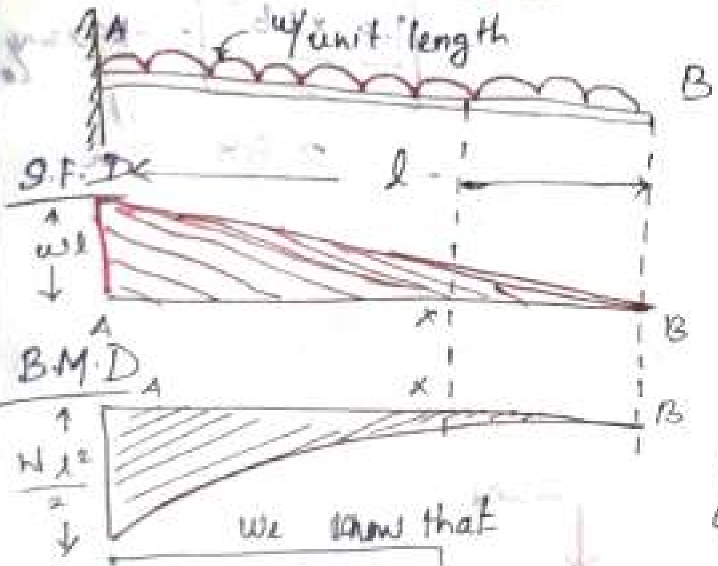
When the load applied, it creates bending like down ward direction hence it's 'Negative'

$M_C = -(1.5 \times 0.5) = -0.75\ kN\text{-}m$

$M_A = -[(1.5 \times 1.5) + 2 \times 1]$

$\boxed{M_A = -4.25\ kN.m}$

CANTILEVER with UDL:



Consider a cantilever AB of length l and carrying UDL of w/unit length

w.k.t

shear force at any section X at a distance 'x' from B

Thus shear force at B is zero and increases in a straight line at 'A' as shown in fig

we know that

$\boxed{F_x = +Wx}$ → shear force

At B, shear force is zero and increased by st. line

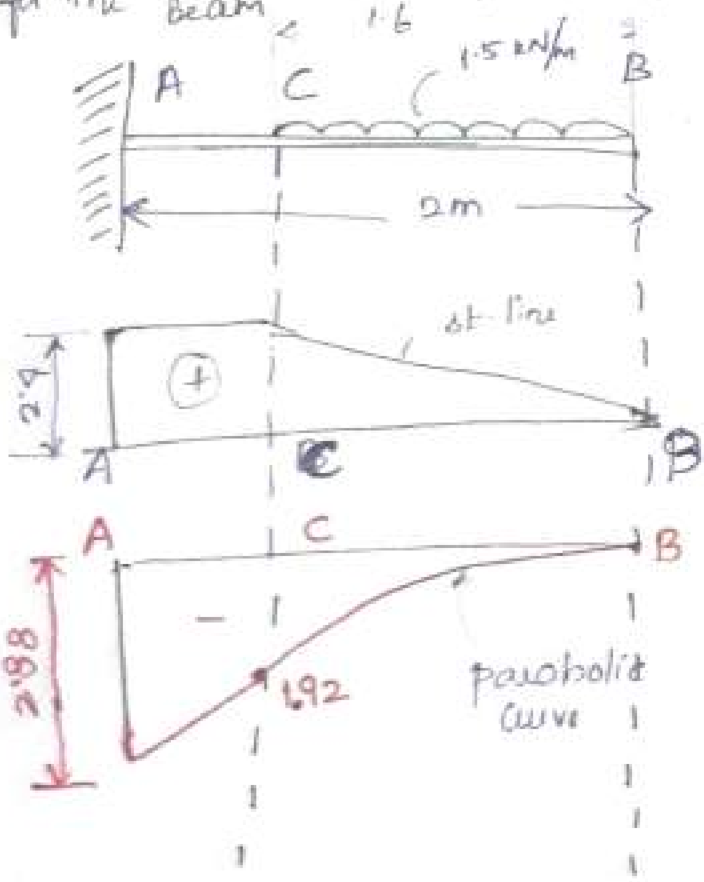
w.k.t B.M at X, therefore

$$M_x = -w \cdot x \cdot \frac{x}{2} = -\frac{wx^2}{2}$$

we also see that B.M is zero (where $x=0$) and increases in the form of parabolic curve to $-\frac{wl^2}{2}$ at B [where $x = l$]

Problem

A Cantilever beam AB 2 m long carries a UDL of 1.5 kN/m over a length of 1.6m from the free end. Draw S.F.D & B.M.D for the beam



