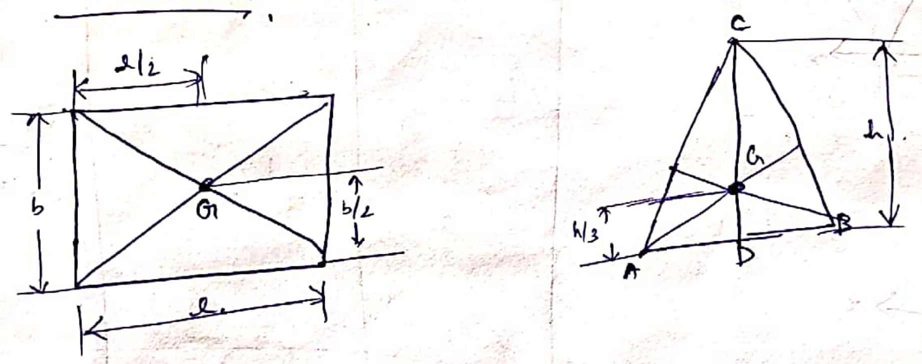
CENTROID & CENTRE OF GRAVITY:-

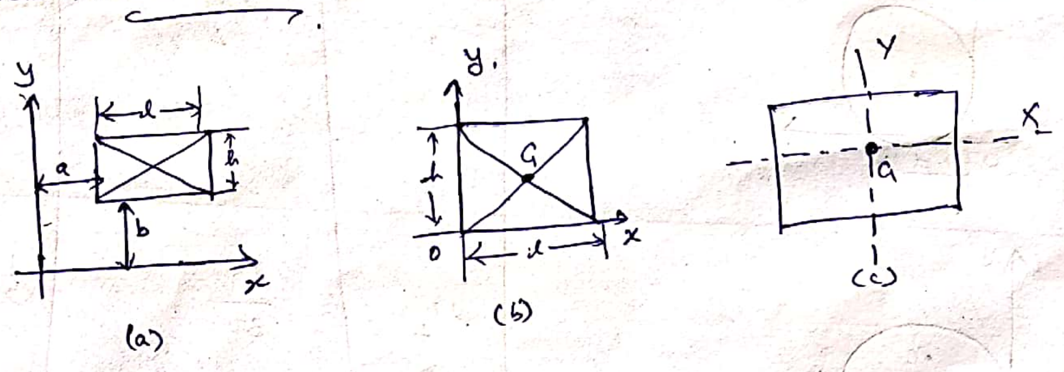
* The C.O.G. of a body is defined as the Point through which the entire weight of body acts, when this is reffered to weightless laminas (or) Plane areas (Plane areas have no mass) is called the Centroid of the area. (or) in otherwords, Centroid is the term refered to one & two Dimensional figures & C.O.G is refered to three D Figures. Both are represented by G.

* It is to be noted that, every body has one & only one, centre of gravity, which may be within the body or even outside of the body.

CENTROID OF PLANE FIGURES:



REFERENCE AXES & CENTROIDAL AXES:-



CENTROID OF COMPOSITE PLANE FIGURES:-

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 \ldots + a_nx_n}{a_1+a_2+a_3 \ldots + a_n}$$

$||^{ly}$ $$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 \ldots + a_ny_n}{a_1+a_2+a_3 \ldots + a_n}$$

