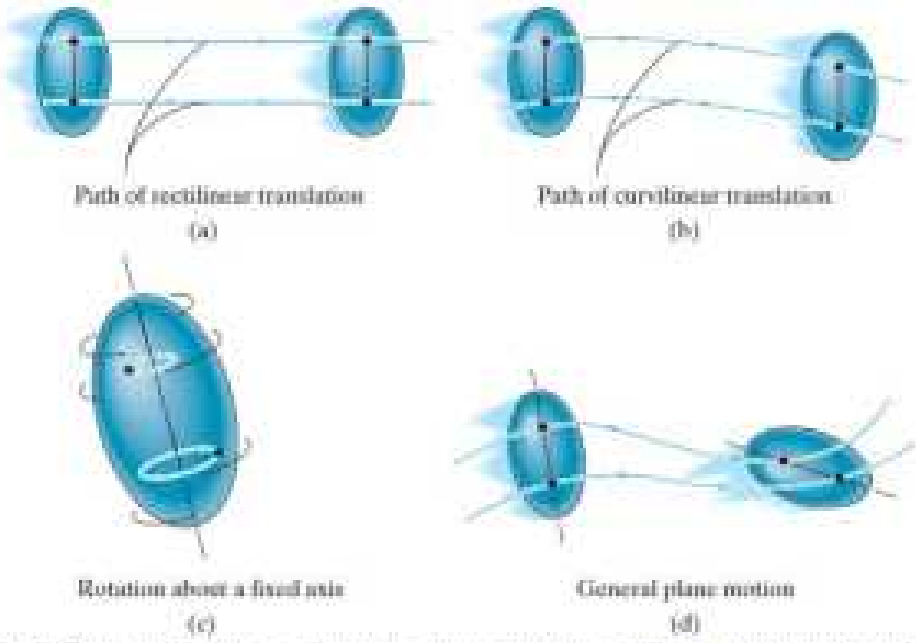
Planar Rigid-Body Motion

This study is important for the design of gears, cams, and mechanisms used for many mechanical operations. Once the kinematics is thoroughly understood, then we can apply the equations of motion, which relate the forces on the body to the body's motion. The *planar motion* of a body occurs when all the particles of a rigid body move along paths which are equidistant from a fixed plane.

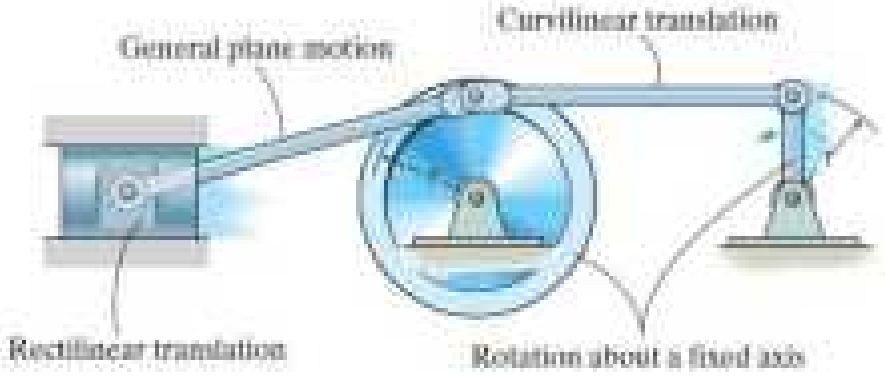


Translation. This type of motion occurs when a line in the body remains parallel to its original orientation throughout the motion. When the paths of motion for any two points on the body are parallel lines, the motion is called *rectilinear translation*, Fig. *a*. If the paths of motion are along curved lines, the motion is called *curvilinear translation*, Fig.*b*.

Rotation about a fixed axis. When a rigid body rotates about a fixed axis, all the particles of the body, except those which lie on the axis of rotation, move along circular paths, Fig.*c*.

General plane motion. When a body is subjected to general plane motion, it undergoes a combination of translation *and* rotation, Fig.*d*. The translation occurs within a reference plane, and the rotation occurs about an axis perpendicular to the reference plane.

In the following sections we will consider each of these motions in detail. Examples of bodies undergoing these motions are shown in Fig



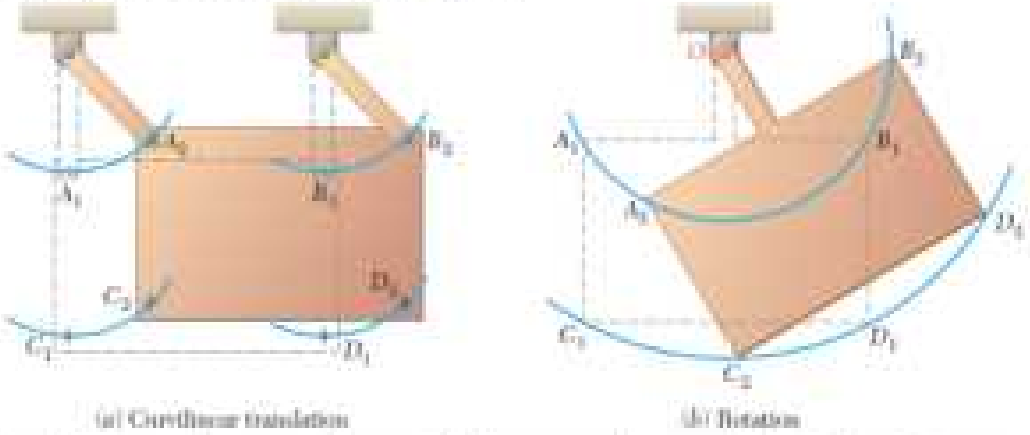
Objectives
- Describe the five basic types of rigid body motion: translation, rotation about a fixed axis, general plane motion, motion about a fixed point, and general motion.
- Use angular kinematic relationships involving θ, ω, and α to determine the angular motion of a rigid body.
- Identify the directions of terms in the relative velocity and relative acceleration equations.
- Calculate the linear velocity and acceleration of any point on a rigid body undergoing translation, fixed axis rotation, or general plane motion.

