INTRODUCTION

THIN CYLINDER:

- ✓ The cylinder which have thickness is less than 1/10 to 1/20 of its Diameter, that cylinder is called as thin cylinder.
- ✓ Thin cylinder is only resist to the internal Pressure.
- ✓ Thin cylinder failure due to internal fluid pressure by the formation of circumferential stress and longitudinal stress.
- ✓ The internal pressure which is acting radially inside the thin cylinder is known as radial pressure in thin cylinder.

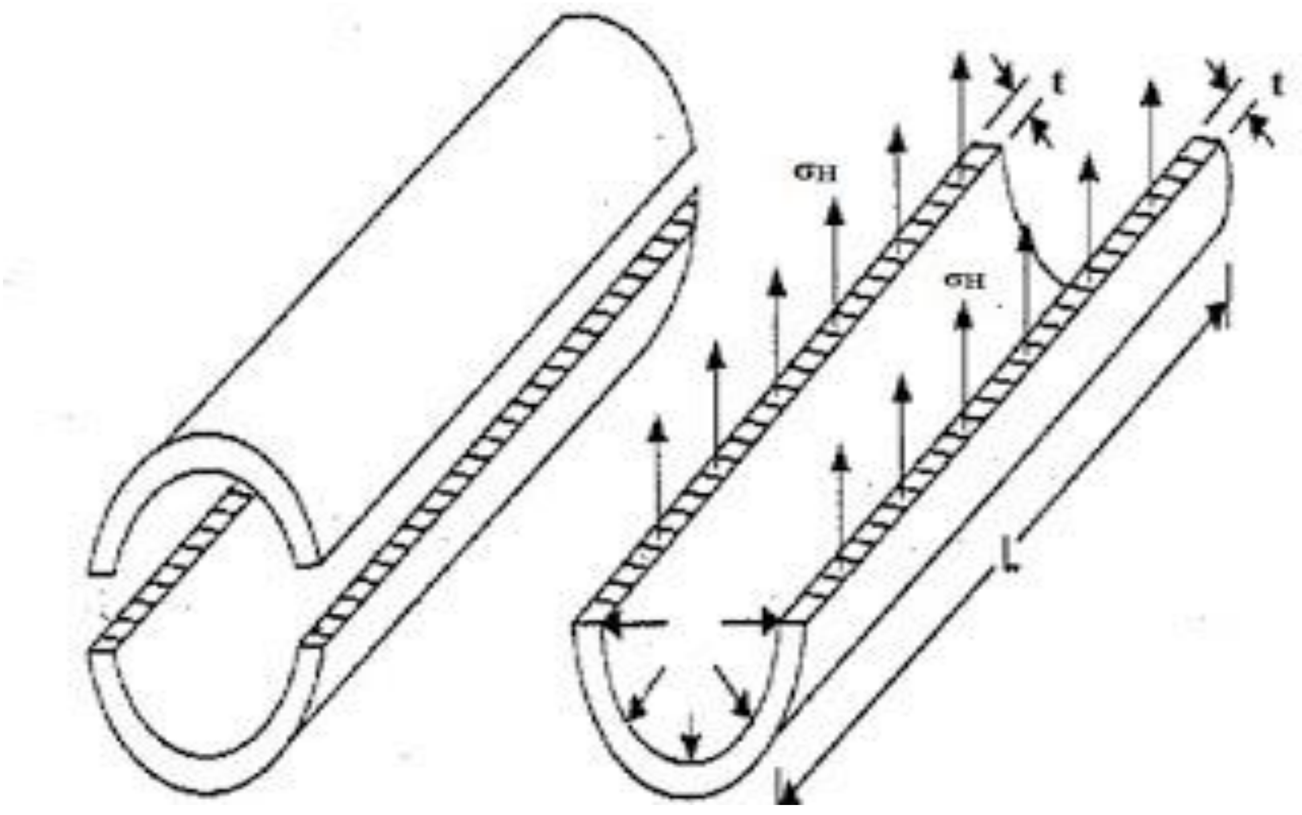
STRESSES IN A THIN CYLINDRICAL SHELL:

There will be two types of stresses, which will be developed in the wall of thin cylindrical shell and these stresses are as mentioned here.