Given

length = 2 m

UDL = 1.5 kN/m

length of UDL = 1.6 m

Shear force diagram

$F_B = 0$

$F_c = +W \cdot x = +1.5 \times 1.6$

$\boxed{F_c = +2.4 \text{ kN}}$

$F_A = F_B + F_c$

$\boxed{F_A = +2.4 \text{ kN}}$

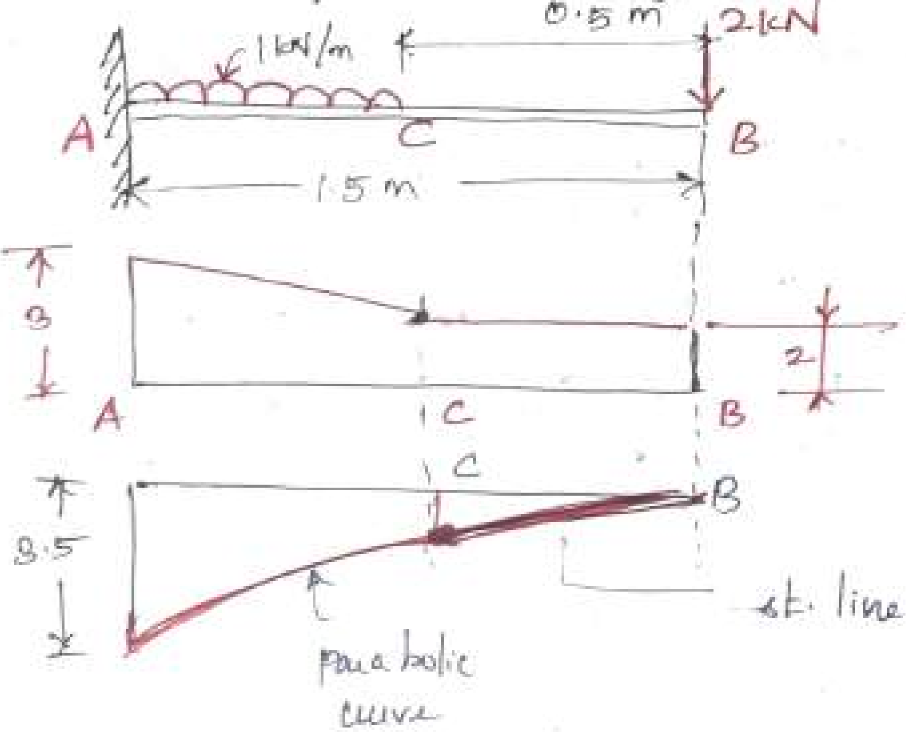
For B.M. Diagram

$$M_B = 0 \; ; \quad M_C = \frac{-W \cdot a^2}{2} = -\left[\frac{1.5 \times 1.6^2}{2}\right] = -1.92 \text{ kN-m}$$

$$M_A = -\left[(1.5 \times 1.6)\left[0.4 + \frac{1.6}{2}\right]\right] = -2.88 \text{ kN/m}$$

(1.e) d

A cantilever beam of 1.5 m span is loaded as shown in fig. Draw the shear force and bending Moment



$F_B = 2$ kN

$F_C = [2 \times \cancel{0.5}] + [1 \times 1] = 1 + 1 = 2$ kN

B.M. Diagram

$M_B = 0$

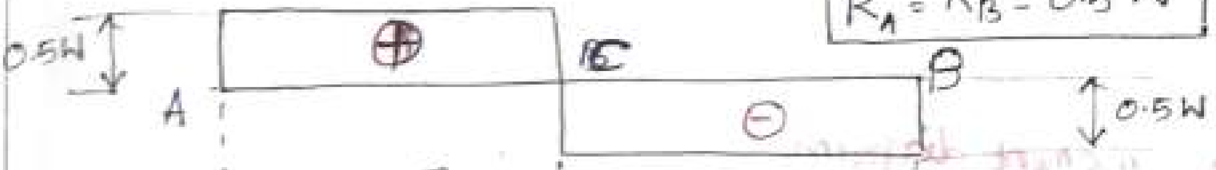
$M_C = -[2 \times 0.5] = -1$ kN-m

$M_A = -\left[(2 \times 1.5) + (1 \times 1) \times \frac{1}{2}\right] = -3.5$ kN-m

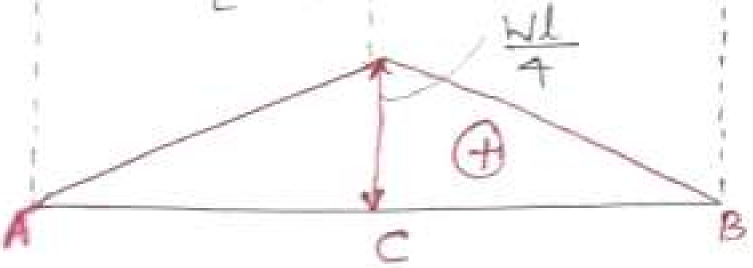
SSB with a Point Load at its midpoint

— [SSB]

$\boxed{R_A = R_B = 0.5W}$



[SFD]



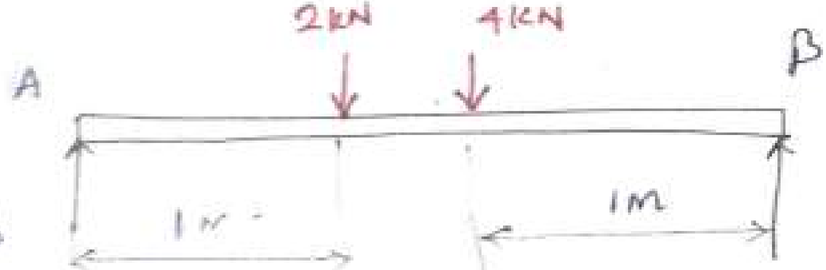
[B.M.D]

<u>wkt</u> Bending moment at A & B is zero.
It increases by a straight line. Therefore B.M. at C

$$M_C = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$$

<u>Problems</u>

A SSB AB of span 2.5m is carrying two point loads as shown in fig



Let us find out the Reactions

R_A & R_B. Taking moments about A

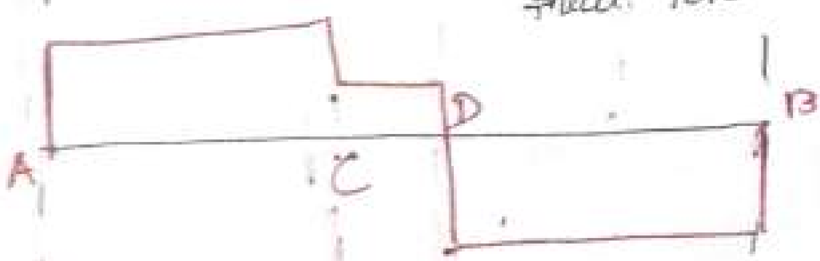
$R_B \times 2.5 = (2\times1)+(4\times1.5)$

