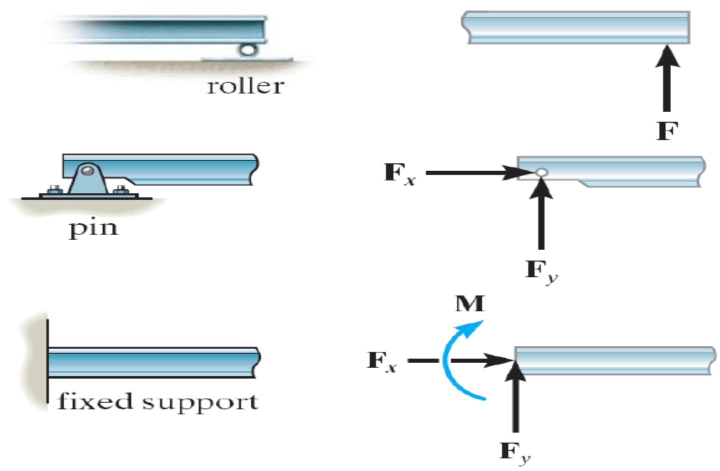
EQUILIBRIUM OF RIGID BODIES IN 2D

Rigid Body Equilibrium

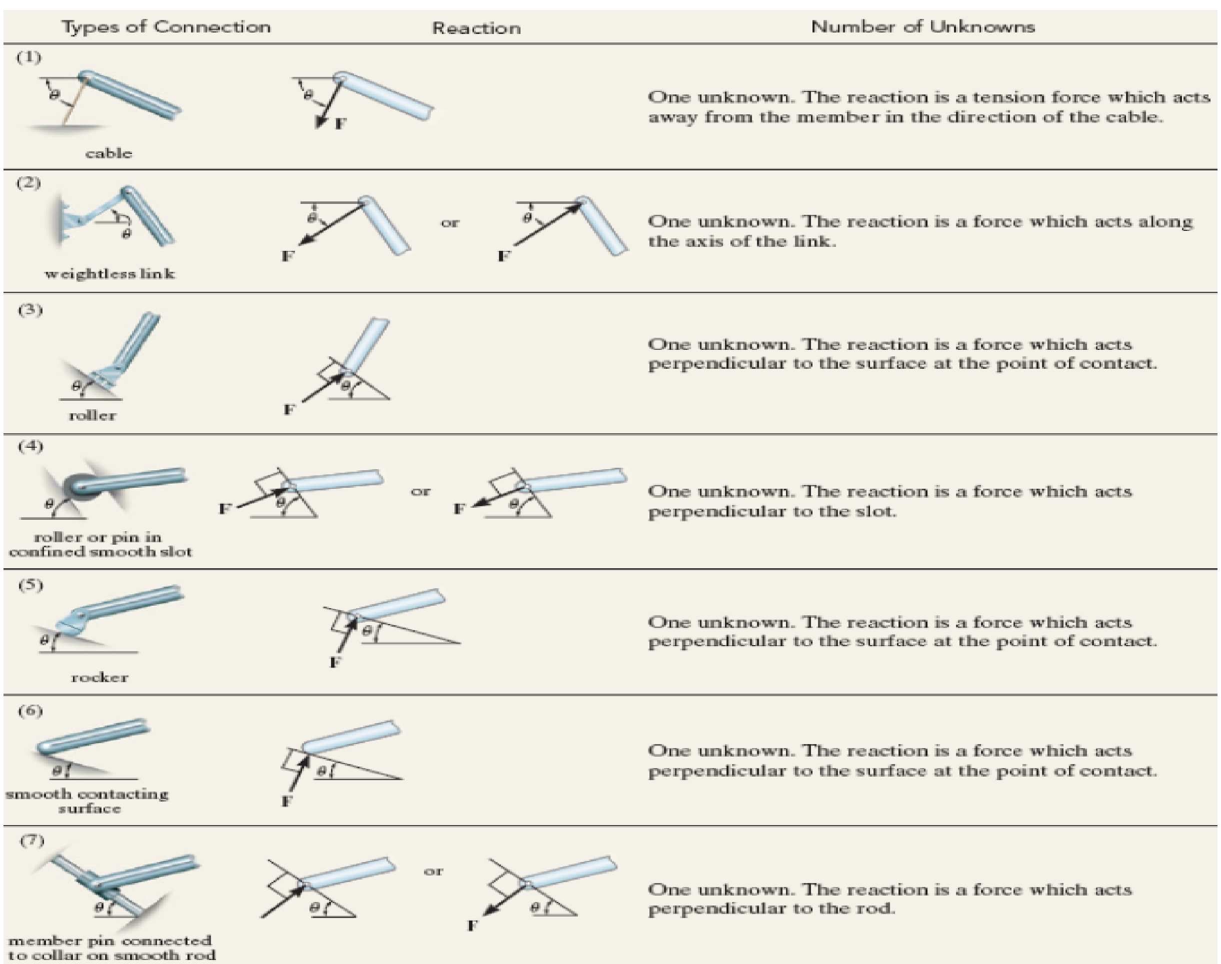
Support Reactions

Prevention of Translation or Rotation of a body

Restraints

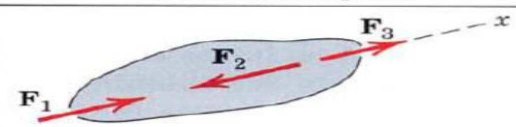
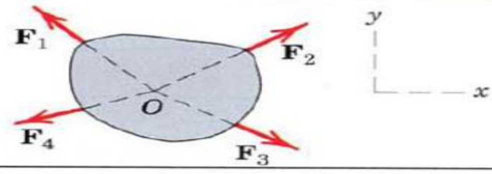
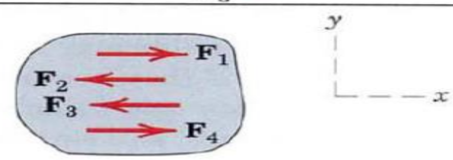
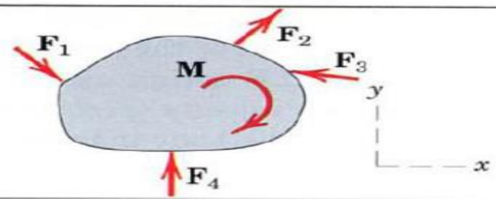


RIGID BODY DYNAMICS:

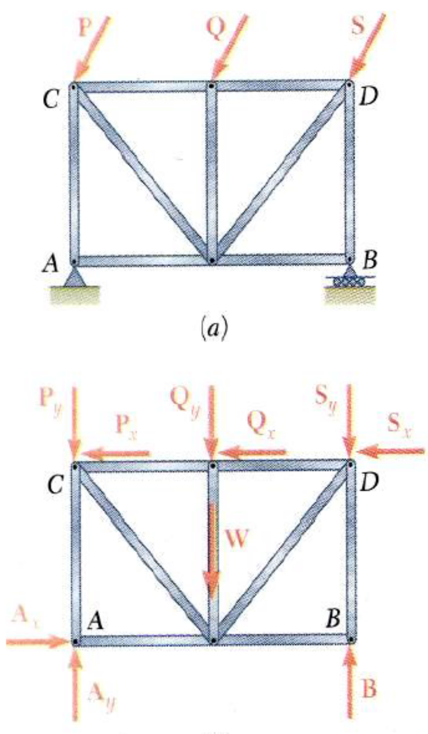


Types of Connection	Reaction	Number of Unknowns
(1) cable	F	One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2) weightless link	F or F	One unknown. The reaction is a force which acts along the axis of the link.
(3) roller	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) roller or pin in confined smooth slot	F or F	One unknown. The reaction is a force which acts perpendicular to the slot.
(5) rocker	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6) smooth contacting surface	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7) member pin connected to collar on smooth rod	or F	One unknown. The reaction is a force which acts perpendicular to the rod.

RIGID BODY EQUILIBRIUM:

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Equilibrium of a Rigid Body in Two Dimensions



