1. A vertical plate 30 cm high and 1 m wide and maintained at a uniform temperature of 120^oC is exposed to quiescent air at 30^oC. Calculate the average heat transfer coefficient and the total heat transfer rate from the plate to air.

Sol: Given: $L = 30\,cm$; $W = 1\,m$. $T_w = 120°C$ $T_\infty = 30°C$

To find: (a) $h = ?$ (b) $Q = ?$

$$T_{film} = \frac{T_w + T_\infty}{2} = \frac{120+30}{2} = 75°C$$

Properties of air @ 75°C from data handbook

$\rho = 1.015\,kg/m^3$; $\mu = 20.9 \times 10^{-6}\,Pa\,s$ $\nu = 20.6 \times 10^{-6}\,m^2/s$.

$Pr = 0.693$ $C_p = 1.009\,kJ/kg\,K$ $K = 0.03025\,W/m\text{-}K$.

$$\beta = \frac{1}{(T_{film}+273)} = \frac{1}{(75+273)} = 2.87 \times 10^{-3}/°K.$$

$$Gr = \frac{\beta g \Delta T L^3}{\nu^2} = \frac{2.87 \times 10^{-3} * 9.81 * (120-30) * 0.3^3}{(20.6 \times 10^{-6})^2}$$

$Gr \simeq 1.6 \times 10^8$

$Ra = Gr\,Pr = 1.6 \times 10^8 * 0.693 = 1.16 \times 10^8 < 10^9.$

The flow is laminar.

$$Nu_x = 0.508\,Pr^{0.5}(0.952 + Pr)^{-0.25} \cdot Gr_x^{0.25}$$
$$= 42.$$

$Nu_{avg} = \frac{4}{3} Nu_x = \frac{4}{3} \times 42 = 56$

$Nu = \frac{hL}{k} \rightarrow h_{avg} = \frac{Nu_{avg} k}{L} = \frac{56 \times 0.02025}{0.2}$

(a) $h_{avg} = 5.61 \ W/m^2\text{-}K.$

(b) $Q = h_{avg} \, A \, (T_w - T_\infty)$

$= 5.61 \times (0.2 \times 1)(120 - 30)$

$Q = 151 \ W.$

Case 1:-

Try calculating the $Nu \rightarrow h \rightarrow Q$ using Churchill & Chu correlation given in pg.136 of data book.

$$\overline{Nu} = \left[0.825 + \frac{0.387 \, (Gr \, Pr)^{0.167}}{\left[1 + \left(\frac{0.492}{Pr} \right)^{0.5625} \right]^{0.296}} \right]^2.$$

$\overline{Nu} = 63 \longrightarrow \overline{h} = 6.38 \ W/m^2K \longrightarrow Q = 172 \ W.$

Case 2: With radiation:

$Q_{rad} = \sigma A (T_w^4 - T_\infty^4) = 5.67 \times 10^{-8} \times (0.2 \times 1)(393^4 - 303^4)$

$= 262.28 \ W.$

$Q_{total} = Q_{conv} + Q_{rad} = 172 + 263 = 435 \ W.$

2. A flat electrical heater of 0.4 m X 0.4 m size is placed vertically in still air at 20°C. The heat generated is $1200W/m^2$. Determine the value of convective heat transfer coefficient and the average plate temperature.

Sol: $A = 0.4 \times 0.4\,m$; $L = 0.4\,m$; $B = 0.4\,m$. $q = 1200\,W/m^2$.

$T_\infty = 20°C$

To find $h, T_w = ?$ **Iteration 1:**

Assuming an average value of $h = 6\,W/m^2K$.