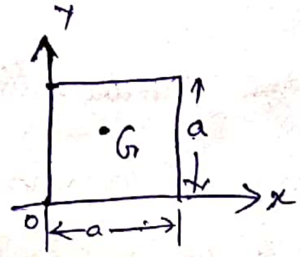
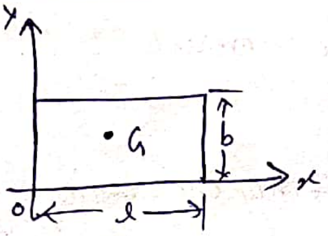
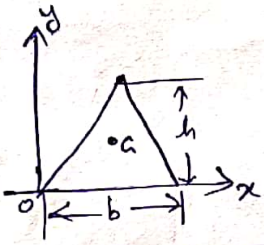
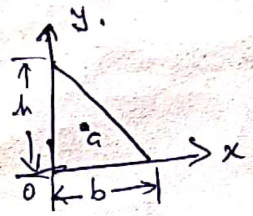
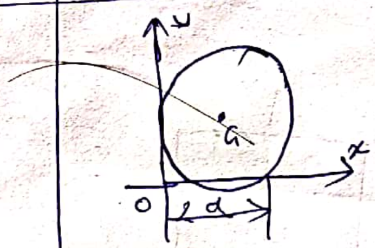
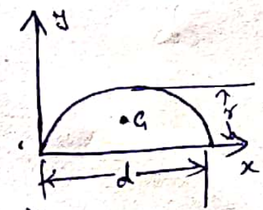
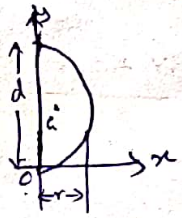
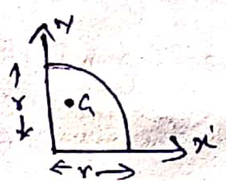
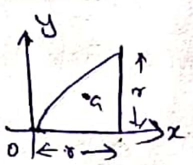
$a_1, a_2, \ldots a_n$ → Area of simple Plane figures 1, 2, 3 of Composite Plane figures.

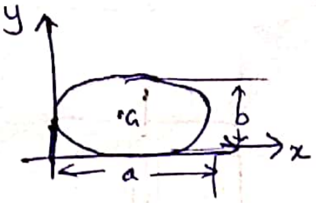
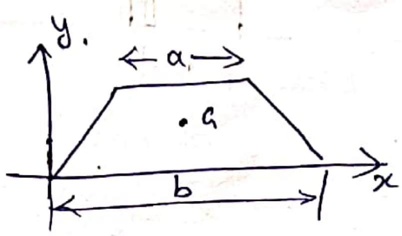
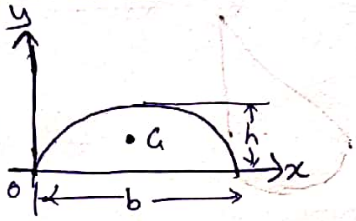
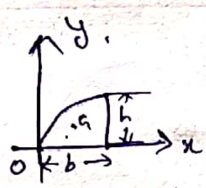
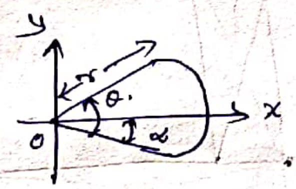
$x_1, x_2, \ldots x_n$ → horizontal distance of centroid of simple figures 1, 2, ... from vertical reference axis oy.

$y_1, y_2 \ldots y_n$ → Vertical " " " " from horizontal " " ox.

$\bar{x}$ & $\bar{y}$ → Horiz & vertical dist of Centroid of Composite plane figure from the vertical & horizontal reference axes.

CENTROID OF SIMPLE PLANE FIGURES

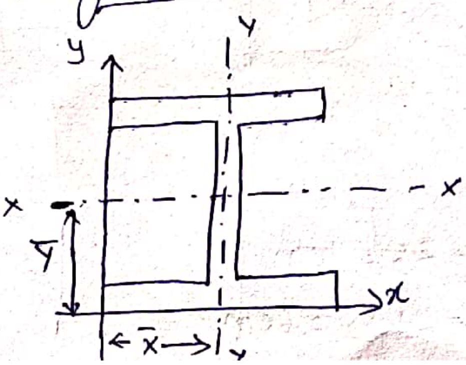
S.No	NAME	SHAPE	$\bar{x}$	$\bar{y}$	AREA
1	Square		$a/2$	$a/2$	a^2
2	RECTANGLE		$l/2$	$b/2$	$lb.$
3	TRIANGLE (ISOSCELES)		$b/2$	$h/3$	$\frac{1}{2}bh$
4	TRIANGLE (RIGHT-ANGLED)		$b/3$	$h/3$	$\frac{1}{2}bh$
5	CIRCLE		$d/2$	$d/2$	πr^2 (or) $\frac{\pi d^2}{4}$.
6	SEMI-CIRCLE		$d/2$	$\frac{4r}{3\pi}$	$\frac{1}{2} \times \frac{\pi d^2}{4}.$
7	SEMI-CIRCLE		$\frac{4r}{3\pi}$	$\frac{d}{2}$	$\frac{1}{2} \times \frac{\pi d^2}{4}.$
8)	QUADRANT		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{1}{4} \times \pi r^2$
			$\left[r - \frac{4r}{3\pi}\right]$	$\frac{4r}{3\pi}$	

