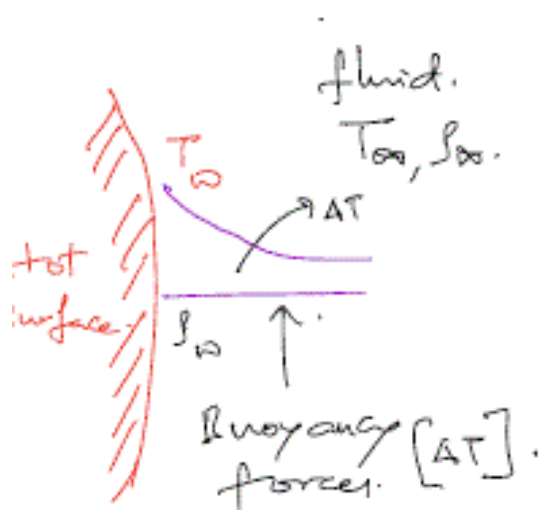




What we will look into?

- 1) Single-phase fluid flow
- 2) Correlation
- 3) No-bulk flow in free convection.
- 4) Flows caused by buoyancy.
- 5) Density difference due to temp. difference.
- 6) Fluid flow & Heat transfer are connected.
- 7) No-noise [quiet operation]
- 8) Heat transfer coefficients are low.
- 9) Area requirements are large.
- 10) Orientation of surface plays a role.
- 11) Controlling the process is difficult [natural].



- 1)  $T_w > T_\infty$  &  $\rho_w < \rho_\infty$ .
- 2) Buoyancy forces set up the natural convection currents.
- 3) Fluid flow & Heat transfer are linked.

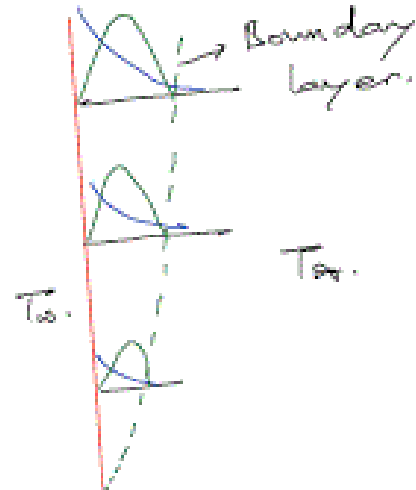
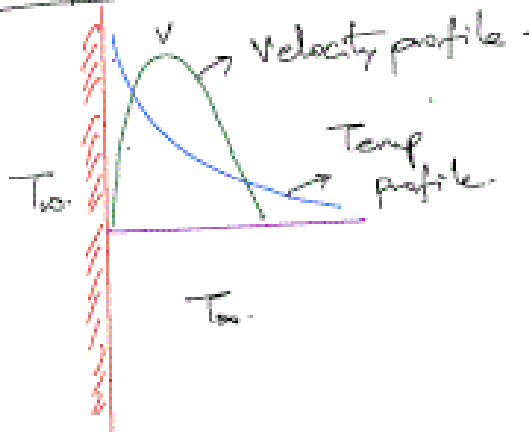


Dimensionless numbers

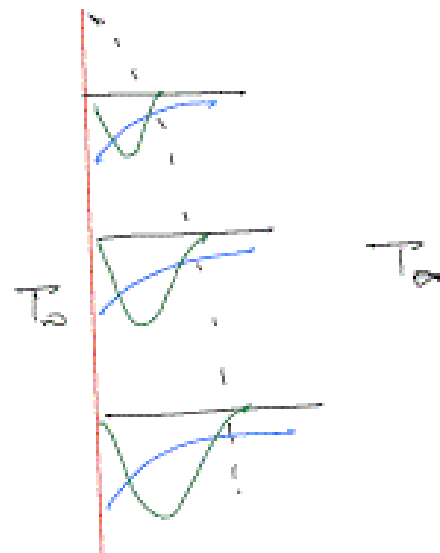
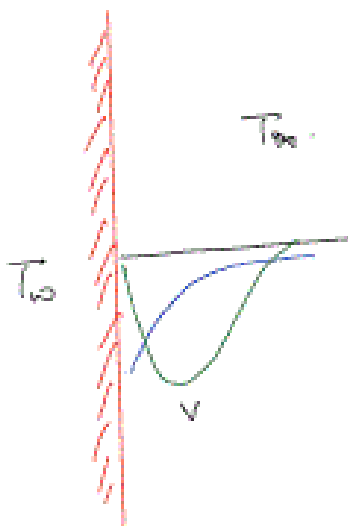
①  $Nu = \frac{hk}{k}$     ②  $Pr = \frac{\mu C_p}{k}$     ③  $Gr = \frac{g\beta\Delta TL^3}{\nu^2}$

Vertical plate:

Hot plate:  $T_w > T_\infty$



Cold plate:  $T_w < T_\infty$





Correlations from data hand book.

