WEDGE FRICTION

A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used for slight adjustements in the position of a body *i.e.* for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weights as shown in Fig. 9.10.

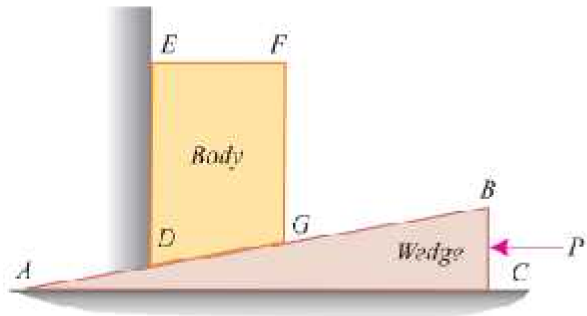


Fig.

It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes. Thus these problems may be solved either by the equilibrium method or by applying Lami's theorem. Now consider a wedge *ABC*, which is used to lift the body *DEFG*.

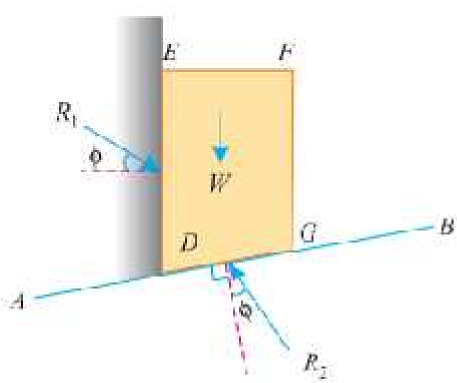
Let W = Weight fo the body *DEFG*,

P = Force required to lift the body, and

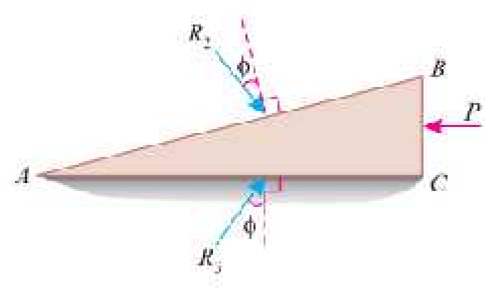
μ = Coefficient of friction on the planes *AB*, *AC* and *DE* such that

$\tan \phi = \mu$.

A little consideration will show that when the force is sufficient to lift the body, the sliding will take place along three planes *AB*, *AC* and *DE* will also occur as shown in Fig. 9.11 (*a*) and (*b*).



(*a*) Forces on the body *DEFG*



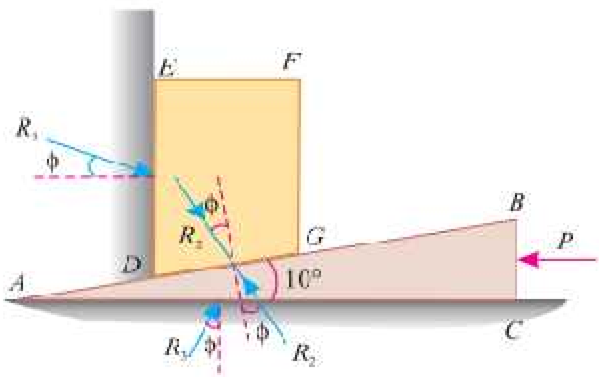
(*a*) Forces on the wedge *ABC*

Graphical method

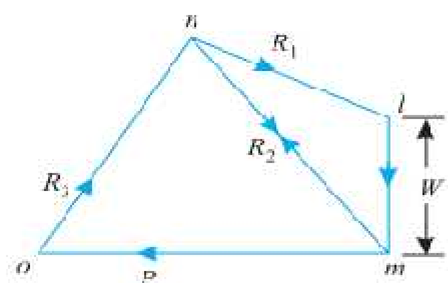
1. First of all, draw the space diagram for the body *DEFG* and the wedge *ABC* as shown in Fig. 9.12 (*a*). Now draw the reactions R_1, R_2 and R_3 at angle f with normal to the faces *DE*, *AB* and *AC* respectively (such that $\tan \phi = \mu$).

2. Now consider the equilibrium of the body *DEFG*. We know that the body is in equilibrium under the action of
   (*a*) Its own weight (*W*) acting downwards
   (*b*) Reaction R_1 on the face *DE*, and
   (*c*) Reaction R_2 on the face *AB*.