CIRCUMFERENTIAL STRESS OR HOOP STRESS:

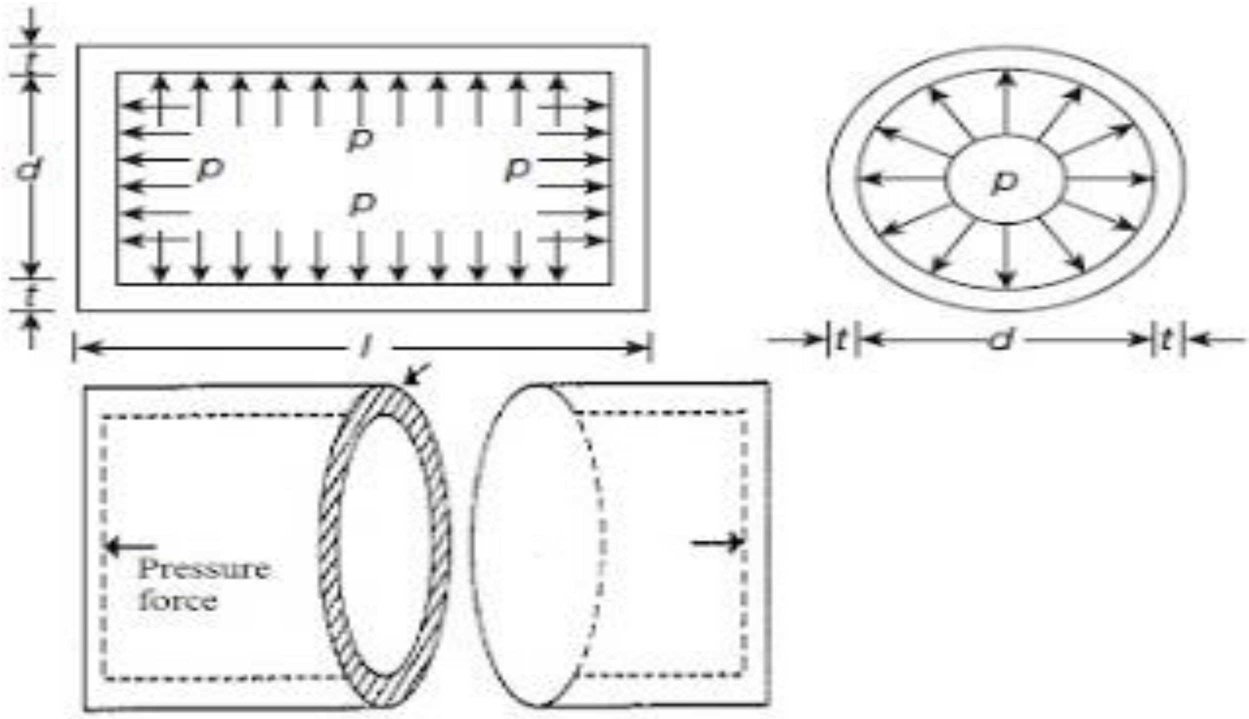
Stress acting along the circumference of thin cylinder will be termed as circumferential stress or hoop stress.

If fluid is stored under pressure inside the cylindrical shell, pressure will be acting vertically upward and downward over the cylindrical wall.



LONGITUDINAL STRESS:

- ✓ Stress acting along the length of thin cylinder will be termed as longitudinal stress.
- ✓ If fluid is stored under pressure inside the cylindrical shell, pressure force will be acting along the length of the cylindrical shell at its two ends.



EXPRESSION FOR CIRCUMFERENTIAL or HOOP STRESS:

$$\sigma_1 = \frac{p\,d}{2\,t}$$

Where, p – Internal fluid pressure

d – Internal diameter if the cylinder

t – Thickness of the wall of the cylinder

σ_1 – Circumferential or hoop stress in the material.

EXPRESSION FOR LONGITUDINAL STRESS:

$$\sigma_2 = \frac{p\,d}{4\,t}$$

σ_2 - Longitudinal stress in the material

Longitudinal stress= Half of the circumferential stress.

PROBLEMS

1.A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm^2. Determine the longitudinal stress developed in the pipe and circumferential stress developed in the pipe.

GIVEN DATA:

Diameter - 1.5 m

Thickness - 1.5 cm = 0.015 m

Internal fluid pressure of 1.2 N/mm^2

SOLUTION:

1. LONGITUDINAL STRESS,

$$\sigma_2 = \frac{p\,d}{4\,t} = \frac{1.2 \times 1.5}{4 \times 0.015} = \mathbf{30\ N/mm^2}$$

2. CIRCUMFERENTIAL STRESS,

$$\sigma_1 = \frac{p\,d}{2\,t} = \frac{1.2 \times 1.5}{2 \times 0.015} = \mathbf{60\ N/mm^2}$$

2. A cylinder of internal diameter 2.5 m and of thickness 5 cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm^2, determine the internal pressure of the gas.

GIVEN DATA:

Internal diameter - 2.5 m

Thickness - 5 cm = 0.05 m

Tensile stress - 80 N/mm^2

As tensile stress is given, hence this should be equal to circumferential stress (σ_1), σ_1 = 80 N/mm^2

SOLUTION:

1.INTERNAL PRESSURE OF THE GAS,

$$\sigma_1 = \frac{p\,d}{2\,t}$$

$$p = \frac{\sigma_1 \times 2\,t}{d} = \frac{80 \times 2 \times 0.05}{2.5} = \mathbf{3.2\ N/mm^2}$$

3. A cylinder of internal diameter 0.5 m contains air at a pressure of 7 N/mm^2. If the maximum permissible stress induced in the material is 80 N/mm^2, find the thickness of the cylinder.