$\therefore\ q = h\Delta T \rightarrow \Delta T = \frac{q}{h} = \frac{1200}{6} = 200°C$

$T_w - T_\infty = 200 \rightarrow T_w = 200 + 20 = 220°C$

$\therefore\ T_f = \frac{220+20}{2} = 120°C \qquad \beta = \frac{1}{(120+273)} = 2.54\times10^{-3}/°C$

Property values of air @ 120°C

$\rho = 0.876\,kg/m^3$; $\nu = 26.625\times10^{-6}\,m^2/s$; $Pr = 0.685$

$k = 0.03338\,W/mK$; $C_p = 1011\,J/kgK$.

$$Gr = \frac{g\beta\Delta T L^3}{\nu^2} = \frac{9.81\times2.54\times10^{-3}\times1200\times0.4^3}{(26.625\times10^{-6})^2}$$

$$Gr = 2.69\times10^9$$

$Ra_L = Gr.Pr = 2.29\times10^3\times0.688 = 1.85\times10^9 > 10^9$ (Turbulent)

$$Nu_{x=L} = 0.17\,(Gr_L Pr)^{0.25} = 0.17\times(1.85\times10^9)^{0.25}.$$

$Nu_{x=L} = 35.26$.

$$\overline{Nu}_L = \frac{5}{4} Nu_{x=L} = \frac{5}{4}\times35.26 = 44.075$$

$$\bar{h}_L = \frac{\overline{Nu}_L * K}{L} = \frac{44.075 * 0.03338}{0.4} = 3.68 \ W/m^2K.$$

Iteration-2: Assume $h = 4.5 \ W/m^2K$.

$$q = h A T \rightarrow \Delta T = \frac{q}{h} = \frac{1200}{4.5} = 267$$

$$T_w = 267 + 20 = 287°C \rightarrow T_f = \frac{287+20}{2} = 153.5 \approx 150°C$$

Properties of air @ 150°C

$\rho = 0.83 \ kg/m^3$ $\nu = 28.5 \times 10^{-6} \ m^2/s$ $Pr = 0.683$ $K = 0.0352 \ W/mK$.

$$Gr = \frac{2.34 \times 10^{-3} * 9.81 * 267 * 0.4^3}{(28.5 \times 10^{-6})^2} = 0.484 \times 10^9.$$

$$Ra = Gr\,Pr = 0.484 \times 10^9 * 0.683 = 0.33 \times 10^9 < 10^9 \text{ (Laminar)}$$

$$Nu_x = 0.6 (Gr Pr)^{0.2} = 0.6 (0.33 \times 10^9)^{0.2} = 30.34.$$

$$\overline{Nu}_L = \frac{4}{3} * 30.34 = 40.45$$

$$\bar{h}_L = \frac{\overline{Nu}_L * k}{L} = \frac{40.45 * 0.0352}{0.4} = 3.6 \ W/m^2K.$$

Iteration 3: Assume $h = 4 \ W/m^2K$.

$\Delta T = \frac{q}{h} = \frac{1200}{4} = 300°C$ $T_w = 320°C$ $T_f = 170°C$

$\rho = 0.79 \ kg/m^3$; $\nu = 24.9 \times 10^{-6} \ m^2/s$; $Pr = 0.681$; $K = 0.037 \ W/mK$

$$Gr Pr = \frac{2.26 \times 10^{-3} * 9.81 \times 300 \times 0.4^3}{(24.9 \times 10^{-6})^2} * 0.681 = 0.46 \times 10^9 < 10^9 \text{ (Laminar)}$$

$Nu_x = 0.6 (Gr Pr)^{0.2} = 32.5$; $\overline{Nu}_L = \frac{4}{3} Nu_x = 43.35$

$$h_L = \frac{Nu_L k}{L} = \frac{43.35 * 0.037}{0.4} = 4 \ W/m^2K.$$

3. A vertical plate is maintained at 40°C in 20°C still air. Determine the height at which the boundary layer will turn turbulent if turbulence sets in at $Gr * Pr = 10^9$. Repeat the problem for water flow at film temperature of 30°C. Determine the value of boundary layer thickness and average convection coefficient at the location where flow turns turbulent.

