In the last chapter, we have discussed the principles of friction of various types. Though these principles have a number of applications in Engineering-science, yet the following are important from the subject point of view:

1. Ladder friction
2. Wedge friction
3. Screw friction.

LADDER FRICTION

The ladder is a device for climbing or scaling on the roofs or walls. It consists of two long uprights of wood, iron or rope connected by a number of cross pieces called rungs. These runing serve as steps.

Consider a ladder AB resting on the rough ground and leaning against a wall, as shown in Fig.

As the upper end of the ladder tends to slip downwards, therefore the direction of the force of friction between the ladder and the wall (F_w) will be upwards as shown in the figure. Similarly, as the lower end of the ladder tends to slip away from the wall, therefore the direction of the force of friction between the ladder and the floor (F_f) will be towards the wall as shown in the figure.

Since the system is in equilibrium, therefore the algebraic sum of the horizontal and vertical components of the forces must also be equal to zero.

Note: The normal reaction at the floor (R_f) will act perpendicular of the floor. Similarly, normal reaction of the wall (R_w) will also act perpendicular to the wall.

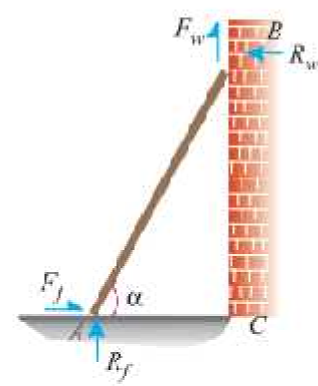


Fig. . Ladder friction

Example *A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3.*

What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium in this position.

Solution. Given: Length of the ladder (l) = 3.25 m; Weight of the ladder (w) = 250 N; Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μ_f) = 0.3.

Frictional force acting on the ladder.

The forces acting on the ladder are shown in Fig. 9.2.

let F_f = Frictional force acting on the ladder at the Point of contact between the ladder and floor, and

R_f = Normal reaction at the floor.

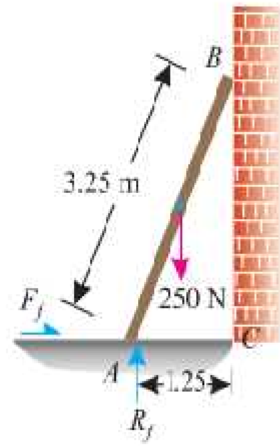


Fig.

Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall. Resolving the forces vertically,

$$R_f = 250 \text{ N}$$

From the geometry of the figure, we find that

$$BC = \sqrt{(3.25)^2 - (1.25)^2} = 3.0 \text{ m}$$

Taking moments about B and equating the same,

$$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625) = (250 \times 1.25) - 156.3 = 156.2 \text{ N}$$

$$\therefore \quad F_f = \frac{156.2}{3} = 52.1 \text{ N} \qquad \textbf{Ans.}$$

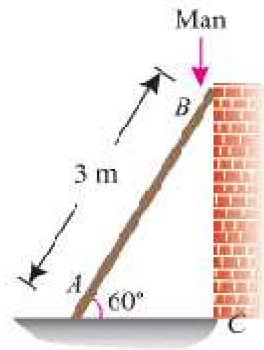
Equilibrium of the ladder

We know that the maximum force of friction available at the point of contact between the ladder and the floor

$$= \mu R_f = 0.3 \times 250 = 75 \text{ N}$$

Thus we see that the amount of the force of friction available at the point of contact (75 N) is more than the force of friction required for equilibrium (52.1 N). Therefore the ladder will remain in an equilibrium position. **Ans.**

Example A uniform ladder 3 m long weighs 200 N. It is placed against a wall making an angle of 60° with the floor as shown in Fig.



The coefficient of friction between the wall and the ladder is 0.25 and that between the floor and ladder is 0.35. The ladder, in addition to its own weight, has to support a man of 1000 N at its top at B. Calculate:

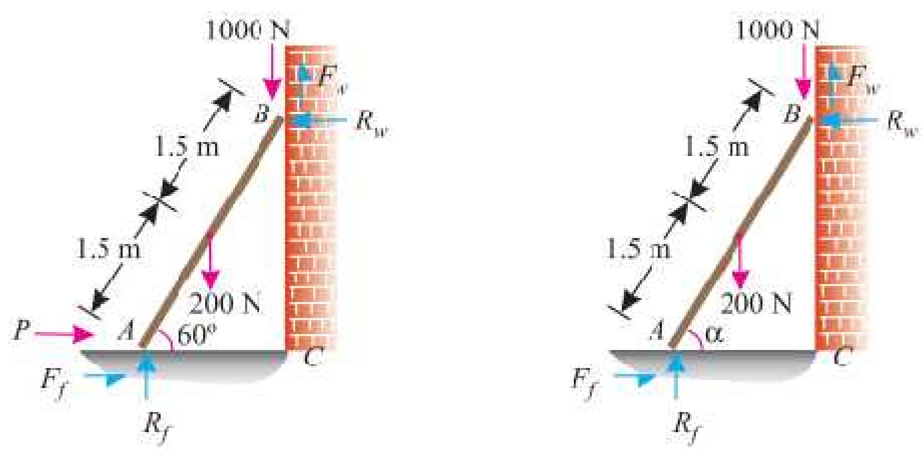
(i) *The horizontal force P to be applied to ladder at the floor level to prevent slipping.*