GIVEN DATA:

Internal diameter - 0.5 m

Pressure - 7 N/mm^2

Maximum permissible stress - 80 N/mm^2

As maximum permissible stress is given, hence this should be equal to circumferential stress (σ_1).

σ_1 - 80 N/mm^2

SOLUTION:

1.THICKNESS OF THE CYLINDER,

$$\sigma_1 = \frac{p\,d}{2\,t}$$

$$t = \frac{p\,d}{2 \times \sigma_1} = \frac{7 \times 0.5}{2 \times 80}$$

t = **0.021875 m or 2.18 cm.**

4. A thin cylinder of internal diameter 1.25 m contains a fluid at an internal pressure of 2 N/mm^2. Determine the maximum thickness of the cylinder if i) longitudinal stress is not to exceed 30 N/mm^2 ii) circumferential stress is not to exceed 45 N/mm^2.

GIVEN DATA:

Internal diameter, d - 1.25 m

Internal fluid pressure, p - 2 N/mm^2

Longitudinal stress, σ_2 = 30 N/mm^2

circumferential stress, σ_1 = 45 N/mm^2.

SOLUTION:

1.MAXIMUM THICKNESS OF THE CYLINDER if σ_1 = 45 N/mm^2,

$$\sigma_1 = \frac{p\,d}{2\,t}$$

$$t = \frac{p\,d}{2 X \sigma_1} = \frac{2 X 1.25}{2 X 45}$$

t = **0.0277 m or 2.77 cm.**

2. MAXIMUM THICKNESS OF THE CYLINDER if σ_2 = 30 N/mm^2,

$$\sigma_2 = \frac{p\,d}{4\,t}$$

$$t = \frac{p\,d}{4 X \sigma_1} = \frac{2 X 1.25}{4 X 30}$$

t = **0.0208 m or 2.08 cm.**

The longitudinal or circumferential stresses induced in the material are inversely proportional to the thickness of the cylinder. Hence the stress induced will be less if the value of ‘t’ is more. Hence take the maximum value of ‘t’.

t = **2.77 cm.**

5. A water main 80 cm diameter contains water at a pressure head of 100 m. If the weight density of water is 9810 N/m^3, find the thickness of the metal required for the water main. Given the permissible stress as 20 N/mm^2.

GIVEN DATA:

Diameter of main, d - 1.25 m

Pressure head of water, h = 100 m

Permissible stress, σ_1 = 20 N/mm^2

SOLUTION:

1.THICKNESS OF THE METAL,

Pressure of water inside the water main, $P = w \times h = 9810 \times 100$

$$= 981000\ N/m^2 = 0.981\ N/mm^2.$$

$$\sigma_1 = \frac{p\,d}{2\,t}$$

$$t = \frac{p\,d}{2 \times \sigma_1} = \frac{0.981 \times 80}{2 \times 20} = \mathbf{2\ cm.}$$

EFFICIENCY OF A JOINT

The cylindrical shells such as boilers are having two types of joints namely longitudinal joint and circumferential joint.

η_l - Efficiency of a longitudinal joint and

η_c - Efficiency of a circumferential joint.

Circumferential stress, $\sigma_1 = \dfrac{p\,d}{2\,t \times \eta_l}$

Longitudinal stress, $\sigma_2 = \dfrac{p\,d}{4\,t \times \eta_c}$

Efficiency of a joint means the efficiency of a longitudinal joint.

6. A boiler is subjected to an internal steam pressure of 2 N/mm^2. The thickness of boiler plate is 2 cm and permissible tensile stress is 120 N/mm^2. Find out the maximum diameter, when efficiency of longitudinal joint is 90 % and that of circumferential joint is 40 %.

GIVEN DATA:

Internal steam pressure, $p = 2\ N/mm^2$

Thickness of boiler plate, $t = 2$ cm

Permissible tensile stress = $120\ N/mm^2$

$\eta_l = 90$ %

$\eta_c = 40$ %.

SOLUTION:

In case of a joint, the permissible stress may be circumferential stress or longitudinal stress.

1.MAXIMUM DIAMETER FOR CIRCUMFERENTIAL STRESS,

$$\sigma_1 = \frac{p\,d}{2\,t \times \eta_l}$$

$$120 = \frac{2 \times d}{2 \times 0.90 \times 2}$$

$$d = \frac{120 \times 2 \times 0.90 \times 2}{2} = \mathbf{216\ cm.}$$

2. MAXIMUM DIAMETER FOR LONGITUDINAL STRESS,

$$\sigma_2 = \frac{p\,d}{4\,t \times \eta_c}$$

$$120 = \frac{2 \times d}{4 \times 0.40 \times 2}$$

$$d = \frac{120 \times 4 \times 0.40 \times 2}{2} = \mathbf{192\ cm.}$$

The longitudinal or circumferential stresses induced in the material are directly proportional to diameter. Hence the stress induced will be less if the value of ‘d’ is less. Hence take the minimum value of d.