Fluid properties are evaluated at mean fluid temperature

$$T_f = \frac{T_w + T_\infty}{2}$$

Vertical plate:

$$Nu = C \cdot Gr^a Pr^b$$

↑  
Experiments

$10^6 > Gr Pr > 10^9 \rightarrow$  Laminar.

$Gr Pr > 10^9 \rightarrow$  Turbulent

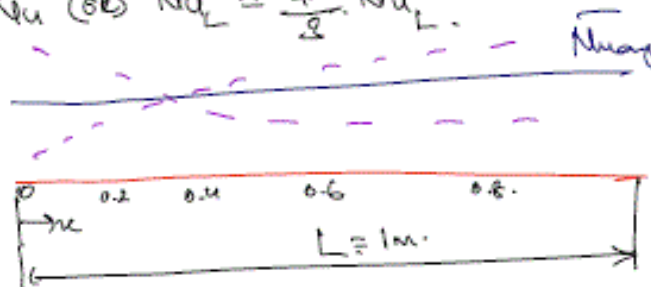
Constant heat flux:

Constant temp

$T_w = \text{constant}$

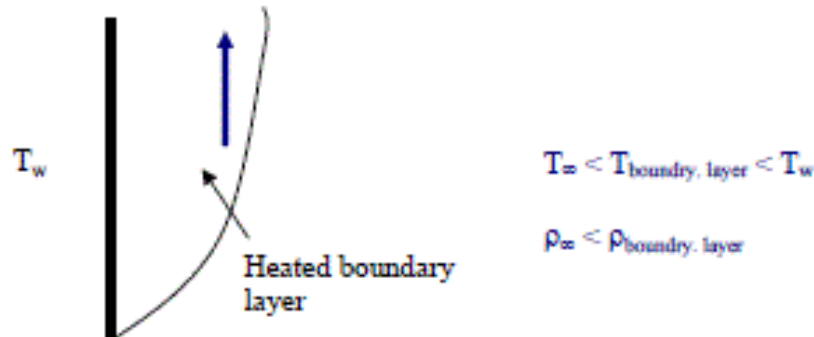
$Nu_x = C Gr_x^a Pr^b \rightarrow$  Local nusselt numbers.

$$\overline{Nu} \text{ (or) } \overline{Nu}_L = \frac{4}{3} Nu_L$$



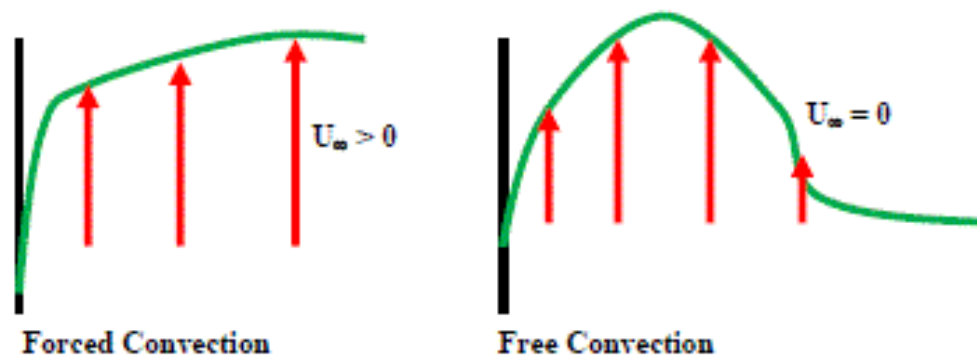


Free convection is sometimes defined as a convective process in which fluid motion is caused by buoyancy effects.