$\boxed{R_B = 3.2 \text{ kN}}$

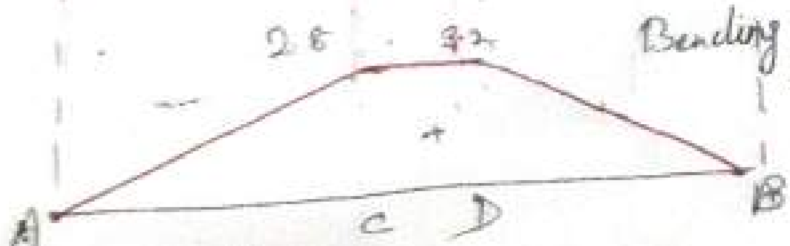
$R_A = (2+4) - 3.2$

$\boxed{R_A = 2.8 \text{ kN}}$

Shear Force Diagram



Bending moment diagram



i Shear force diagram

$F_A = +R_A = 2.8\,kN$

$F_C = +2.8 - 2 = 0.8\,kN$

$F_D = 0.8 - 4 = -3.2\,kN$

$F_B = -3.2\,kN$

Bending moment diagram

It is shown in fig

$M_A = 0;\ M_C = 2.8\,kN.m$

$M_D = 3.2 \times 1 = 3.2\,kN\text{-}m$

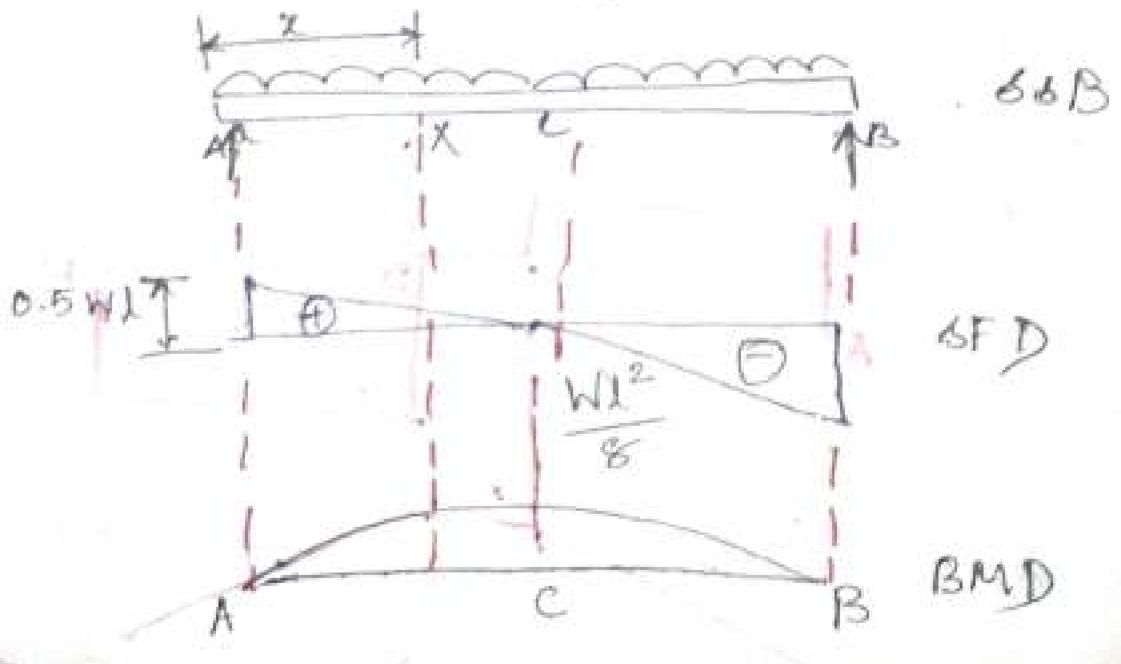
$M_B = 0$

The value of M_D also be found out from the Reaction R_A as follows

$M_D = (2.8 \times 1.5) - (2 \times 0.5) = 3.2\,kN\,m$

SSB with UDL

A beam AB of length l and carrying UDL/unit Length

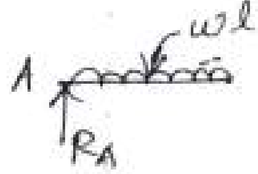


since the load is uniformly distributed of w/unit length
Therefore the Reactions at supports A.

$$R_A = R_B = \frac{Wl}{2}$$

wkt shear force at section 'X' at a distance 'x' from A.