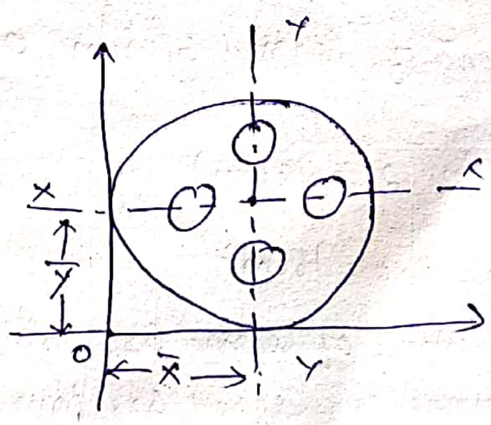
10	ELLIPSE		$a/2$	$b/2$	$\pi ab/4$.
11.	TRAPEZIUM		$b/2$	$\left[\frac{b+2a}{b+a}\right]\frac{h}{3}$	$\frac{1}{2}(a+b)h$.
12.	PARABOLA		$b/2$	$2/3\,h$	$2/3\,bh$.
13.	SEMI-PARABOLA		$5/8\,b$	$2b/5$	$2/3\,bh$
14	SECTOR OF CIRCLE		$\frac{2r\sin\alpha}{3\alpha}$	0	$\frac{\theta}{360}\times\pi r^2$

AXIS OF SYMMETRY:-

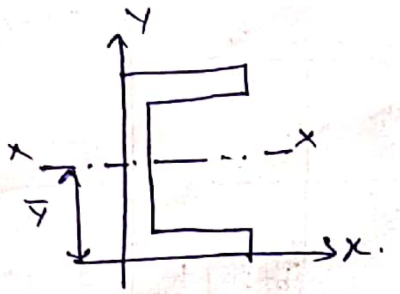
If a composite plane figure has an axis of symmetry (i.e.), an axis about which similar configuration is seen on either side), centroid lies on it. This concept reduce the work of locating the centroid. Axis of symmetry concept is illustrated by some examples given below.

(i) Symmetrical about both the axis.

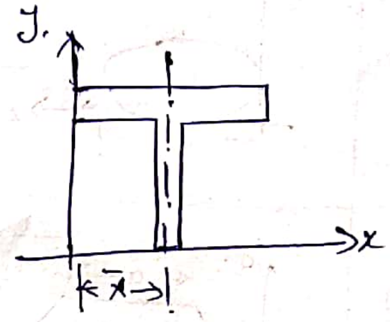




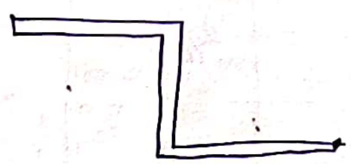
(ii) SYMMETRICAL ABOUT X AXIS



(iii) SYMMETRICAL ABOUT Y AXIS

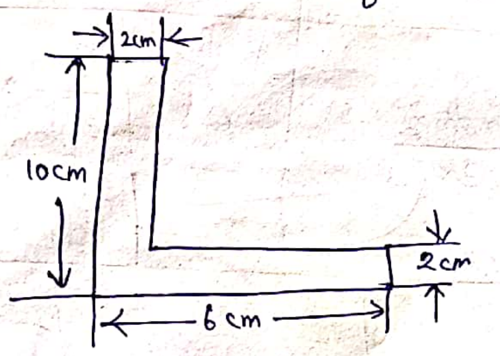


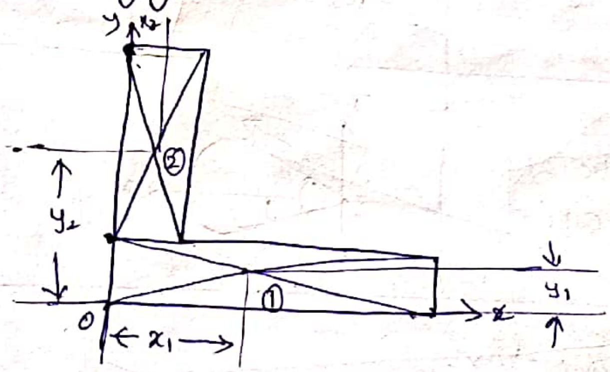
(iv) NOT SYMMETRICAL ABOUT ANY AXIS:



(5) ~~Calculate the~~

Locate the Centroid of L-section shown in fig ~~1a~~.