### Velocity Profiles

Compare the velocity profiles for forced and natural convection shown below:



### Coefficient of Volumetric Expansion

The thermodynamic property which describes the change in density leading to buoyancy in the Coefficient of Volumetric Expansion,  $\beta$ .

$$\beta \equiv -\frac{1}{\rho} \cdot \left. \frac{\partial \rho}{\partial T} \right|_{P=\text{Const.}}$$

### Evaluation of $\beta$

- Liquids and Solids:  $\beta$  is a thermodynamic property and should be found from Property Tables. Values of  $\beta$  are found for a number of engineering fluids in Tables given in Handbooks and Text Books.
- Ideal Gases: We may develop a general expression for  $\beta$  for an ideal gas from the ideal gas law:



$$P = \rho \cdot R \cdot T$$

Then,

$$\rho = P/R \cdot T$$

Differentiating while holding P constant:

$$\left. \frac{d\rho}{dT} \right|_{P \text{ const.}} = - \frac{P}{R \cdot T^2} = - \frac{\rho \cdot R \cdot T}{R \cdot T^2} = - \frac{\rho}{T}$$

Substitute into the definition of  $\beta$

$$\beta = \frac{1}{T_{\text{ref}}}$$

Ideal Gas

### Grashof Number

Because  $U_{\infty}$  is always zero, the Reynolds number,  $[\rho \cdot U_{\infty} \cdot D] / \mu$ , is also zero and is no longer suitable to describe the flow in the system. Instead, we introduce a new parameter for natural convection, the Grashof Number. Here we will be most concerned with flow across a vertical surface, so that we use the vertical distance,  $z$  or  $L$ , as the characteristic length.

$$Gr \equiv \frac{g \cdot \beta \cdot \Delta T \cdot L^3}{\nu^2}$$

Just as we have looked at the Reynolds number for a physical meaning, we may consider the Grashof number:

$$Gr \equiv \frac{\rho^2 \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{\mu^2} = \frac{\left( \frac{\rho \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{L^3} \right) \cdot (\rho \cdot U_{\text{max}}^2)}{\mu^2 \cdot \frac{U_{\text{max}}^2}{L^3}} = \frac{\left( \frac{\text{Buoyant Force}}{\text{Area}} \right) \cdot \left( \frac{\text{Momentum}}{\text{Area}} \right)}{\left( \frac{\text{Viscous Force}}{\text{Area}} \right)^2}$$

### Free Convection Heat Transfer Correlations

The standard form for free, or natural, convection correlations will appear much like those for forced convection except that (1) the Reynolds number is replaced with a Grashof number and (2) the exponent on Prandtl number is not generally 1/3 (The von Karman boundary layer analysis from which we developed the 1/3 exponent was for forced convection flows):

$$Nu_x = C \cdot Gr_x^m \cdot Pr^n \quad \text{Local Correlation}$$

$$Nu_L = C \cdot Gr_L^m \cdot Pr^n \quad \text{Average Correlation}$$

Quite often experimentalists find that the exponent on the Grashof and Prandtl numbers are equal so that the general correlations may be written in the form:



$$Nu_x = C \cdot [Gr_x \cdot Pr]^m$$

Local Correlation

$$Nu_L = C \cdot [Gr_L \cdot Pr]^m$$

Average Correlation

This leads to the introduction of the new, dimensionless parameter, the Rayleigh number,  $Ra$ :

$$Ra_x = Gr_x \cdot Pr$$

$$Ra_L = Gr_L \cdot Pr$$

So that the general correlation for free convection becomes:

$$Nu_x = C \cdot Ra_x^m$$

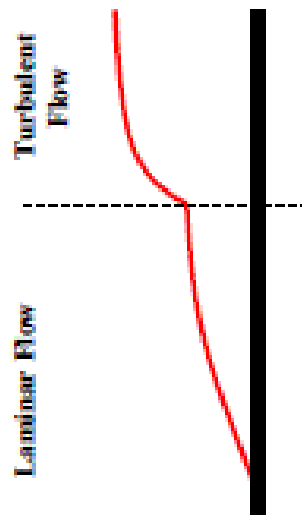
Local Correlation

$$Nu_L = C \cdot Ra_L^m$$

Average Correlation

### Laminar to Turbulent Transition

Just as for forced convection, a boundary layer will form for free convection. The boundary layer, which acts as a thermal resistance, will be relatively thin toward the leading edge of the surface resulting in a relatively high convection coefficient. At a Rayleigh number of about  $10^9$  the flow over a flat plate will become transitional and finally become turbulent. The increased turbulence inside the boundary layer will enhance heat transfer leading to relative high convection coefficients because of better mixing.



