Displacements, Velocity and acceleration, their relationship

Displacement(s)

$$Speed = \frac{Distance\text{ travelled}}{Time\ taken} \qquad \text{unit}(\frac{m}{s}, \frac{km}{hr})$$

$$Velocity = \frac{Distance\text{ travelled in particular direction}}{time\text{ taken}} \qquad \text{(m/s) [u,v] (vector quantity)}$$

Acceleration[a], (vector quantity)

$$= \frac{Change\text{ of velocity}}{time\ taken}$$

$$= \frac{final\text{ velocity-initial velocity}}{time\ taken} \qquad (m/s^2)$$

Negative acceleration is called as retraction[when final velocity<Initial velocity]

Average velocity

$$Average\ velocity = \frac{Change\text{ of position}}{Change\text{ of time}} = \frac{\Delta x}{\Delta t}$$

Average speed

$$Average\ speed = \frac{Total\ distance\ travelled}{Total\ time\ taken}$$

[If particle starts from a point and then if returns to the same point, average velocity is zero. But average speed is not zero.]

Instantaneous velocity

For any particle, the instantaneous velocity at any instant of time is the limit of average velocity as the increment of time approaches zero.

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Mathematical expressions for velocity and acceleration

S=Distance travelled by a particle in a straight line

T= time taken by the particle to travel this distance 's'

Mathematically velocity[v] $= \frac{ds}{dt}$,

Acceleration[a]= $= \frac{dv}{dt} = = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d_2 s}{dt^2}$

Types of rectilinear motion

[uniform acceleration, variable acceleration]

Rectilinear motion with uniform acceleration

Equation of motion in a straight line

u=initial velocity[m/s]

v=final velocity[m/s]

s=distance [m]

t=-time taken by the particle, to change from u to v[sec]

a=acceleration of particle [m/s²]

change of velocity=final Velocity-Initial velocity=(v-u)

$$acceleration\ (or\ rate\ of\ change\ of\ velocity) = \frac{Change\ of\ velocity}{time\ taken}$$

$$a = \frac{v-u}{t}$$

$(v-u) = at$ or v=u+at

Average velocity= $= \frac{initial\ velocity + final\ velocity}{2} = \frac{u+v}{2}$

Distance travelled by the particle in 't' sec, s=average velocity × time $= \left(\frac{u+v}{2}\right)t$

Substitute the above value of t in equation $s = ut + \frac{1}{2}at^2$

$$= u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 = (v-u)\left[\frac{u}{a} + \frac{1}{2a}(v-u)\right]$$

$$= (v-u)\left[\frac{2u+v-u}{2a}\right] = \frac{(v-u)(v+u)}{2a} = \frac{v^2-u^2}{2a}$$

$$v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as$$

Equation of motion

(i) $v=u+at$,(ii) $S = ut + \frac{1}{2}at^2$,(iii) $v^2 = u^2 + 2as$

[If the body starts from rest, its initial velocity is zero(ie,u=0)]
[If the body comes to rest, its final velocity is zero(ie,V=0)]

Distance travelled in n^{th} second

U=initial velocity of the particle
V=final velocity of the particle
a=constant acceleration of the particle
S_n =Distance travelled in n sec
S_{n-1} =Distance travelled in (n-1) sec

$$S^{nth} = u + \frac{a}{2}(2n-1)$$

Rectilinear motion

S.no	Rectilinear motion		
	Horizontal motion	Vertical downward motion	Vertical upward motion
1.	$v = u + at$	$v = u + gt$	$v = u - gt$
2.	$s = ut + \frac{1}{2}at^2$	$h = ut + \frac{1}{2}gt^2$	$h = ut - \frac{1}{2}gt^2$
3.	$v^2 = u^2 + 2as$	$v^2 = u^2 + 2gh$	$v^2 = u^2 - 2gh$
4.	$S_n = u + \frac{a}{2}(2n-1)$	$h_n = u + \frac{g}{2}(2n-1)$	$h_n = u + \frac{g}{2}(2n-1)$

[i) when a body starts moving vertically downwards, its initial velocity=0,
ii) when a body is projected vertically upwards, at the highest point, its final velocity, v=0]

upward motion $\Rightarrow$ (i) Maximum height attained by upward particle $h_{max} = \frac{u^2}{2g}$

(ii) Time taken by the particle in reach maximum height $t = \frac{u}{g}$

(iii)Total time taken to return the surface (T)=2 $\times$ Time up, $T = \frac{2u}{g}$

$u = \sqrt{2g \times h_{max}}$ (if h_{max} is known), $u = t \times g$ (if time is known),

Downward motion $\Rightarrow$

Striking velocity of the particle, moving downwards, from the position of rest

$v^2 = u^2 + 2gh \Rightarrow v = \sqrt{2gh}$

Rectilinear motion with variable acceleration

(i)Displacement is function of time $s = f(t) \Rightarrow$

$$\text{Velocity } v = \frac{ds}{dt}, \Rightarrow v = \frac{d}{dt}[f(t)]$$

$$\text{Acceleration, a} = \frac{dv}{dt} = \frac{d^2}{dt^2}[f(t)]$$

(ii) Acceleration *is function of time a=* $f(t)$

$$\text{Velocity V} = \int f(t)dt$$

$$\text{Displacement, S} = \int v dt = \iint f(t)dt$$

(iii) Acceleration *is function of velocity*

(iii) Acceleration *is function of distance*