Maximum diameter of the boiler is equal to the minimum value of diameter.

Hence maximum diameter, **d = 192 cm.**

If d = 216 cm, σ_2 will be more than the given permissible stress.

$$\sigma_2 = \frac{p\,d}{4\,t \times \eta_c} = \frac{2 \times 216}{4 \times 2 \times 0.4} = \mathbf{135\ N/mm^2}$$

7. A cylinder of thickness 1.5 cm has to withstand maximum internal pressure of 1.5 N/mm^2. If the ultimate tensile stress in the material of the cylinder is 300 N/mm^2, factor of safety 3 and joint efficiency 80 %, determine the diameter of the cylinder.

GIVEN DATA:

ȣ

Thickness of cylinder, t = 1.5 cm

Internal pressure, p = 1.5 N/mm^2

Ultimate tensile stress = 300 N/mm^2

Factor of safety = 3

Ƞ = 80 %

Joint efficiency means the longitudinal joint efficiency, η_l = 80 %

SOLUTION:

$$\text{Working stress, } \sigma_1 = \frac{\text{Ultimate tensile stress}}{\text{Factor of safety}} = \frac{300}{3} = 100\ N/mm^2$$

$$\sigma_1 = \frac{p\,d}{2\,t \times \eta_l}$$

$$100 = \frac{1.5 \times d}{2 \times 0.80 \times 1.5}$$

$$d = \frac{100 \times 2 \times 0.80 \times 1.5}{1.5} = \mathbf{160\ cm\ or\ 1.6\ m.}$$

EFFECT OF INTERNAL PRESSURE ON THE DIMENSIONS OF A THIN CYLINDRICAL SHELL:

$$\text{Change in diameter, } \delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$$

$$\text{Change in length, } \delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$\text{Volumetric Strain, } \frac{\delta V}{V} = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

Change in volume, $\delta V = V\,[\,2\,e_1 + e_2]$

Change in volume, $\delta V = V \left(2\frac{\delta d}{d} + \frac{\delta L}{L} \right)$

8. Calculate the change in diameter, change in length and change in volume of a thin cylindrical shell 100 cm diameter, 1 cm thick and 5 m long when subjected to internal pressure of 3 N/mm^2. Take the value of E = 2 X 10^5 N/mm^2 and poisson's ratio, μ = 0.3.

GIVEN DATA:

Diameter of shell, d = 100 cm

Thickness of shell, t = 1 cm

Length of shell, L = 5 m = 500 cm

Internal pressure, p = 3 N/mm^2

Young's modulus, E = 2 X 10^5 N/mm^2

Poisson's ratio, μ = 0.3.

SOLUTION:

Change in diameter, $\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$

$$= \frac{3 \times 100^2}{2 \times 1 \times 2 \times 10^5}\left(\frac{1 - 0.3}{2}\right)$$

= **0.06375 cm.**

Change in length, $\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$

$$= \frac{3 \times 100 \times 500}{2 \times 1 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$$

= **0.075 cm.**

Change in volume, $\delta V = V \left(2\frac{\delta d}{d} + \frac{\delta L}{L} \right)$

$V = (\pi d^2/4) \times L = (\pi \times 100^2 /4) \times 500 = 3926990.817\ cm^3$

$$\delta V = 3926990.817 \times \left(\frac{2 \times 0.06375}{100} + \frac{0.075}{500} \right)$$

$= \mathbf{5595.96\ cm^3}$

9. A cylindrical thin drum 80 cm in diameter and 3 m long has a shell thickness of 1 cm. If the drum is subjected to an internal pressure of 2.5 N/mm². Determine the change in diameter, change in length and change in volume. Take the value of E = 2 X 10⁵ N/mm² and poisson's ratio, μ = 0.25.

GIVEN DATA:

Diameter of drum, d = 80 cm

Thickness of shell, t = 1 cm

Length of shell, L = 3 m = 300 cm

Internal pressure, p = 2.5 N/mm^2

Young's modulus, E = 2 X 10^5 N/mm^2

Poisson's ratio, μ = 0.25.