PORTION - 1 (Size 6cm x 2cm)

Area, $a_1 = 6 \times 2 = 12\ cm^2$

Horizontal distance of centroid G_1 from OY axis, $X_1 = 6/2 = 3cm$.

Vertical distance of centroid G_1 from OX axis, $Y_1 = 2/2 = 1cm$.

Portion (2) (Size 2cm x 8cm)

Area, $a_2 = 2 \times 8 = 16cm^2$

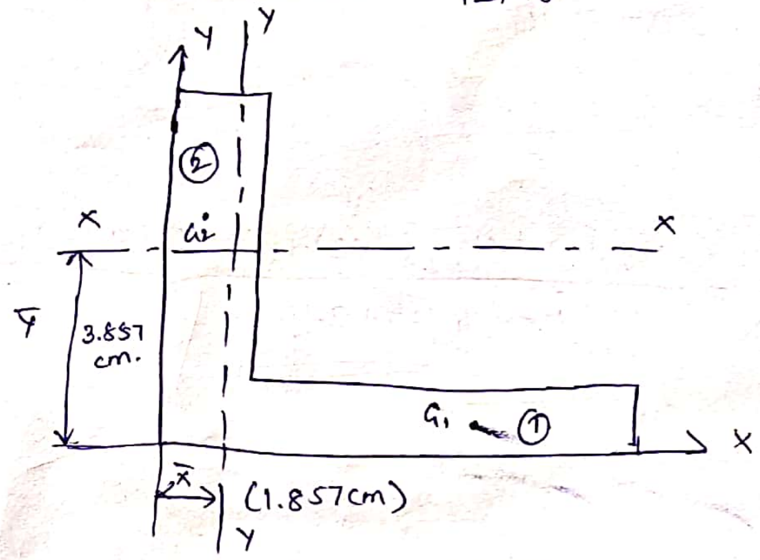
Horizontal distance of Centroid G_2, from OY axis, $X_2 = 2/2 = 1cm$.

Vertical distance of Centroid G_2, from OX axis, $Y_2 = 2 + 8/2 = 6cm$

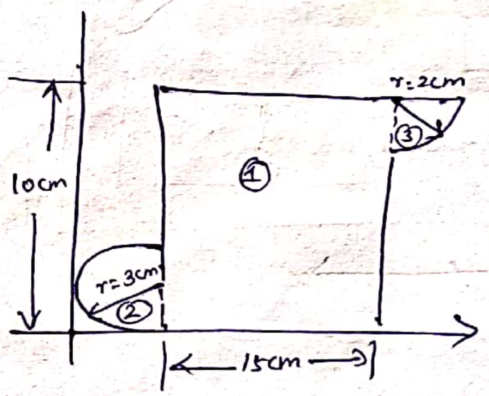
Using the relation,

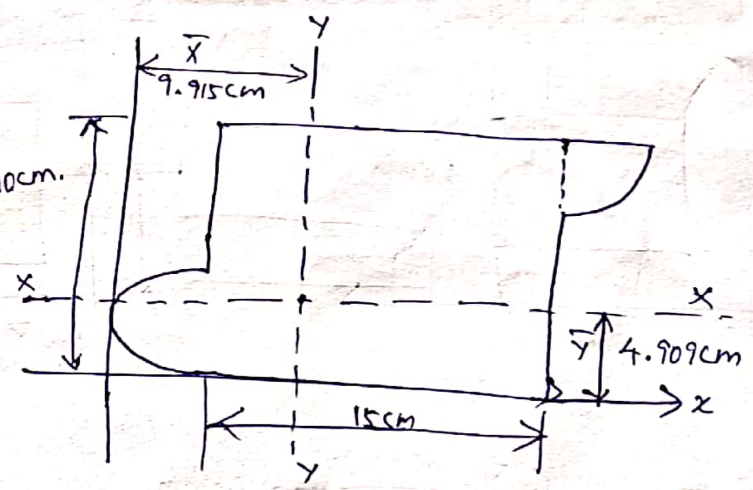
$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = \frac{(12\times3)+(16\times1)}{12+16} = 1.857\,cm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{(12\times1)+(16\times6)}{12+16} = 3.857\,cm.$$



5) Locate the centroid of fig.