$$F_x = R_A - \omega x \quad — ②$$



where, $\boxed{R_A = 0.5\,\omega l}$ sub in ②

$\therefore F_x = 0.5\,\omega l - \omega \cdot x =$
where $x = 0$, and decreases uniformly by a straight line law to zero at midpoint, beyond which continues to decreases uniformly to $-0.5\,\omega l$

wkt The bending moment at any section at a distance 'x' from A

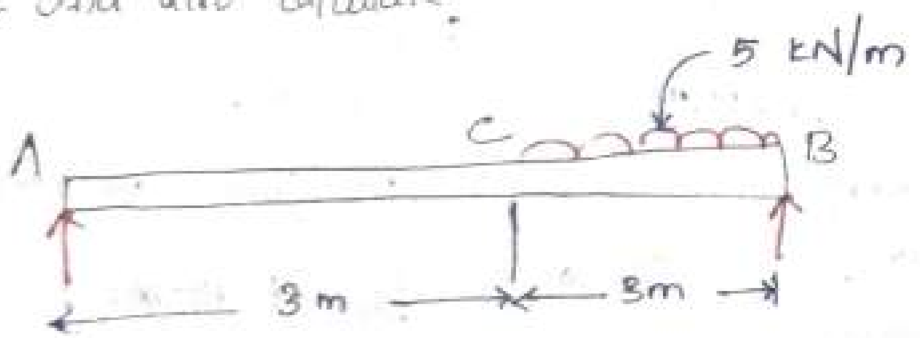
$$M_x = R_A \cdot x - \frac{\omega x^2}{2} = \frac{Wl}{2}x - \frac{\omega x^2}{2}$$

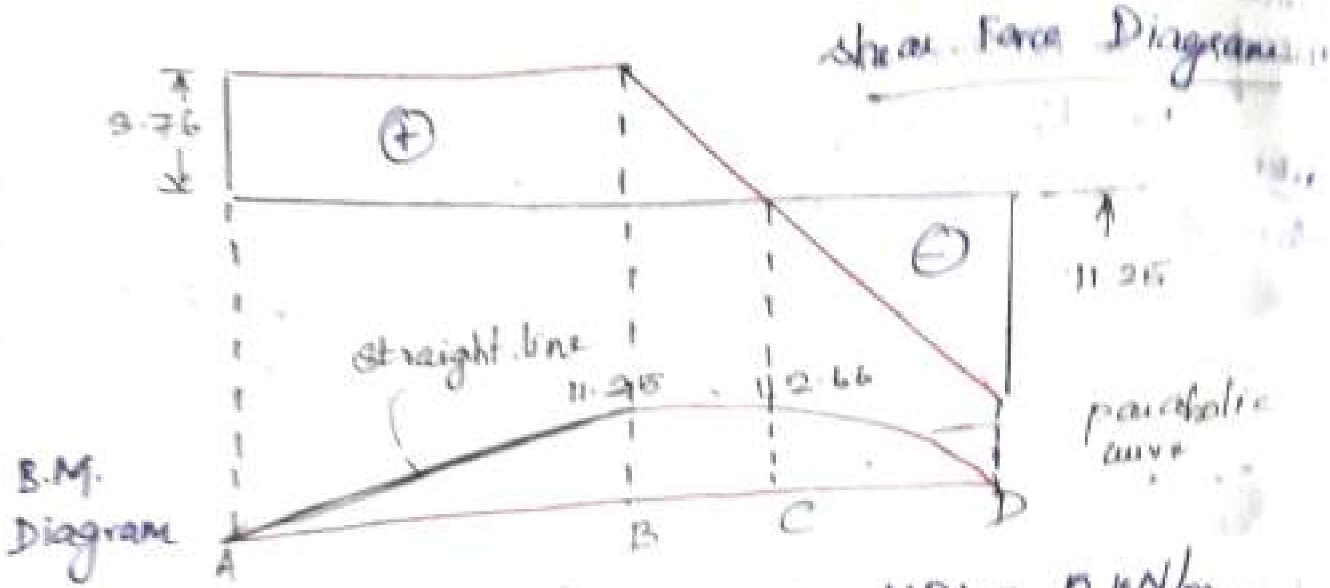
we also see that B M is zero at A and B [where $X = 0$; $x = l$; increases in the form of parabolic curve at C (ie) The mid point of the beam, where shear forces changes its sign, thus the B.M at C

$$M_c = \frac{Wl}{2}\left[\frac{l}{2}\right] - \frac{W}{2}\left[\frac{l}{2}\right]^2 = \frac{Wl^2}{4} - \frac{Wl^2}{8}$$

$$\boxed{M_c = Wl^2/8}$$

① A SSB 6m long is carrying UDL of 5kN/m over the length of 3m from the right end. Draw the SF & BM digram and calculate the maximum B.M on the section and also calculate.





Given span of length (l) = 6m. UDL = 5 kN/m
length of the beam (a) = 3 m

First of all, to find the Reactions R_A and R_B

Taking moments about A and eq. the same.

$$R_B \times 6 = (5\times3)\times4.5 = 67.5$$

$$R_B = 67.5/6 = 11.25\ kN$$

$$R_A = (5\times3) - 11.25 = 3.75\ kN$$

Shear Force Diagram

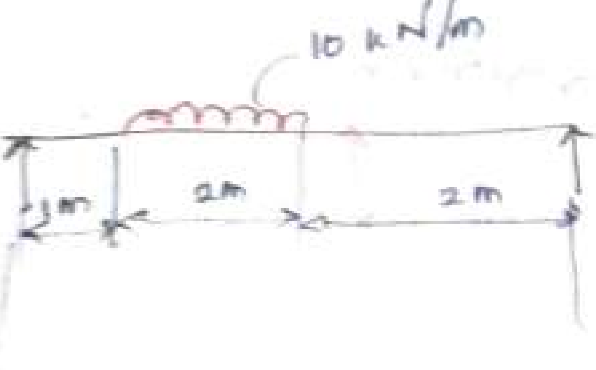
$F_A = +R_A = +3.75\ kN$ | $F_B = +3.75 - (5\times3)$
$F_C = +3.75\ kN$ | $= -11.25\ kN$