SOLUTION:

Change in diameter, $\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$

$$= \frac{2.5 \times 80^2}{2 \times 1 \times 2 \times 10^5}\left(\frac{1 - 0.25}{2}\right)$$

$= \mathbf{0.035\ cm.}$

Change in length, $\delta L = \frac{pd\,L}{2tE}\left(\frac{1}{2} - \mu\right)$

$$= \frac{2.5 \times 80 \times 300}{2 \times 1 \times 2 \times 10^5}\left(\frac{1}{2} - 0.25\right)$$

= 0.0375 **cm.**

Change in volume, $\delta V = V\left(2\frac{\delta d}{d} + \frac{\delta L}{L}\right)$

$V = (\pi d^2/4) \times L = (\pi \times 80^2/4) \times 300 = 1507964.473\ cm^3$

$\delta V = 1507964.473 \times \left(2 \times \frac{0.035}{80} + \frac{0.0375}{300}\right)$

$= \mathbf{1507.96\ cm^3}$

10. A cylindrical vessel whose ends are closed by means of rigid flange plates is made of steel plate 3 mm thick. The length and the internal diameter of the vessel are 50 cm and 25 cm respectively. Determine the longitudinal and circumferential stresses in the cylindrical shell due to an internal fluid pressure of 3 N/mm$^{2.}$ Also calculate the increase in length, diameter and volume of the vessel. Take the value of E = 2 X 10^5 N/mm^2 and poisson's ratio, μ = 0.3.

GIVEN DATA:

Thickness, t = 3 mm = 0.3 cm

Length of cylindrical vessel, L = 50 cm

Internal diameter, d = 25 cm

Internal fluid pressure, p = 3 N/mm^2

Young's modulus, E = 2 X 10^5 N/mm^2

Poisson's ratio, μ = 0.25.

SOLUTION:

1.CIRCUMFERENTIAL STRESS,

$\sigma_1 = \frac{p\,d}{2\,t} = \frac{3 \times 25}{2 \times 0.3} = \mathbf{125\ N/mm^2}$

2. LONGITUDINAL STRESS,

$\sigma_2 = \frac{p\,d}{4\,t} = \frac{3 \times 25}{4 \times 0.3} = \mathbf{62.5\ N/mm^2}$

Change in diameter, $\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$

$$= \frac{3 \times 25^2}{2 \times 0.3 \times 2 \times 10^5}\left(1 - \frac{0.3}{2}\right)$$

$$= \mathbf{0.0133\ cm.}$$

Change in length, $$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{3 \times 25 \times 50}{2 \times 0.3 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$$

$$= \mathbf{0.00625\ cm.}$$

Change in volume, $$\delta V = V\left(2\frac{\delta d}{d} + \frac{\delta L}{L}\right)$$

$V = (\pi d^2/4) \times L = (\pi \times 25^2/4) \times 50 = 24543.69\ cm^3$

$$\delta V = 24543.69 \times \left(2 \times \frac{0.0133}{25} + \frac{0.00625}{50}\right)$$

$= \mathbf{29.18\ cm^3}$

11. A cylindrical vessel is 1.5 m diameter and 4 m long is closed at ends by rigid plates. It is subjected to an internal pressure of 3 N/mm^2. If the maximum principal stress is not to exceed 150 N/mm^2, find the thickness of the shell. Assume E = 2 X 10^5 N/mm^2 and poisson's ratio, μ = 0.25. Find the changes in diameter, length and volume of the shell.

GIVEN DATA:

Diameter of vessel, d = 1.5 m = 1500 mm

Length of cylindrical vessel, L = 4 m = 4000 mm

Internal pressure, p = 3 N/mm^2

Maximum principal stress = 150 N/mm^2

Young's modulus, E = 2 X 10^5 N/mm^2

Poisson's ratio, μ = 0.25.

SOLUTION:

1.THICKNESS OF THE SHELL,

Maximum principal stress means circumferential stress.

$\sigma_1 = 150\ N/mm^2$

$$\sigma_1 = \frac{p\,d}{2\,t}$$

$$t = \frac{p\,d}{2 \times \sigma_1} = \frac{3 \times 1500}{2 \times 150}$$

t = **15 mm.**

2. CHANGES IN DIAMETER, LENGTH & VOLUME,

Change in diameter, $\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$

$$= \frac{3 \times 1500^2}{2 \times 15 \times 2 \times 10^5}\left(1 - \frac{0.25}{2}\right)$$

= **0.984 mm.**

Change in length, $\delta L = \frac{pd\,L}{2tE}\left(\frac{1}{2} - \mu\right)$

$$= \frac{3 \times 1500 \times 4000}{2 \times 15 \times 2 \times 10^5}\left(\frac{1}{2} - 0.25\right)$$

= **0.75 mm.**

Volumetric Strain, $\frac{\delta V}{V} = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$

Change in volume, $\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times V$

$$= \frac{3 \times 1500}{2 \times 2 \times 10^5 \times 15}\left(\frac{5}{2} - 2 \times 0.25\right) \times V$$

$$V = (\pi d^2/4) \times L = (\pi \times 1500^2 /4) \times 4000 = 7.0685 \times 10^9 \text{ mm}^3$$

$$\delta V = \frac{3 \times 1500}{2 \times 2 \times 10^5 \times 15}\left(\frac{5}{2} - 2 \times 0.25\right) \times 7.0685 \times 10^9$$

$$= \mathbf{10602875\ mm^3}$$

WIRE WINDING OF THIN CYLINDERS:

