Probability :

Probability is a concept which we use to deal with uncertainty.

* we use probability in daily life to make decisions when you don't know for sure what the outcome will be.

For example,

1. Most probably it will rain today
2. I doubt that he will win the race.
3. Chances are high that the prices of Petrol will go up.

Applications :

* Modelling of text and web data
* Speech recognition
* Robotics
* Network traffic and system Reliability modeling
* Probabilistic analysis of algorithms and graphs
* Machine learning and data mining
* Cryptography

Terms related with Probability:

Experiment:

An experiment which, though repeated under essentially identical conditions does not give unique results but may result in any one of the several possible outcomes

Outcome:

A result of an experiment is called an outcome.

Sample Space:

A Sample Space is the collection of all possible outcomes.

Trial:

Performing an experiment is known as trial

Event:

The outcomes of the experiment are known as Event.

Types of Events:

* Mutually Exclusive Events:

If the occurence of one event excludes the occurence of another event, such events are mutually exclusive events

* Exhaustive Events:

A set of events is called exhaustive if all the events together consume the entire sample space.

* Independent event:

If the occurence of one event has no influence over the occurence of the other event.

Probability of an Event :

$$P(A) = \frac{\text{Favourable number of cases}}{[\text{Exhaustive number of cases. (or) Total number of cases}]}$$

1]. Find the probability of getting
i). 4
ii). an odd number on a die.

Soln. :

$S = \{1, 2, 3, 4, 5, 6\}$

$P(A) = \frac{1}{6}$; $P(B) = \frac{3}{6} = \frac{1}{2}$.

2] If a coin is tossed, then what is the probability of getting head ?

$S = \{H, T\}$

$P(\text{Head}) = \frac{1}{2}$

3]. If you flip a balanced coin twice, what is the probability of getting atleast one head ?

$S = \{HH, HT, TH, TT\}$

2^n

$2^2 = 4$

$P(\text{atleast one head}) = \frac{3}{4}$

Axioms of probability:

1. For any event A, $P(A) \geq 0$
2. Probability of the sample space S is $P(S) = 1$
3. If $A_1, A_2, \ldots$ are disjoint events, then

$$P[A_1 \cup A_2 \cup \ldots] = P[A_1] + P[A_2] + \ldots \quad [\because A_1 \cap A_2 = \phi]$$

Results:

i). $P[\phi] = 0$ impossible event

ii). $P[\bar{A}] = 1 - P[A]$

iii) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

iv). $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

v). $P[A \cup B] = P(A) + P(B)$ A & B are mutually exclusive events

vi). $P(A \cap B) = P(A) \cdot P(B)$ A & B are independent

vii). $P[A \cup B] = P(A) + P(B) - P(A \cap B)$

viii). $P[A^c \cup B^c] = P(A \cap B)^c$; $P(A^c \cap B^c) = P(A \cup B)^c$

ix). $P(A + B) = P(A \cup B)$; $P(AB) = P(A \cap B)$

problems.

1. If $P(A) = 0.4$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$. Find $P(\bar{A} \cap \bar{B})$ and $P(\bar{A} \cup \bar{B})$

Soln.:

$$P(\bar{A}) = 1 - P(A) = 1 - 0.4 = 0.6$$

$$P(\bar{B}) = 1 - P(B) = 1 - 0.7 = 0.3$$

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$= 1 - [0.4 + 0.7 - 0.3] = 1 - 0.8 = 0.2$

$P(\bar{A} \cap \bar{B}) = 0.2$

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

$= 1 - 0.3$

$P(\bar{A} \cup \bar{B}) = 0.7$

2. If A and B are even with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$. Find $P(A^c \cap B^c)$

Soln:

$P(A^c \cap B^c) = P(A \cup B)^c$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left[\frac{3}{8} + \frac{1}{2} - \frac{1}{4}\right]$

$= 1 - \left(\frac{3+4-2}{8}\right) = 1 - \frac{5}{8} = \frac{8-5}{8}$

$= \frac{3}{8}$

3. Event A & B are $P(A+B) = \frac{3}{4}$, $P(AB) = \frac{1}{4}$. $P(\bar{A}) = \frac{2}{3}$. Find $P(B)$

Soln.