(ii) *If the force P is not applied, what should be the minimum inclination of the ladder with the horizontal, so that there is no slipping of it with the man at its top.*

Solution. Given: Length of the ladder (l) = 3 μ ; Weight of the ladder (W) = 200 N; Coefficient of friction between the wall and the ladder (μ_w) = 0.25 and coefficient of friction between the floor and ladder (μ_f) = 0.35.

The forces acting in both the cases are shown in Fig. 9.6 (*a*) and (*b*).

First of all, cosider the ladder inclined at an angle of 60° and subjected to a horizontal force (P) at the floor as shown in Fig. 9.6 (*a*).



(i) Horizontal force (P) applied to the ladder at floor level to prevent slipping

Resolving the forces horizontally,

$$P + F_f = R_w \quad \text{... (i)}$$

and now resolving the forces vertically,

$$R_f + F_w = 1000 + 200 = 1200 \text{ N} \quad \text{...(ii)}$$

Taking moments about A and equating the same.

$$(200 \times 1.5 \cos 60°) + 1000 \times 3 \cos 60°)$$
$$= (F_w \times 3 \cos 60°) + (R_w \times 3 \sin 60°)$$

Dividing both sides by the cos 60°,

$$300 + 3000 = (3 \times F_w) + (3 \times R_w \tan 60°)$$

$$\therefore \quad 1100 = F_w + R_w \tan 60° \quad \text{...(iii)}$$

We know that $F_w = \mu_w \times R_w = 0.25\, R_w$ ...($\because \mu_w = 0.25$)

Substituting this value of F_w in equation *(iii)*,

$$1100 = (0.25\, R_w) + (R_w \tan 60°) = R_w (0.25 + 1.732) = R_w \times 1.982$$

$$\therefore \quad R_w = \frac{1100}{1.982} = 555 \text{ N}$$

and $F_w = 0.25\, R_w = 0.25 \times 555 = 138.7$ N

Now substituting the value of F_w in equation *(ii)*,

$$R_f + 138.7 = 1200$$

$$\therefore \quad R_f = 1200 - 138.7 = 1061.3 \text{ N}$$

and $F_f = \mu_f R_f = 0.35 \times 1061.3 = 371.5$ N

Now substituting the value of F_f in equation *(i)*,

$$P + 371.5 = 555$$

$$\therefore \quad P = 555 - 371.5 = 183.5 \text{ N}$$ **Ans.**

(ii) *Inclination of the ladder with the horizontal for no slipping*

Now consider the ladder inclined at angle (α) and without any horizontal force acting at the floor as shown in Fig. 9.6 (*b*).

Resolving the forces horizontally,

$$R_w = F_f = \mu_f \times R_f = 0.35 \times R_f \quad ...(iv)$$

and now resolving the forces vertically,

$$R_f + F_w = 1000 + 200 = 1200 \text{ N}$$

We know that $F_w = \mu_w \times R_w = 0.25(0.35\, R_f) = 0.09\, R_f$ ...($\because R_w = 0.35\, R_f$)

or $R_f + 0.09\, R_f = 1200$

$$\therefore \quad R_f = \frac{1200}{1.09} = 1101 \text{ N}$$

and $R_w = 0.35 R_f = 0.35 \times 1101 = 385.4$ N

Similarly $F_w = 0.09\, R_f = 0.09 \times 1101 = 99.1$ N

Taking moments about *A* and equating the same,

$$(1000 \times 3 \cos \alpha) + (200 \times 1.5 \cos \alpha) = (F_w \times 3 \cos \alpha) + (R_w \times 3 \sin \alpha)$$

Dividing both sides by 3 cos α,

$$1000 + 100 = F_w + R_w \tan \alpha$$

$$1100 = 99.1 + 385.4 \tan \alpha$$

$$385.4 \tan \alpha = 1100 - 99.1 = 1000.9$$

$$\therefore \quad \tan \alpha = \frac{1000.9}{385.4} = 2.5970 \quad \text{or} \quad \alpha = 68.9° \quad \textbf{Ans.}$$