- We know that the hoop stress is two times the longitudinal stress in a thin cylinder, when the cylinder is subjected to internal fluid pressure.
- Hence the failure of a thin cylinder will be due to hoop stress.
- Also, the hoop stress which is tensile in nature is directly proportional to the fluid pressure inside the cylinder.
- In case of cylinders which have to carry high internal fluid pressures, some methods of reducing the hoop stresses have to be devised.
- One method is to wind strong steel wire under tension on the walls of the cylinder.
- The effect of the wire is to put the cylinder wall under an initial compressive stress.

THIN SPHERICAL SHELLS:

- A thin spherical shell of internal diameter 'd' and thickness 't' is subjected to an internal fluid pressure 'p'.
- The fluid inside the shell has the tendency to split the shell into two hemi spheres along x-x axis.
- The force P which has a tendency to split the shell.

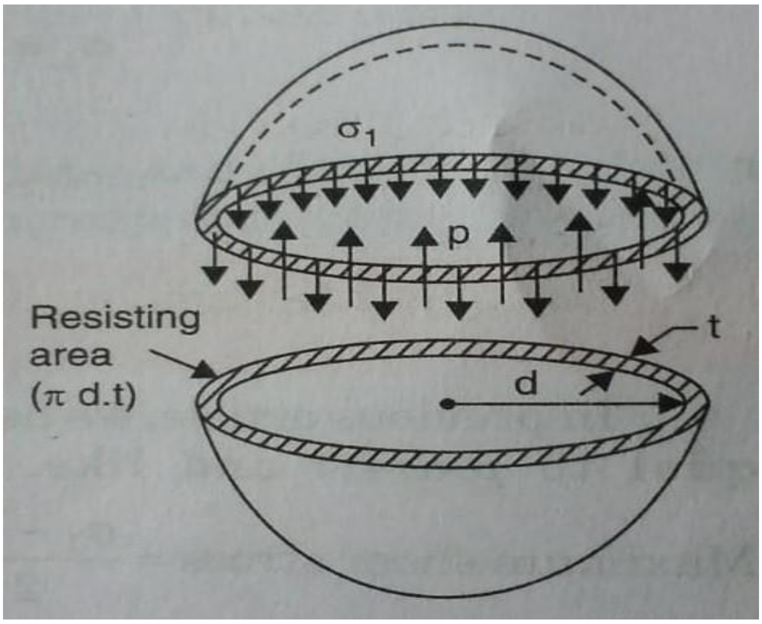
$$= p \times (\pi \times d^2/4)$$

The area resisting this force, $A = \pi \times d \times t$

Hoop stress induced in the material of the shell is given by,

$$\sigma_1 = \frac{\text{Force (P)}}{\text{Area resisting this force (A)}}$$

$$\sigma_1 = \frac{p \times (\pi \times d^2/4)}{\pi \times d \times t} = \frac{p\,d}{4\,t}$$



PROBLEMS:

1.A vessel in the shape of a spherical shell of 1.2 m internal diameter and 12 mm shell thickness is subjected to pressure of 1.6 N/mm^2 . Determine the stress induced in the material of the vessel.

GIVEN DATA:

Internal diameter of shell, d = 1.2 m = 1200 mm

Thickness of shell, t = 12 mm

Pressure, p = 1.6 N/mm^2

SOLUTION:

1. STRESS INDUCED IN THE MATERIAL OF THE VESSEL,

$$\sigma_1 = \frac{p\,d}{4\,t} = \frac{1.6 \times 1200}{4 \times 12} = \mathbf{40\ N/mm^2}$$

2.A spherical vessel 1.5 m diameter is subjected to an internal pressure of 2 N/mm^2. Find the thickness of the plate required if maximum stress is not to exceed 150 N/mm^2 and joint efficiency is 75 %.

GIVEN DATA:

Diameter of spherical vessel, d = 1.5 m = 1500 mm

Internal pressure = 2 N/mm^2

Maximum stress, σ_1 = 150 N/mm^2

Joint efficiency, η = 75 %.