Bending Moment diagram

The BM is shown in fig $M_A = 0;\ M_B = 0$

$M_C = 37.5\times3 = 11.25\,M$

3 A SSB 5m long is loaded with UDL of 10 kN/m over a length 2m. Draw SFD and BMD for the beam indicated the value of maximum B.M



Given

$l = 5\ m$

UDL $(\omega) = 10\ kN/m$

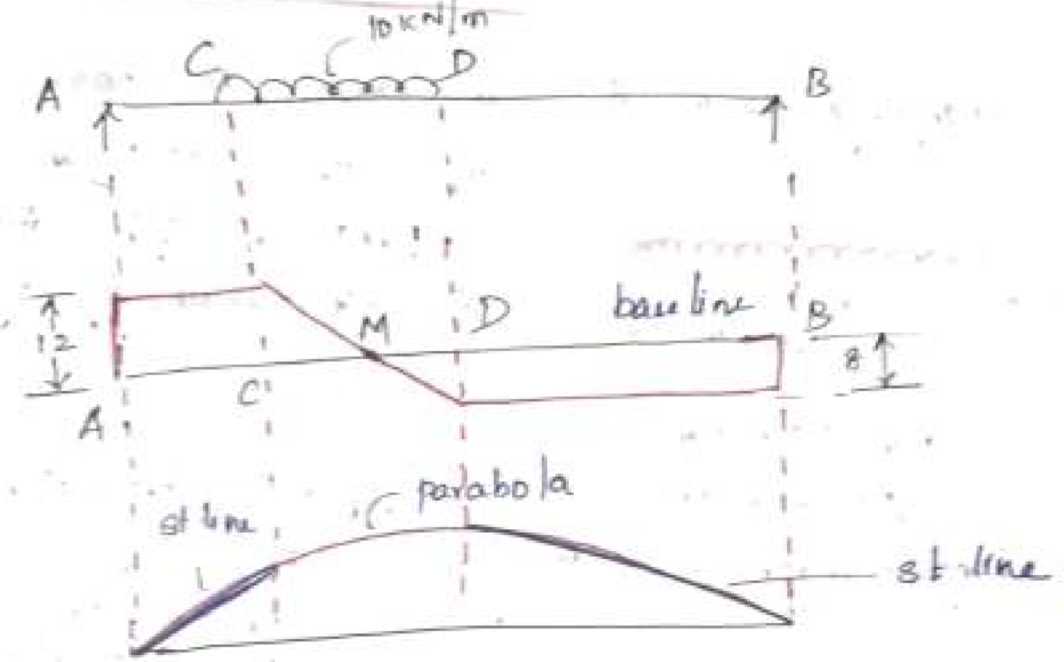
load at distance = 2m

- let's find out the reaction R_A & R_B. Taking moments about A

$R_B \times 5 = (10 \times 2) \times 2 = 40$

$R_B = 40/5 = 8$ kN ; $R_A = (10 \times 2) - 8 = 12$ kN

Shear Force Diagram



$F_A = +R_A = +12$ kN; $F_C = +12$ kN; $F_D = 12 - (10 \times 2) = -8$ kN

$F_B = -8$ kN

B.M. Diagram

The B.M is shown in fig. The values are tabulated here

$M_A = 0$; $M_C = 12 \times 1 = 12$ kN-m ; $M_D = 8 \times 2 = 16$ kN-m

The maximum B.M will occur at M, where the shear force changes sign

w.k.t $x/12 = 2 - x/8$; $8x = 24 - 12x$

$20x = 24$; $x = 24/20 = 1.2$ m

$M_m = 12(1 + 1.2) - 10 \times 1.2 \times \frac{1.2}{2} = 19.2$ kN-m

Overhanging Beam:

It is SSB which overhangs (extend in the form of cantilever) from its support.

An overhanging beam is the SSB which may overhang on one side only (or) both the sides of supports.

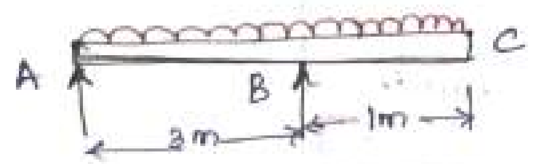
Point of Contraflexure:

It is thus obvious that overhanging beam, there will be a point, where the B.M. will change sign from negative to positive (or) vice versa. Such a point, where the B.M changes sign, is known as **point of contraflexure**.

PROBLEMS:

An overhanging beam ABC is loaded as shown in fig

Draw the SFD and BMD and find the point of contraflexure, if any.