Given. $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$

$P(\bar{A}) = \frac{2}{3}$ | $P(\bar{A}) = 1 - P(A) \Rightarrow P(A) = 1 - P(\bar{A})$

$P(A) = 1 - \frac{2}{3} = \frac{1}{3}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B) = P(A \cup B) - P(A) + P(A \cap B)$$

$$= \frac{3}{4} - \frac{1}{3} + \frac{1}{4} = \frac{9-4+3}{12}$$

$$P(B) = \frac{8}{12} = \frac{2}{3}$$

4]. A bag containing 6 red, 4 black and 7 blue, 10 white. Five balls are drawn at random. what is the probability that two of them are red and one is black, two is blue.

Soln.

6R 4BLA 7B 10W = 27

$$\text{Prob} = \frac{{}^{6}C_2 \times {}^{4}C_1 \times {}^{7}C_2}{{}^{27}C_5}$$

$$= \frac{6 \times 5 \times 4 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1 \times 27 \times 26 \times 25 \times 24 \times 23} \quad (nC_1 = n)$$

$$= \frac{14}{897} = 0.015$$

5]. A bag containing 5 white balls, 6 green balls. Three balls are drawn with replacement. what is the chance that

i). All are same color

ii). They are alternatively different color.

i). All are same color

Prob = Either All are white or All are Green

$$= \frac{5C_3}{11C_3} + \frac{6C_3}{11C_3}$$

$$= \frac{\cancel{5}\times\cancel{4}^{2}\times\cancel{3}\times\cancel{3}\times\cancel{2}\times 1}{\cancel{3}\times\cancel{2}\times 1\times 11\times\cancel{10}^{\cancel{2}}\times\cancel{9}^{3}} + \frac{\cancel{6}\times\cancel{5}\times 4\times\cancel{3}\times\cancel{2}\times 1}{\cancel{3}\times\cancel{2}\times 1\times 11\times\cancel{10}^{\cancel{2}}\times\cancel{9}^{3}}$$

$$= \frac{2}{33} + \frac{4}{33} = \frac{6}{33} = \frac{2}{11} = 0.18$$

ii). All are alternative different color

$$P(A) = P(WGW) + P(GWG)$$

$$= \frac{5C_2\ 6C_1}{11C_3} + \frac{6C_2\ 5C_1}{11C_3}$$

$$= \frac{\cancel{5}\times 4\times\cancel{6}^{\cancel{3}}\times\cancel{3}\times\cancel{2}\times 1}{\cancel{2}\times 1\times 11\times\cancel{10}^{\cancel{2}}\times\cancel{9}} + \frac{\cancel{6}\times\cancel{5}\times 5\times\cancel{3}\times\cancel{2}\times 1}{\cancel{2}\times 1\times 11\times\cancel{10}^{\cancel{2}}\times\cancel{9}}$$

$$= \frac{4}{11} + \frac{5}{11} = \frac{9}{11} = 0.8$$

Without Replacement:

i). $$P(A) = \frac{5C_1}{11C_1}\times\frac{4C_1}{10C_1}\times\frac{3C_1}{9C_1} + \frac{6C_1}{11C_1}\times\frac{5C_1}{10C_1}\times\frac{4C_1}{9C_1}$$

$$= \frac{\cancel{5}\times\cancel{4}^{\cancel{2}}\times\cancel{3}}{11\times\cancel{10}^{\cancel{2}}\times\cancel{9}_{3}} + \frac{\cancel{6}^{\cancel{2}}\times\cancel{5}\times\cancel{4}^{2}}{11\times\cancel{10}^{\cancel{2}}\times\cancel{9}_{3}}$$

$$= \frac{2}{33} + \frac{4}{33} = \frac{6}{33} = \frac{2}{11} = 0.18$$

ii). All are alternative different color

$$= P(\omega G_1 \omega) + P(G_1 \omega G_1)$$

$$= \frac{{}^5C_1}{{}^{11}C_1} \cdot \frac{{}^6C_1}{{}^{10}C_1} \cdot \frac{{}^4C_1}{{}^9C_1} + \frac{{}^6C_1}{{}^{11}C_1} \cdot \frac{{}^5C_1}{{}^{10}C_1} \cdot \frac{{}^5C_1}{{}^9C_1}$$

$$= \frac{5 \times 6 \times 4}{11 \times 10 \times 9} + \frac{6 \times 5 \times 5}{11 \times 10 \times 9}$$

$$= \frac{4}{33} + \frac{5}{33} = \frac{9}{33} = \frac{3}{11} = 0.27$$