SOLUTION:

$$\sigma_1 = \frac{p\,d}{4\,t \times \eta}$$

$$t = \frac{2 \times 1500}{4 \times 150 \times 0.75} = \mathbf{6.67\ mm}$$

CHANGE IN DIMENSIONS OF A THIN SPHERICAL SHELL DUE TO AN INTERNAL PRESSURE:

Strain, $\frac{\delta d}{d} = \frac{pd}{4tE}(1 - \mu)$

Change in diameter, $\delta d = \frac{pd}{4tE}(1 - \mu) \times d$

Volumetric strain, $\frac{\delta V}{V} = 3 \times \frac{pd}{4tE}(1 - \mu)$

Change in volume, $\delta V = 3 \times \frac{pd}{4tE}(1 - \mu) \times V$

3.A spherical shell of internal diameter 0.9 m and of thickness 10 mm is subjected to an internal pressure of 1.4 N/mm^2. Determine the increase in diameter and increase in volume. Take E = 2 X 10^5 N/mm^2 and poisson's ratio, μ = 1/3.

GIVEN DATA:

Internal diameter of spherical shell, d = 0.9 m = 900 mm

Thickness, t = 10 mm

Internal pressure, p = 1.4 N/mm^2

E = 2 X 10^5 N/mm^2

Poisson's ratio, μ = 1/3.

SOLUTION:

1. INCREASE IN DIAMETER,

Increase in diameter, $\delta d = \frac{pd}{4tE}(1 - \mu) \times d$

Increase in diameter, $\delta d = \frac{1.4 \times 900}{4 \times 10 \times 2 \times 10^5}(1 - (1/3)) \times 900$

= **0.0945 mm.**

2. INCREASE IN VOLUME,

Change in volume, $\delta V = 3 \times \frac{pd}{4tE}(1 - \mu) \times V$

Volume, $V = \pi \times d^3/6 = \pi \times 900^3/6$ = **381.7 mm^3.**

Change in volume, $\delta V = \frac{3 \times 1.4 \times 900}{4 \times 10 \times 2 \times 10^5}(1 - (1/3)) \times 381.7$

= **12028.5 mm^3.**

THICK CYLINDER:

- The cylinder which have Thickness is more than 1/20 of its diameter that Cylinder is called as thick Cylinder.
- If the ratio of thickness to internal diameter is more than 1/20, then cylindrical shell is known as thick cylinders.

STRESSES PRODUCED DUE TO INTERNAL FLUID PRESSURE:

- ✓ Radial pressure p (Compressive)
- ✓ Circumferential stress or Hoop stress σ_1 (Tensile)
- ✓ Longitudinal stress σ_2 (Tensile)

Radial Pressure: $p_x = \frac{b}{x^2} - a$ …………….(1)

Hoop Stress: $\sigma_x = \frac{b}{x^2} + a$ ………………..(2)

Above 2 equations are called Lame's equations.

The constants 'a' and 'b' are obtained from boundary conditions.

1. At $x = r_1$, $p_x = p_0$ or the pressure of fluid inside the cylinder.
2. At $x = r_2$, $p_x = 0$ or atmospheric pressure.

PROBLEMS:

1.Determine the maximum and minimum stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm². Also sketch the radial pressure distribution and hoop stress distribution across the section.

GIVEN DATA:

Internal diameter, $d_1 = 400$ mm

Internal radius, $r_1 = 400/2 = 200$ mm

External diameter, $d_2 = 400 + 2 \times 100 = 600$ mm

External radius, $r_2 = 600/2 = 300$ mm

Fluid pressure, $p_0 = 8$ N/mm^2

SOLUTION:

1. MAXIMUM AND MINIMUM STRESS,

The radial pressure $p_x = \frac{b}{x^2} - a$ ……………(1)

Now apply the boundary conditions to the above equation.

The boundary conditions are:

1. At $x = r_1 = 200$ mm, $p_x = p_0 = 8$ N/mm^2
2. At $x = r_2 = 300$ mm, $p_x = 0$.

Substituting these boundary conditions in equation (1), we get

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a \text{ …………(2)}$$

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a \ldots\ldots\ldots\ldots.(3)$$