- For all forces and moments acting on a two-dimensional structure,

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

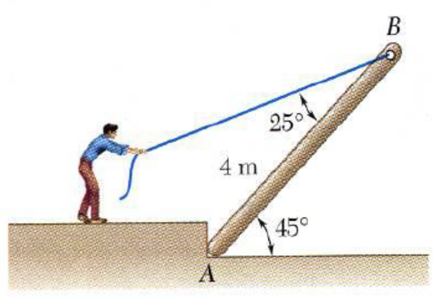
where A is any point in the plane of the structure.

- The 3 equations can be solved for no more than 3 unknowns.

- The 3 equations can not be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

Rigid Body Equilibrium: Example



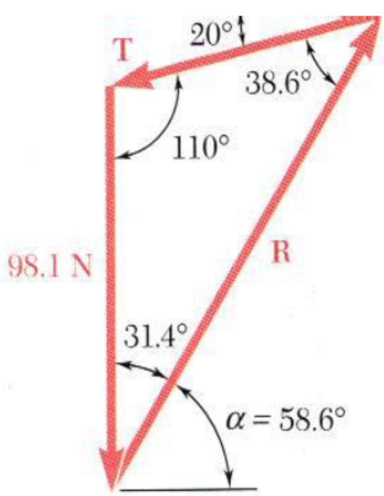
A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the **tension in the rope** and the **reaction at A**.

Solution:

- Create a **free-body diagram** of the joist.
  - The joist is a **3 force body** acted upon by the **rope**, its **weight**, and the **reaction at A**.
- The **three forces** must be **concurrent** for **static equilibrium**.
  - Reaction **R** must pass through the intersection of the lines of action of the weight and rope forces.
  - Determine the direction of the reaction force **R**.
- Utilize a **force triangle** to determine the magnitude of the reaction force **R**.

Rigid Body Equilibrium: Example



- Determine the magnitude of the reaction force **R**.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\text{N}}{\sin 38.6^\circ}$$

$$T = 81.9\,\text{N}$$

$$R = 147.8\,\text{N}$$
