The angular velocity of the disk is defined by $\omega = (5t^2 + 2)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = 0.5$ s.

SOLUTION

$$\omega = (5\,t^2 + 2)\ \text{rad/s}$$

$$\alpha = \frac{d\omega}{dt} = 10\,t$$

$$t = 0.5\ \text{s}$$

$$\omega = 3.25\ \text{rad/s}$$

$$\alpha = 5\ \text{rad/s}^2$$

$$v_A = \omega r = 3.25(0.8) = 2.60\ \text{m/s}$$

$$a_t = \alpha r = 5(0.8) = 4\ \text{m/s}^2$$

$$a_n = \omega^2 r = (3.25)^2(0.8) = 8.45\ \text{m/s}^2$$

$$a_A = \sqrt{(4)^2 + (8.45)^2} = 9.35\ \text{m/s}^2$$

The angular acceleration of the disk is defined by $\alpha = 3t^2 + 12$ rad/s, where t is in seconds. If the disk is originally rotating at $\omega_0 = 12$ rad/s, determine the magnitude of the velocity and the n and t components of acceleration of point A on the disk when $t = 2$ s.

Solution

Angular Motion. The angular velocity of the disk can be determined by integrating $d\omega = \alpha\, dt$ with the initial condition $\omega = 12$ rad/s at $t = 0$.

$$\int_{12\text{ rad/s}}^{\omega} d\omega = \int_0^{2s} (3t^2 + 12)\,dt$$

$$\omega - 12 = (t^3 + 12t)\Big|_0^{2s}$$

$$\omega = 44.0 \text{ rad/s}$$

Motion of Point A. The magnitude of the velocity is

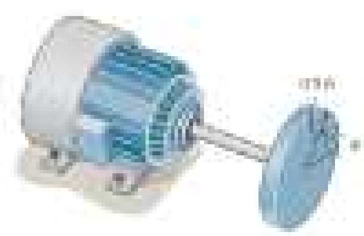
$$v_A = \omega r_A = 44.0(0.5) = 22.0 \text{ m/s}$$

At $t = 2$ s, $\alpha = 3(2^2) + 12 = 24$ rad/s^2. Thus, the tangential and normal components of the acceleration are

$$(a_A)_t = \alpha r_A = 24(0.5) = 12.0 \text{ m/s}^2$$

$$(a_A)_n = \omega^2 r_A = (44.0^2)(0.5) = 968 \text{ m/s}^2$$

The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.



SOLUTION

Angular Displacement: At $t = 90$ s,

$$\theta = 20(90) + 4(90^2) = (34200 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 5443 \text{ rev}$$

Angular Velocity: Applying Eq. 16-1, we have

$$\omega = \frac{d\theta}{dt} = 20 + 8t\Big|_{t=90\text{ s}} = 740 \text{ rad/s}$$

Angular Acceleration: Applying Eq. 16-2, we have

$$\alpha = \frac{d\omega}{dt} = 8 \text{ rad/s}^2$$