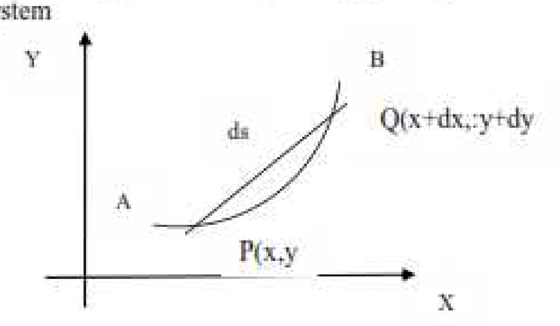
Relative motion –Curvilinear motion

Two system $\Rightarrow$ (i) Cartesian system (ii) polar system

(i)Cartesian system



Velocity of the particle $\Rightarrow$

Horizontal component of velocity [parallel to OX], $V_x = \underset{\partial t \to 0}{Lt} \frac{\partial x}{\partial t} \Rightarrow V_x = \frac{dx}{dt}$

Vertical component of velocity [parallel to OY], $V_y = \underset{\partial t \to 0}{Lt} \frac{\partial y}{\partial t} \Rightarrow V_y = \frac{dy}{dt}$

The resultant velocity of particle at any point,

$$V_y = \underset{\partial t \to 0}{Lt} \frac{\partial y}{\partial t} \Rightarrow V_y = \frac{dy}{dt}$$

$$V = \sqrt{(V_x)^2 + (V_y)^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

The angle of inclination of resultant velocity with x axis $\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

Acceleration of the particle

Let the velocity of particle changes from P(x,y) to Q(x+dx,y+dy) in a small interval ∂t

Acceleration of the particle along OX,

$$a_x = \underset{\partial t \to 0}{Lt} \frac{\partial v_x}{\partial t} \Rightarrow a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Acceleration of the particle along OY

$$a_y = \underset{\partial t \to 0}{Lt} \frac{\partial v_y}{\partial t} \Rightarrow a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2}$$

Resultant acceleration, $a = \sqrt{(a_x)^2 + (a_y)^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

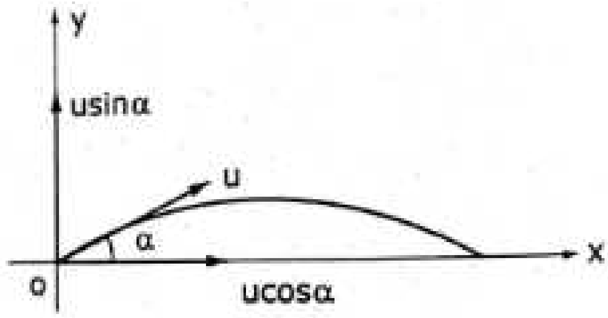
Let the angle of inclination of acceleration with x axis is ϕ, $\phi = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

Acceleration –Normal and tangential components

Tangential acceleration, $a_t = \frac{dv}{dt}$

Normal acceleration, $a_n = \frac{v^2}{r}$

Projectile Motion



Projectile

Angle of projection

Velocity of projectile

Component of velocity along OX axis $= u\cos\alpha$ [horizontal velocity]

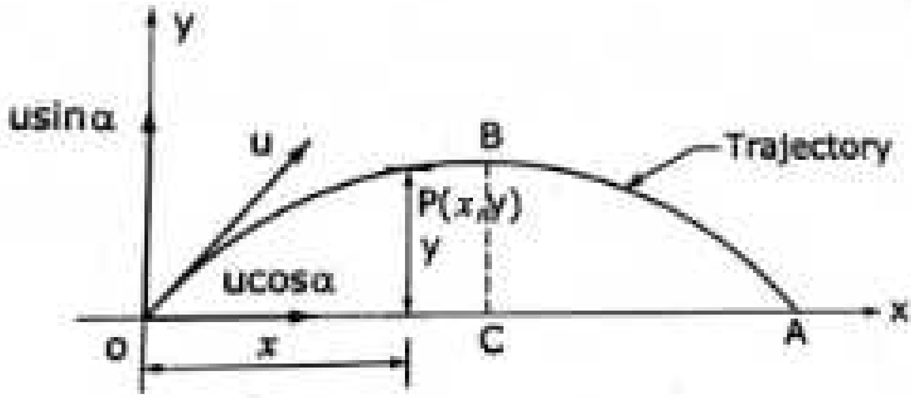
Component of velocity along OY axis $= u\sin\alpha$ [Vertical velocity]