Subtracting equation 3 from equation 2, we get

$$8 - 0 = \frac{b}{40000} - a - \frac{b}{90000} + a$$

$$8 = \frac{9b - 4b}{360000} = \frac{5\,b}{360000}$$

$$b = \frac{360000 \text{ X } 8}{5} = \mathbf{576000}$$

Substituting this value in equation (3), we get

$$0 = \frac{576000}{90000} - a$$

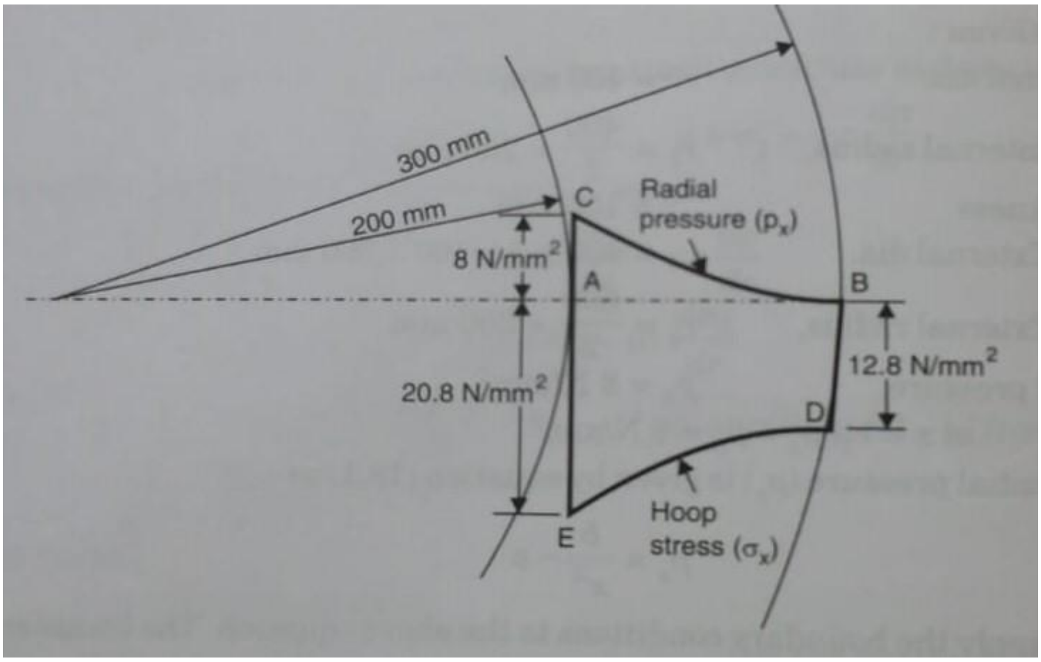
$$a = 576000/90000 = \mathbf{6.4}$$

The values of ‘a’ and ‘b’ are substituted in the hoop stress.

$$\text{Hoop Stress: } \sigma_x = \frac{b}{x^2} + a = \frac{576000}{x^2} + 6.4$$

$$\text{At } x = 200 \text{ mm, } \sigma_{200} = \frac{576000}{200^2} + 6.4 = \mathbf{20.8\ N/mm^2}$$

$$\text{At } x = 300 \text{ mm, } \sigma_{300} = \frac{576000}{300^2} + 6.4 = \mathbf{12.8\ N/mm^2}$$


300 mm
200 mm
C
Radial
pressure (p_x)
8 N/mm^2
A
B
12.8 N/mm^2
20.8 N/mm^2
D
Hoop
stress (σ_x)
E

REFERENCE BOOKS:

1. Bansal R.K., “Strength of Materials”, 5th Edition, Laxmi Publications, 2013.
2. Ramamrutham R., “Strength of Materials”,14th Edition, Dhanpat Rai Publications, 2011.
3. Punmia B.C., “Strength of Materials”, Laxmi Publications, 2011.
4. Subramanian R., “Strength of Materials - Theory and Problems”, Oxford University Press, 2010.
5. Rajput R.K., “Strength of Materials”, S. Chand and Company Ltd., New Delhi, 2007.
6. Timoshenko S.P. and Gere J.M., “Mechanics of Materials”, 6th Edition, Tata McGraw Hill.

QUESTION BANK:

1. Distinguish thin and thick cylinder.
2. Name the stresses set up in a thin cylinder subjected to internal fluid pressure.
3. What are circumferential and longitudinal stresses?
4. What do you mean by Lame's equation?
5. Name the stresses set up in a thick cylinder subjected to internal fluid pressure.
6. A cylinder of internal diameter 2.5 m and of thickness 5 cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm2, find the internal pressure of the gas.
7. A vessel in the shape of a spherical shell of 1.2 m internal diameter and 12 mm shell thickness is subjected to pressure of 1.6 N/mm2. Find the stress induced in the material of the vessel.
8. A spherical vessel 1.5 m diameter is subjected to an internal pressure of 2 N/mm2. Find the thickness of the plate required if maximum stress is not to exceed 150 N/mm2 and joint efficiency is 75 %.
9. A cylinder pipe of diameter 2 m and thickness 2 cm is subjected to an internal fluid pressure of 1.5 N/mm2, find the longitudinal and circumferential stress developed in the pipe material.
10. A thin cylinder of internal diameter 2 m contains a fluid at an internal pressure of 3 N/mm2. Determine the maximum thickness of the cylinder if i)longitudinal stress is not to exceed 30 N/mm2 ii)circumferential stress is not to exceed 40 N/mm2.
11. A thin cylindrical shell of 120 cm diameter, 1.5 cm thick and 6 m long is subjected to internal fluid pressure of 2.5 N/mm2.If E = 2 X105 N/mm2 and poisson's ratio = 0.3, determine i) Change in diameter ii) change in length iii) change in volume.
12. Determine the thickness of metal necessary for a cylindrical shell of internal diameter 150 mm to withstand an internal pressure of 50 N/mm2. The maximum hoop stress in the section is not to exceeed 150 N/mm2.