   Now, in order to draw the vector diagram for the above mentioned three forces, take some suitable point *l* and draw a vertical line *lm* parallel to the line of action of the weight (*W*) and cut off *lm* equal to the weight of the body to some suitable scale. Through *l* draw a line parallel to the reaction R_1. Similarly, through *m* draw a line parallel to the reaction R_2, meeting the first line at *n* as shown in Fig. 9.12 (*b*).



(*a*) Space diagram



(*b*) Vector diagram

3. Now consider the equilibrium of the wedge *ABC*. We know that it is equilibrium under the action of
   (*a*) Force acting on the wedge (*P*),
   (*b*) Reaction R_2 on the face *AB*, and
   (*c*) Reaction R_3 on the face *AC*.

   Now, in order to draw the vector diagram for the above mentioned three forces, through *m* draw a horizontal line parallel to the force (*P*) acting on the wedge. Similarly, through *n* draw a line parallel to the reaction R_3 meeting the first line at *O* as shown in Fig. 9.12 (*b*).
4. Now the force (*P*) required on the wedge to raise the load will be given by *mo* to the scale.

Analytical method

1. First of all, consider the equilibrium of the body *DEFG*. And resolve the forces W, R_1 and R_2 horizontally as well as vertically.
2. Now consider the equilibrium of the wedge *ABC*. And resolve the forces *P*, R_2 and R_3 horizontally as well as vertically.

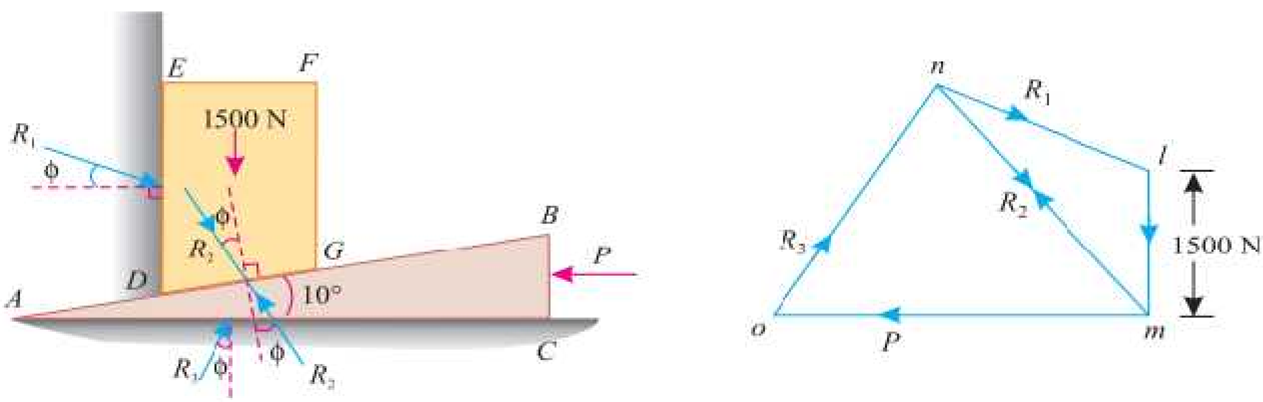
Example *A block weighing 1500 N, overlying a 10° wedge on a horizontal floor and leaning against a vertical wall, is to be raised by applying a horizontal force to the wedge.*

Assuming the coefficient of friction between all the surface in contact to be 0.3, determine the minimum horizontal force required to raise the block.

Solution. Given: Weight of the block (W) = 1500 N; Angle of the wedge (α) = 10° and coefficient of friction between all the four surfaces of contact (μ) = 0.3 = $\tan \phi$ or $\phi = 16.7°$.

Let P = Minimum horizontal force required to raise the block.

The example may be solved graphically or analytically. But we shall solve it by both the methods.



(*a*) Space diagram (*b*) Vector diagram

Fig. 9.13.

