UNIT-3

Free Electron theory of metals

Classical free electron theory is based on the following postulates:

1. A solid metal is composed of atoms and the atoms have nucleus, around which there are revolving electrons.

2. In a metal the valance electrons of atoms are free to move throughout the volume of the metal like gas molecules of a perfect gas in a container

3. The free electrons move in a random directions and collide with either positive ions fixed to the lattice or other free electrons and collisions are elastic in nature i.e. there is no loss of energy.

4. The movement of free electrons obeys the classical kinetic theory of gasses. The mean K.E. of a free electron is equal to that of gas molecule $\left(\frac{3}{2}KT\right)$.

5. The electron velocities in a metal obey Maxwell-Boltzman distribution of velocities.

6. The free electrons move in a uniform potential field due to ions fixed in the lattice

7. When an electric field is applied to the metal the free electrons are accelerated. The accelerated electrons move in opposite direction of the applied.

8. The electric conduction is due to the free electrons only.

ROOT MEAN SQUARE (R.M.S.) VELOCITY:

Let $\overline{C}$ be the r.m.s velocity of the free electron. then the

$$Kinetic\ energy = \frac{1}{2}m\overline{C}^2$$

But according to the classical free electron theory the mean

$$\text{Kinetic Energy} = \left(\frac{3}{2}KT\right).$$

$$\therefore \frac{1}{2}m\overline{c}^2 = \frac{3}{2}KT$$

$$\Rightarrow \boxed{\overline{c} = \sqrt{\frac{3KT}{m}}}$$ where $\overline{c}$ = root mean square velocity

MEAN FREE PATH (λ) AND MEAN COLLISION TIME (τ_C)

The average distance travelled by an electron between two successive collisions in the presenceof applied filed is known as 'Mean free path (λ)'.

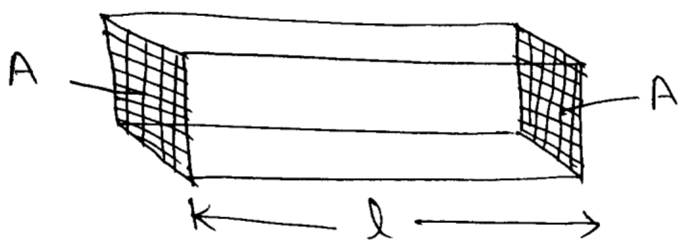
The time taken by an electron between two successive collisions is known as "Mean CollisionTime(τ_C)" of the electron

$$\boxed{\tau_c = \frac{\lambda}{\bar{c}} = \lambda\sqrt{\frac{m}{3KT}}}$$

DRIFT VELOCITY (v_d):

It is the average velocity acquired by the free electrons of a metal in a particular directionduring the application of the electric field.

ELECTRICAL CONDUCTVITY IN METALS:



Let us consider a conductor of length l and area of cross section A

The volume of the conductor $= Al$

If there are n number of electrons per unit volume of the

metalthen the total number of electrons in the metal =

Aln

If e is the charge of the electron then the total charge q due to all electrons in the conductor isgiven by $q = Aln.e$

Let t be the time taken by the electron to move from one end to other end then

Current $(I) = \frac{charge}{\_\_\_} = \frac{q}{\_} = \frac{Aln\,e}{\_\_\_}$

But $\frac{l}{t} = v_d$

$$\therefore I = Anev_d$$

$$\Rightarrow v_d = \frac{I}{Ane} = \frac{J}{ne}$$

Where J = current density $= \frac{I}{A}$

In a metal the current density J is given bythe equation

$$J = nev_d \text{-----------------}(1)$$

Where n = number of electrons per Unit volume, e = electron charge and v_d = drift velocityIf E is the applied electric field then the electric force acting on a free electron is given by

$$F = eE \text{.............................}(2)$$

From Newton's IInd law $F = ma$ ---------------------------------- (3)

From (2) and (3) $ma = eE$

i.e. $a = \frac{eE}{m}$

but a =drift velocity/collision time $= \frac{v_d}{\tau_c}$

$$v_d = a\tau_c = \frac{eE}{m}\tau_c$$

$$\therefore J = ne.\frac{eE}{m}\tau_c = \frac{ne^2E}{m}\tau_c \text{..}(4)$$

But from microscopic form of ohms law

$$J = \sigma E \text{ ....(5)}$$

On comparing Eq(4)&(5)

$$\therefore \text{ Conductivity } \sigma = \frac{ne^2}{m}\tau_c \text{ or Resistivity}.\rho = \frac{m}{ne^2\tau_c}$$

Conductivity may also be expressed in terms of mobility (μ) which is defined as drift velocityper unit electric field

$$\mu = \frac{v_d}{E} = \frac{e}{m}\tau_c$$

From (4) $\boxed{\sigma = ne\mu}$

RELAXATION TIME(τ_r)

Under the influence of an external electric field free electrons attain a directional velocityof motion. If the field is switched off the velocity starts decreasing exponentially. Such a process that tends to restore equilibrium is called relaxation process.

If v_o is the velocity at $t = 0$ at which the field is switched

off.The velocity at any time is given by

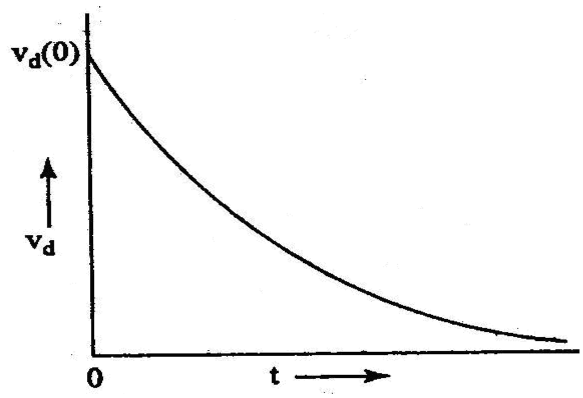
$$v = v_o e^{\frac{-t}{\tau_r}}$$

In the above expression τ_r = relaxation

timeIf $t = \tau_r$

$$v = v_o e^{\frac{-t}{\tau_r}} = v_o e^{-1} = \frac{v_o}{e}$$

$\therefore$ Relaxation time τ_r is defined as the time required for the electron to reduce its velocity to $\frac{1}{e}$ of its initial value. (OR) time taken for the drift velocity to decay $\frac{1}{e}$ of its initial value.



$v_d(0)$
v_d
0
t