we consider the kinematics of **rigid bodies**. We will investigate the relations between the time, the positions, the velocities, and the accelerations of the various particles forming a rigid body.

1. Translation. A motion is said to be a translation if any straight line inside the body maintains the same orientation during the motion. In a translation, all of the particles forming the body move along parallel paths. If these paths are straight lines, the motion is called **rectilinear translation** (Fig.); if the paths are curved lines, the motion is called **curvilinear translation** (Fig.).

2. Rotation About a Fixed Axis. In this motion, the particles forming the rigid body move in parallel planes along circles centered on the same fixed axis (Fig.). If this axis, called the **axis of rotation**, intersects the rigid body, the particles located on the axis have zero velocity and zero acceleration.

Be careful not to confuse rotation with certain types of curvilinear translation. For example, the plate shown in Fig. *a* is in curvilinear translation, with all of its particles moving along *parallel* circles, whereas the plate shown in Fig. *b* is in rotation, with all of its particles moving along *concentric* circles. In the first case, any given straight line drawn on the plate maintains the same direction, whereas in the second case, the orientation of the plate changes throughout the rotation. Because each particle moves in a given plane, the rotation of a body about a fixed axis is said to be a **plane motion.**



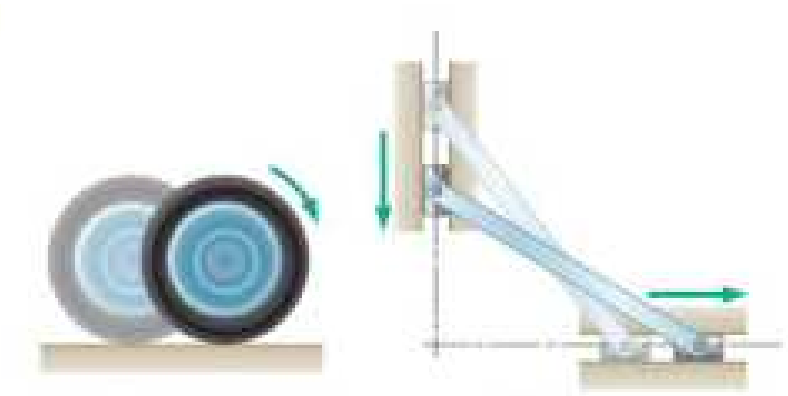
(a) Curvilinear translation (b) Rotation

Fig. *(a) In curvilinear motion, particles move along parallel circles, whereas (b) in fixed-axis rotation, particles move along concentric circles.*

3. General Plane Motion. Many other types of plane motion can occur, i.e., motions in which all the particles of the body move in a single plane. Any plane motion that is neither a rotation nor a translation is referred to as general plane motion. Figure shows two examples of general plane motion.

4. Motion About a Fixed Point. The three-dimensional motion of a rigid body attached at a fixed point O, such as the motion of a top on a rough floor (Fig.), is known as motion about a fixed point.

5. General Motion. Any motion of a rigid body that does not fall in any of these categories is referred to as a general motion.



(a) Rolling wheel (b) Sliding rod

Fig. a) A rolling wheel and (b) a sliding rod are common examples of general plane motion.

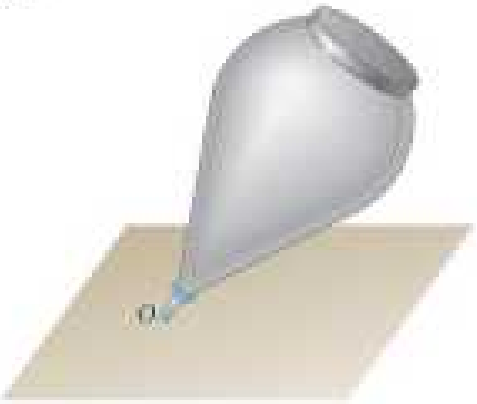


Fig. The motion of a spinning top on a rough surface is an example of three-dimensional motion about a fixed point.

After a brief discussion of the motion of translation, we consider the rotation of a rigid body about a fixed axis. We define the *angular velocity* and the *angular acceleration* of a rigid body rotating about a fixed axis, and you will see how to express the velocity and acceleration of a given point of the body in terms of its position vector and the angular velocity and angular acceleration of the body.