① Air: $T_w = 40°C$, $T_\infty = 20°C$ To find: δ_x, L_c

Properties of air $T_f = \frac{40+20}{2} = 30°C$

$\nu = 16 \times 10^{-6}\ m^2/s$; $\rho = 1.165\ kg/m^3$; $Pr = 0.701$,

$$\beta = \frac{1}{30+273} = \frac{1}{303} = 3.3 \times 10^{-3}\ /°C$$

The flow becomes turbulent @ $GrPr = 10^9$.

$$GrPr = \frac{\beta g \Delta T L_c^3}{\nu^2} * Pr$$

$$10^9 = \frac{3.3 \times 10^{-3} \times 9.81 \times 20 \times L_c^3}{(16 \times 10^{-6})^2} * 0.701$$

$L_c = 0.82\ m.$ $\delta_x = 3.93\, L_c\, Pr^{-0.5} (0.952 + Pr)^{0.25} \cdot Gr_L^{-0.25}$

$\delta_{x=L_c} = 0.022\ m. \rightarrow 22\ mm.$

② Water: $T_f = 30°C$ Properties @ 30°C

$\rho = 998\ kg/m^3$ $\nu = 0.832 \times 10^{-6}\ m^2/s$. $Pr = 5.68$.

$$GrPr = \frac{\beta g \Delta T L_c^3}{\nu^2} * Pr$$

$$10^9 = \frac{3.3 \times 10^{-5} \times 9.81 \times 20 \times L_c^3}{(0.836 \times 10^{-6})^2} * 5.68$$

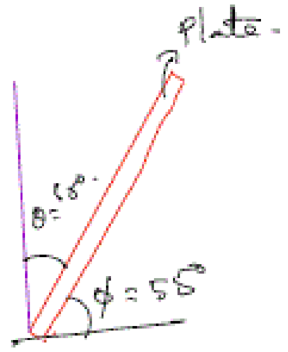
$L_c = 0.057\ m. \rightarrow \delta_{x=L_c} = 1.38\ mm.$

Comment: Critical length reduces for water. So it turns turbulent much faster for water.

19. Consider a surface 0.8 m high, kept at an angle of 55° from the horizontal at a constant wall temperature of 40°C in air at 20°C. Determine the value of convection coefficient and compare the same with that of similar vertical plate.

Sol: $L = 0.8\,m$; $\phi = 55°$; $T_w = 40°C$; $T_\infty = 20°C$

To find ① $h = ?$ ② Compare with vertical plate.



$$T_f = \frac{T_w + T_\infty}{2} = \frac{40+20}{2} = 30°C$$

Properties @ 30°C

$\rho = 1.165\ kg/m^3$; $\nu = 16 \times 10^{-6}\ m^2/s$; $Pr = 0.701$;

$k = 0.02675\ W/m\text{-}K$.

$$\beta = \frac{1}{30+273} = 3.3 \times 10^{-3}/K$$

$$Gr = \frac{\beta g \Delta T L^3}{\nu^2}$$

$$Gr = \frac{3.3\times10^{-3} \times 9.81 \times (40-20) \times 0.8^3}{(16\times10^{-6})^2} = 1.29 \times 10^9.$$

$$Ra = Gr\,Pr = 1.29\times10^9 \times 0.701 = 9.07\times10^8 < 10^9 \text{ (Laminar)}$$

Case a: Vertical

$$Nu = \left[0.825 + \frac{0.387 (Ra)^{0.167}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{0.5624}\right]^{0.296}}\right]^2.$$

$Nu = 120.2$

$$h = \frac{Nu\,k}{L} = \frac{120.2 \times 0.02675}{0.8} = 4.1\ W/m^2K.$$

Case b: Inclined $\theta = 35°$

$$Nu = \left[0.825 + \frac{0.387\,(Ra\cos\theta)^{1/6}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right]^2$$

$Nu = 104.8$

$$h = \frac{Nu\,k}{L} = \frac{104.8 \times 0.02675}{0.8} = 3.5\ W/m^2K.$$

Assignment: ① $\theta = 1°$ to $60°$ [in steps of 1°]
② Calculate 'h'
③ h V/s θ. Compare with h for vertical.

※ For constant heat flux inclined plates

$T_e = T_w - 0.25\,(T_w - T_\infty)$ $\beta = \frac{1}{T_f}$.

Properties at T_e.

Case a: Heated surface up.

Case b: Heated surface down.