PORTION → 1

Area, $a_1 = 15\times10 = 150\,cm^2$

$X_1 = 3+\frac{15}{2} = 10.5\,cm$

$Y_1 = 10/2 = 5\,cm$

PORTION - 2

Area, $A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi\times3^2 = 14.137\,cm^2$

$X_2 = r - \left(\frac{4r}{3\pi}\right) = 3 - \left(\frac{4\times3}{3\pi}\right)$

$= 1.726\,cm.$

$Y_2 = r = 3\,cm.$

PORTION - 3

$A_3 = \frac{1}{4}\pi r^2 = 3.14\,cm^2$

$X_3 = 3+15+\left(\frac{4\times2}{3\pi}\right)$

$= 18.84\,cm$

$Y_3 = 10 - \left(\frac{4\times2}{3\pi}\right) = 9.15\,cm$

using the relation,

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

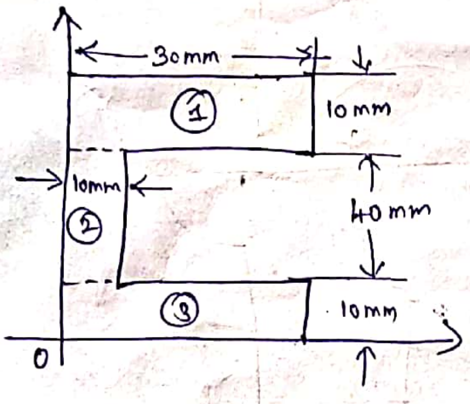
$$= \frac{(150 \times 10.5) + (14.137 \times 1.726) + (3.14 \times 18.84)}{150 + 14.137 + 3.14}$$

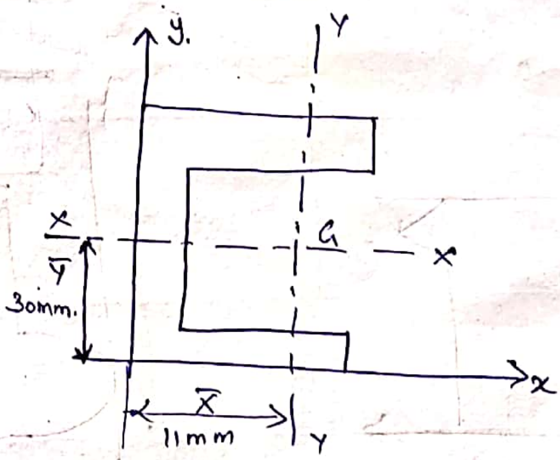
$$\boxed{\bar{X} = 9.915\ cm}$$

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + a_3 Y_3}{a_1 + a_2 + a_3} = \frac{(150 \times 5) + (14.137 \times 3) + (3.14 \times 9.15)}{150 + 14.137 + 3.14}$$

$$\boxed{\bar{Y} = 4.909\ cm}$$

(5) Locate the Centroid of lamina





Due to symmetry, $\bar{y} = \frac{10+40+10}{2} = 30mm.$

to find, $\bar{x}$, use the relation, $\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$

where,

$a_1 = (30 \times 10) = 300\ mm^2$; $x_1 = \frac{30}{2} = 15mm.$

$a_2 = (10 \times 40) = 400\ mm^2$; $x_2 = \frac{10}{2} = 5mm$

$a_3 = (30 \times 10) = 300\ mm^2$; $x_3 = \frac{30}{2} = 15mm.$

$$\bar{x} = \frac{(300 \times 15) + (400 \times 5) + (300 \times 15)}{300 + 400 + 300}$$

$$\boxed{\bar{x} = 11mm}$$

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