$Ra < 10^9$  Laminar flow. [Vertical Flat Plate]

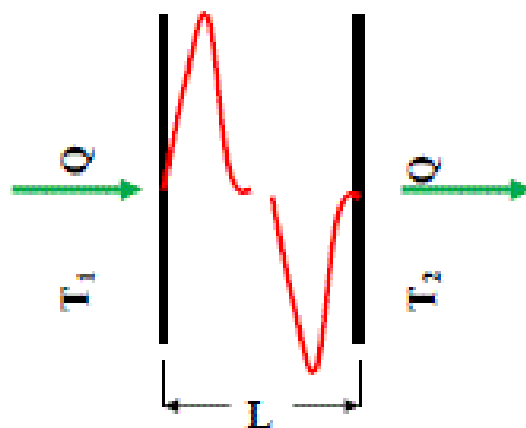
$Ra > 10^9$  Turbulent flow. [Vertical Flat Plate]



Generally the characteristic length used in the correlation relates to the distance over which the boundary layer is allowed to grow. In the case of a vertical flat plate this will be  $x$  or  $L$ , in the case of a vertical cylinder this will also be  $x$  or  $L$ ; in the case of a horizontal cylinder, the length will be  $d$ .

### Critical Rayleigh Number

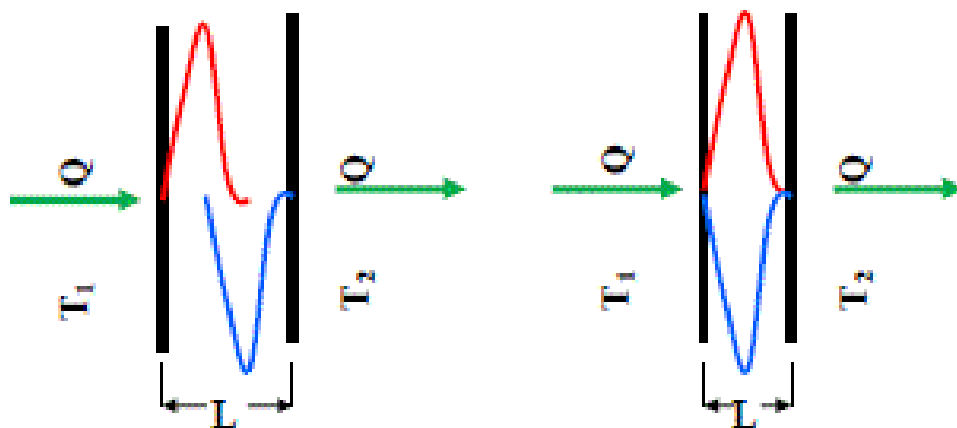
Consider the flow between two surfaces, each at different temperatures. Under developed flow conditions, the interstitial fluid will reach a temperature between the temperatures of the two surfaces and will develop free convection flow patterns. The fluid will be heated by one surface, resulting in an upward buoyant flow, and will be cooled by the other, resulting in a downward flow.



Note that for enclosures it is customary to develop correlations which describe the overall (both heated and cooled surfaces) within a single correlation.

Free Convection Inside an Enclosure (boundary layer limit)

If the surfaces are placed closer together, the flow patterns will begin to interfere:



Free Convection Inside an Enclosure With Partial Flow Interference

Free Convection Inside an Enclosure With Complete Flow Interference (Channel flow limit)



In the case of complete flow interference, the upward and downward forces will cancel, canceling circulation forces. This case would be treated as a pure convection problem since no bulk transport occurs.

The transition in enclosures from convection heat transfer to conduction heat transfer occurs at what is termed the "Critical Rayleigh Number". Note that this terminology is in clear contrast to forced convection where the critical Reynolds number refers to the transition from laminar to turbulent flow.

$$Ra_{crit} = 1000 \quad (\text{Enclosures With Horizontal Heat Flow})$$

$$Ra_{crit} = 1728 \quad (\text{Enclosures With Vertical Heat Flow})$$

The existence of a Critical Rayleigh number suggests that there are now three flow regimes: (1) No flow, (2) Laminar Flow and (3) Turbulent Flow. In all enclosure problems the Rayleigh number will be calculated to determine the proper flow regime before a correlation is chosen.