Given: span (l) = 4 m;
UDL (w) = 4·5;
overhanging length (c) = 1m

S.F.D

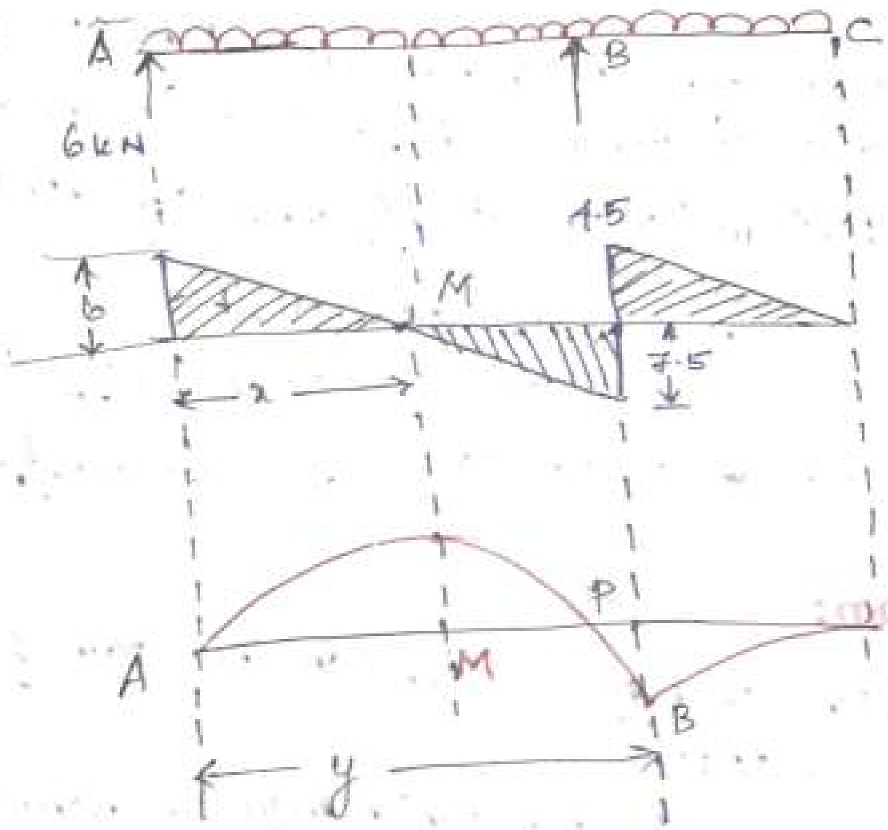
$R_B \times 3 = (4.5 \times 4) \times 2$

$3R_B = 36$

$R_B = 12\ kN$

$R_A = (4.5 \times 4) - 12$

$\boxed{R_A = 6\ kN}$



Shear Force Diagram

$F_A = +R_A = 6kN$

$F_B = +6 - (4.5 \times 3) + 12$

$F_B = 4.5\ kN$

$F_C = +4.5 - (4.5 \times 1)$

$\boxed{F_C = 0}$

B.M. diagram

B.M. diagram is shown in fig and the values are tabulated here.

$M_A = 0$; $M_B = -(4.5 \times 1 \times \frac{1}{2}) = -2.25$ kN-m

$M_C = 0$

wkt the maximum B.M. will occur at M.

$\frac{x}{6} = \frac{3-x}{7.5}$; $7.5x = 18 - 6x$; $\boxed{x = 1.33 \text{ m}}$

$M_m = (6 \times 1.33) - 4.5 \times 1.33 \times \frac{1.33}{2} = 4$ kN-m

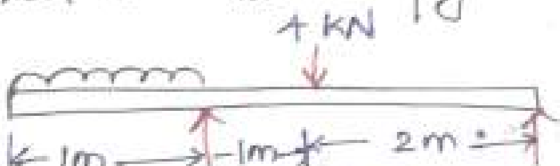
Point of contraflexure:

Let 'P' be the point of contraflexure at a distance 'y' from the support 'A'. wkt. B.M at P

$M_P = 6 \times y - 4.5 \times y \times \frac{y}{2} = 0$

$2.25y^2 - 6y = 0$; $2.25y = 6$: hence $y = 6/2.25 = 2.67$ m

② A Beam ABCD, 4m long is overhanging by 1m and carries load as shown in fig D



Given

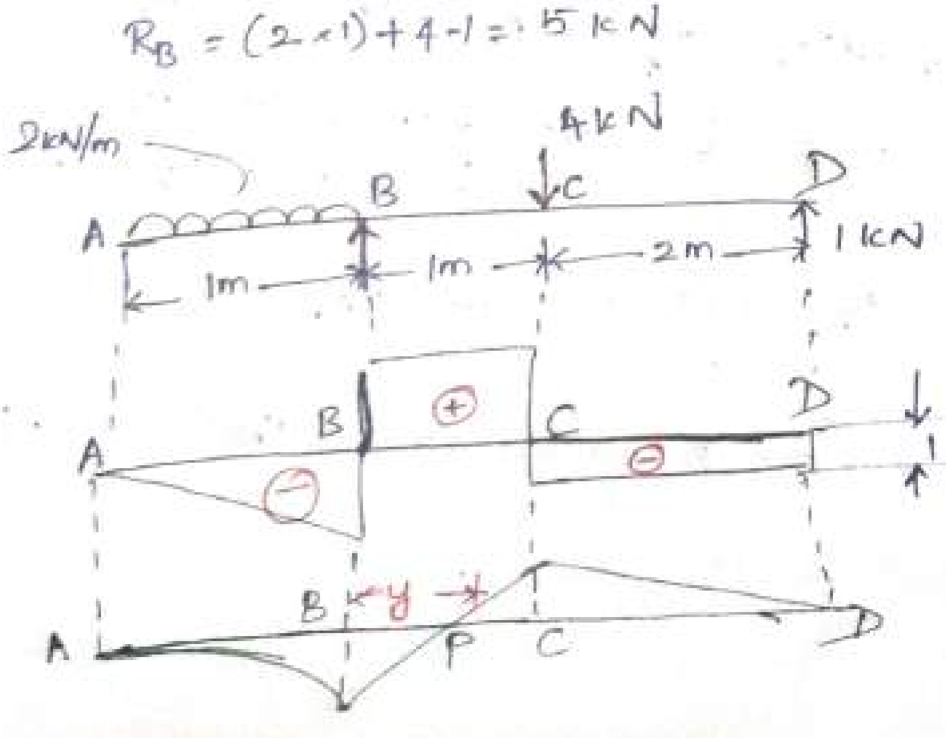
$l = 4$ m; UDL $(w) = 2$ kN/m

Point load$(W) = 4$ kN

First of all, find out the reactions R_B and R_D. Taking moments about 'B' and equating the same.