Trajectory-*the path described by the projectile is called Trajectory*

Time of fight- It is the total time taken by projectile from the instant of projection up to the projectile hits the plane again.

Range :*it is the distance along the plane between the point of projection and the point at which the projectile hits the plane at the end of its journey.*

Path of a projectile



The horizontal distance travelled by projectile, in any time t,

x= velocity × time taken ⇒ $x = u\cos\alpha\, t$ $\quad \Rightarrow t = \dfrac{x}{u\cos\alpha}$

similarly, the vertical distance travelled by the projectile in any time t,

$y = u\sin\alpha t - \dfrac{1}{2}gt^2$

[the above equation is arrived from, $h = ut - \frac{1}{2}gt^2$,here , u= $u\sin\alpha$ and h=y]

Substitute $t = \dfrac{x}{u\cos\alpha}$

$$\therefore \quad y = u\sin\alpha\left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2}\times g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$= \frac{u\sin\alpha x}{u\cos\alpha} - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha} \qquad \therefore \quad y = \tan\alpha x - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}$$

This equation is of the form $y = Ax + Bx^2$,represents a parabola and hence path traversed by the projectile is a parabola.

Initial velocity(u),angle of projection (α)

Time of flight

OB-upward motion of the projectile and BA is the downward motion of the projectile

t-be the time taken by the projectile to reach the maximum height B

for the upward motion from O to B ⇒ $v = u - gt$

v=velocity at B=0

u=initial velocity in the upward direction= $= u\sin\alpha \Rightarrow 0 = u\sin\alpha - gt$

$$\therefore t = \frac{u\sin\alpha}{g}$$

[But, time of flight is the total time taken by projectile from O to A]
Time of flight= time to reach highest point+ time to hit the ground from highest point.
But , the path of the projectile is symmetrical about BC,
Hence, time up = time down

Total time taken (or) Time of flight (T)=2t= $2 \times \frac{u\sin\alpha}{g}$

$$T = \frac{2u\sin\alpha}{g}$$

(ii) maximum height attained
B-highest point [hmax]

Again, consider the vertical motion of the projectile. At the highest point, the velocity of the projectile is zero.

$$h = ut - \frac{1}{2}gt^2 \left[here\ h = h\max; u = u\sin\alpha, t = \frac{u\sin\alpha}{g} \right]$$

$$h\max = \left(u\sin\alpha \times \frac{u\sin\alpha}{g} \right) - \frac{1}{2} \times g \times \left(\frac{u\sin\alpha}{g} \right)^2$$

$$h\max = \frac{u^2\sin^2\alpha}{g} - \frac{1}{2} \times \frac{u^2\sin^2\alpha}{g} = \frac{1}{2} \times \frac{u^2\sin^2\alpha}{g}$$

$$\therefore h\max = \frac{u^2\sin^2\alpha}{2g}$$

(iii) Horizontal range

The distance OA is the horizontal range of the projectile.

Distance OA=component of velocity in the direction OA × total time taken by the projectile from O to A

$$= u\cos\alpha \times T \Rightarrow T = \frac{2u\sin\alpha}{g}$$

$$Range(R) = u\cos\alpha \times \frac{2u\sin\alpha}{g} (\because 2\sin\alpha\cos\alpha = \sin 2\alpha)$$

$$= \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$$Range, R = \frac{u^2\sin 2\alpha}{g}$$