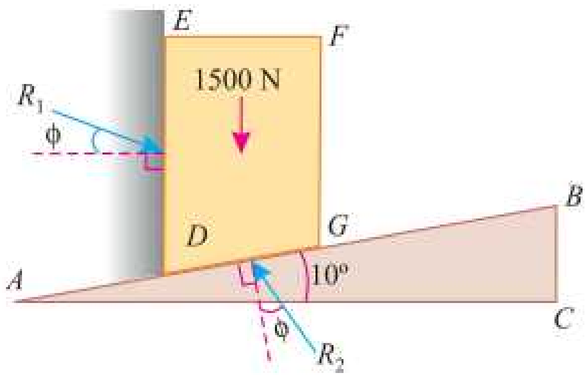
Graphical method

1. First of all, draw the space diagram for the block *DEFG* and the wedge *ABC* as shown in Fig. 9.13 (*a*). Now draw reactions R_1, R_2 and R_3 at angles of ϕ (*i.e.* 16.7° with normal to the faces *DE*, *AB* and *AC* respectively.

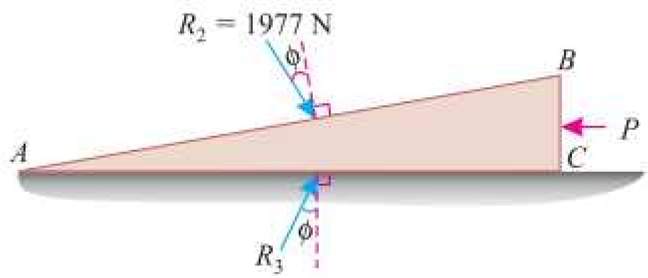
2. Take some suitable point *l*, and draw vertical line *lm* equal to 1500 N to some suitable scale (representing the weight of the block). Through *l*, draw a line parallel to the reaction R_1. Similarly, through *m* draw another line parallel to the reaction R_2 meeting the first line at *n*.
3. Now through *m*, draw a horizontal line (representing the horizontal force *P*). Similarly,through *n* draw a line parallel to the reaction R_3 meeting the first line at *O* as shown in Fig. 9.13(*b*).
4. Now measuring *mo* to the scale, we find that the required horizontal force

$$P = 1420 \text{ N.} \quad \textbf{Ans.}$$

Analytical method



(*a*) Block *DEFG*



(*b*) Wedge *ABC*

First of all, consider the equilibrium ot the block. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.14 (*a*).

1. Its own weight 1500 N acting downwards.
2. Reaction R_1 on the face *DE*.
3. Reaction R_2 on the face *DG* of the block.

Resolving the forces horizontally,

$$R_1 \cos(16.7°) = R_2 \sin(10 + 16.7°) = R_2 \sin 26.7°$$

$$R_1 \times 0.9578 = R_2 \times 0.4493$$

or $$R_2 = 2.132\, R_1$$

and now resolving the forces vertically,

$$R_1 \times \sin(16.7°) + 1500 = R_2 \cos(10° + 16.7°) = R_2 \cos 26.7°$$

$$R_1 \times 0.2874 + 1500 = R_2 \times 0.8934 = (2.132\, R_1)0.8934$$

$$= 1.905\, R_1 \qquad ...(R_2 = 2.132\, R_1)$$

$$R_1(1.905 - 0.2874) = 1500$$

$$\therefore \quad R_1 = \frac{1500}{1.6176} = 927.3 \text{ N}$$

and $$R_2 = 2.132\, R_1 = 2.132 \times 927.3 = 1977 \text{ N}$$

Now consider the equilibrium of the wedge. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.14 (*b*).

1. Reaction R_2 of the block on the wedge.
2. Force (P) acting horizontally, and
3. Reaction R_3 on the face AC of the wedge.

Resolving the forces vertically,

$$R_3 \cos 16.7° = R_2 \cos (10° + 16.7°) = R_2 \cos 26.7°$$

$$R_3 \times 0.9578 = R_2 \times 0.8934 = 1977 \times 0.8934 = 1766.2$$

$$\therefore \quad R_3 = \frac{1766.2}{0.9578} = 1844 \text{ N}$$

and now resolving the forces horizontally,

$$P = R_2 \sin (10° + 16.7°) + R_3 \sin 16.7° = 1977 \sin 26.7° + 1844 \sin 16.7° \text{ N}$$

$$= (1977 \times 0.4493) + (1844 \times 0.2874) = 1418.3 \text{ N} \quad \textbf{Ans.}$$