$R_D \times 3 = (4 \times 1) - (2 \times 1) \times \frac{1}{2} = 3$; $R_D = 3/3 = 1$ kN

$R_B = (2 \times 1) + 4 - 1 = 5$ kN



Shear Force Diagram $F_A = 0$;

$F_B = 0-(2\times1)+5 = +3\,kN$

$F_D = 1kN$; $F_C = +3-4 = -1\,kN$

Bending moment Diagram

$M_A = 0$; $M_B = -(2\times1)\times0.5 = -1\,kN\text{-}m$

$M_C = 1\times2 = 2\,kN$; $M_D = 0$

w.k.t The maximum B.M occurs either at B and C where shear force change its sign. we find that maximum (-ve) occurs at B and maximum (+ve) B.M occurs at C

Point of contraflexure:

Let P be the point of contraflexure at a distance y from the support B. From the geometry of figure B & C, we find that

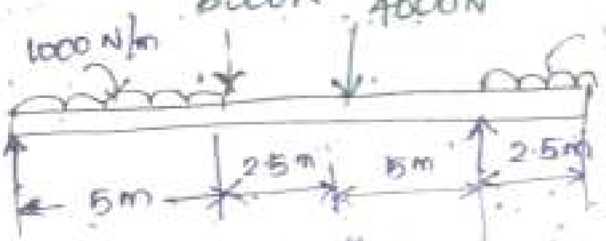
$$\frac{y}{1} = \frac{1-y}{2}$$

$2y = 1-y$

$3y = 1$; $\boxed{y = 1/3}$

$y = 0$

3) Draw the shear force and B.M for the beam shown in fig. Indicate the numerical values of all important section



Given

$l = 15\,m$

UDL $(w_1) = 1000\,N/m$

Point load $(W_1) = 8000N$

Point load $(W_2) = 4000N$

UDL $(w_2) = 1600N/m$

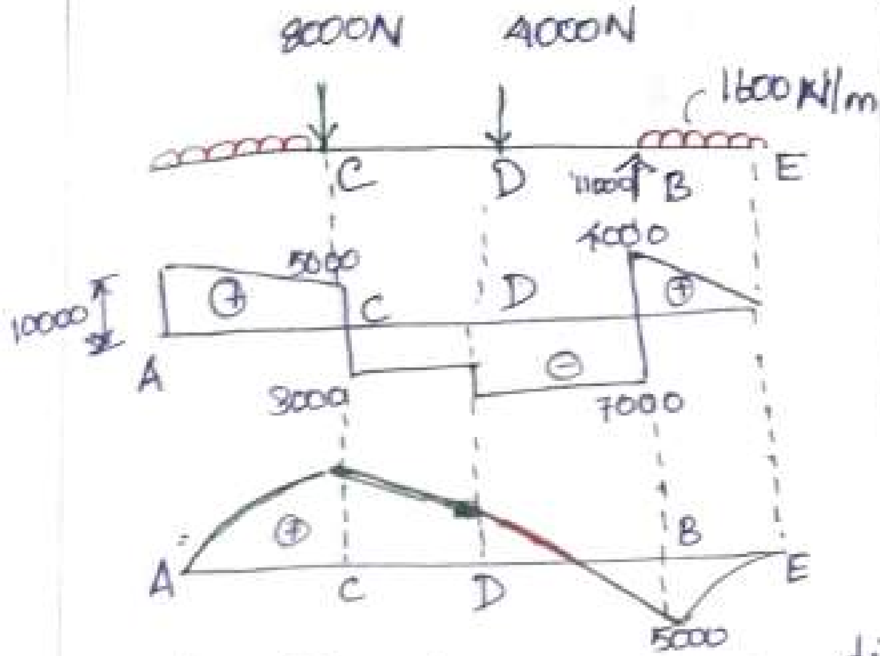
i) Find out the reactions R_A & R_B

$$R_B \times 12.5 = (1600\times2.5)\times13.75 + (4000\times7.5) + (8000\times5) + (1000\times5)\times2.5$$

$$R_B = \frac{137500}{12.6} = 11000N$$

$$R_A = [1000\times5 + 8000 + 4000 + 1600\times2.5] - 11000$$

$R_A = 10000\,N$



Shear force

$F_A = +10000\ N$

$F_C = 10000 - (1000 \times 5) - 800$

$F_C = -3000 N$

$F_D = -7000\ N$

$F_B = -7000 + 11000 = 4000\ N$

$F_E = +4000 - 1600 \times 2.5$

$F_E = 0$

Bending Moment: The B.M. diagram is shown in fig

$M_A = 0; \quad M_C = (10000 \times 5) - (1000 \times 5 \times 2.5)$

$M_C = 37500\ N\text{-}m$

$M_D = (10000 \times 7.5) - (1000 \times 5 \times 5) - (8000 \times 2.5) = 30000\ N\text{-}m$

$M_B = -1600 \times 2.5 \times \frac{2.5}{2} = -5000\ N\text{-}m$

The maximum BM (+ve) or (−ve) will occur at B and C, because shear forces changes sign at both the points:..

A horizontal beam 10 m long is carrying a UDL of 1 kN/m The beam is supported on two supports of 6 m apart. Find the position of the support, so that bending moment on the beam is small as possible Also draw the shear force and BMD

Given $l = 10\ m$ = Total length of beam

UDL load = 1 kN/m

Distance b/w the supports = 6m

Taking moments about,

$R_B \times 6 = 1 \times 10(5-a) = 10(5-a)$

$R_B = \frac{10(5-a)}{6} = \frac{5}{3}(5-a)$

$R_A = 10 - \frac{5}{3}(5-a) = \frac{5}{3}(1+a)$

From the diagram, we find the maximum negative B.M will be either of two supports and the maximum pos. B.M will be in the span AB.

Since the shear force at M is zero, therefore

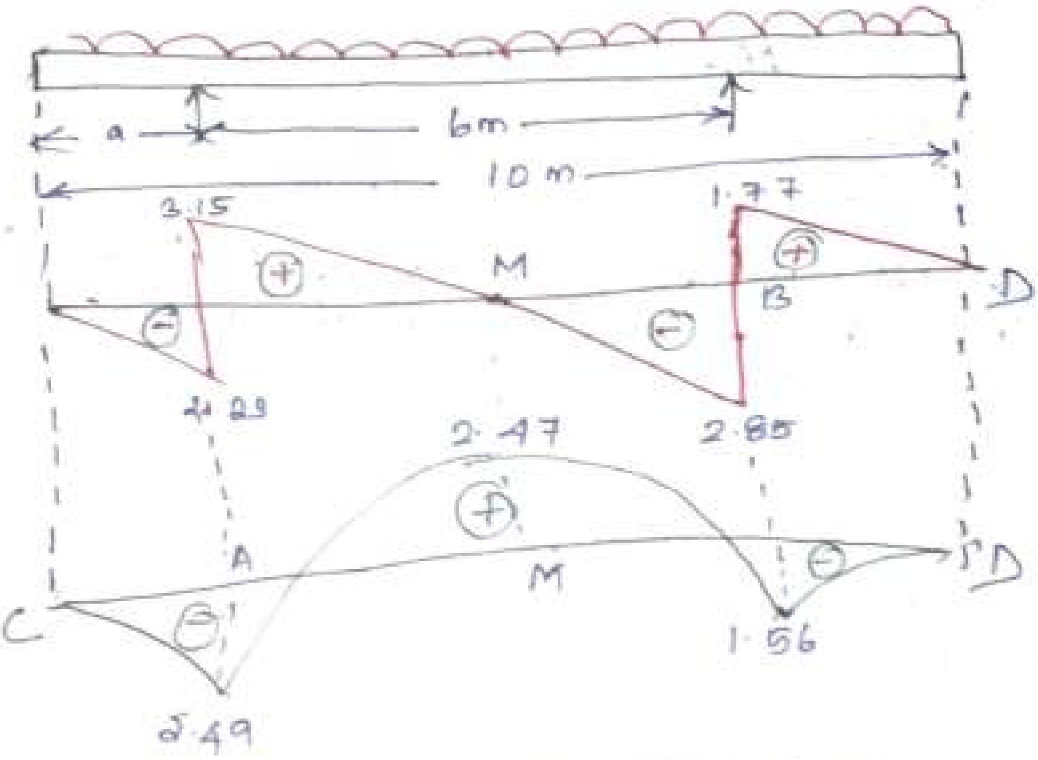
$$1 \cdot x - R_A = 0$$

W.K.T

$$x = R_A = \frac{5}{3}(1+a)$$

The B.M at A

$$M_A = -1 \times a \times \frac{a}{2} = -\frac{x^2}{2} \quad \text{--- (1)}$$



and the B.M, when the shear force is zero.

$$M_m = 1 \times x \times \frac{x}{2} + R_A(x-a) = R_A(x-a) - \frac{x^2}{2}$$

$$= \frac{5}{3}(1+a)\left[\frac{5}{3}(1+a) - a\right] - \frac{1}{2}\left[\frac{5}{3}(1+a)\right]^2$$

$$= \frac{25}{9}(1+a)\left[\frac{5-a}{10}\right]$$

$$= \frac{5}{18}(5+4a-a^2) \quad \text{--- (2)}$$

Equating ① & ② $\frac{a^2}{2} = \frac{5}{18}(5+4a-a^2)$

$\therefore \quad 14a^2 - 20a - 25 = 0$

solving the quadratic eqn for 'a'

$$a = \frac{20 \pm \sqrt{20^2 + (4 \times 14 \times 25)}}{2 \times 14} = 2.23\,m$$

$$x = \frac{5}{3}(1+a) = \frac{5}{3}(1+2.23) = 5.38\,m$$

Now reaction at B

$$R_B = \frac{5}{3}(5-a) = \frac{5}{3}(5-2.23) = 4.62\ kN$$

$$R_A = \frac{5}{3}(1+a) = \frac{5}{3}(1+2.23) = 5.38\ kN$$

Shear Force Diagram

$F_C = 0$

$F_A = 0 - 1 \times 2.23 + 5.38 = 3.15\,kN$

$F_B = 3.15 - 1 \times 6 + 4.62 = +1.77\,kN$

$F_D = 1.77 - 1.77 = 0$

Bending moment Diagram

$M_C = 0$

$M_D = 0$

$M_A = -1 \times 2.23 \times \frac{2.23}{2}$

$= -2.49\,kN\text{-}m$

$M_B = 1 \times 1.77 \times \frac{1.77}{2}$

$M_B = 1.56\ kN\text{-}m$

$M_m = -1 \times 5.38 \times \frac{5.38}{2} + 5.38 \times 3.15$

$M_m = 2.47\ kN.m$

As the matter of fact, the bending moment at the section tends to bend (or) deflect the beam and the internal stresses resists to its bending.

The resistance offered by the internal stresses to the bending is called shearing stress