SNS COLLEGE OF TECHNOLOGY

Coimbatore-26
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT V – STATE VARIABLE ANALYSIS

TOPIC 5: CONCEPTS OF CONTROLLABILITY AND OBSERVABILITY.

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- INTRODUCTION
- BASIC CONCEPTS OF CONTROLLABILITY
- EXAMPLE 1.
- ACTIVITY
- BASIC CONCEPTS OF OBSERVABILITY.
- EXAMPLE 2
- SUMMARY

BASIC CONCEPTS OF CONTROLLABILITY &OBSERVABILITY

A linear system is said to be completely controllable if, for all initial times and all initial states , there exists some input function (or sequence for discrete systems) that drives the state vector to any final state at some finite time.

A linear system is said to be completely observable if, for all initial times, the state vector can be determined from the output function (or sequence), defined over a finite time.

BASIC CONCEPTS OF CONTROLLABILITY

Controllability

A control system is said to be **controllable** if the initial states of the control system are transferred (changed) to some other desired states by a controlled input in finite duration of time.

We can check the controllability of a control system by using **Kalman's test**.

- Write the matrix Qc in the following form.

$$Q_c=[B \;\; AB \;\; A^2B \ldots A^{n-1}B]$$

- Find the determinant of matrix Qc and if it is not equal to zero, then the control system is controllable

BASIC CONCEPTS OF OBSERVABILITY

Observability

A control system is said to be **observable** if it is able to determine the initial states of the control system by observing the outputs in finite duration of time.

We can check the observability of a control system by using **Kalman's test**.

- Write the matrix Qo in following form.

$$Qo=[C^T \quad A^T C^T \quad (A^T)^2 C^T \ldots (A^T)^{n-1} C^T]$$

- Find the determinant of matrix Qo and if it is not equal to zero, then the control system is observable.

EX:CONTROLLABILITY & OBSERVABILITY

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u]$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \end{bmatrix}, D = [0] \quad and \quad n = 2$$

For $n = 2$, the matrix Q_c will be

$$Q_c = \begin{bmatrix} B & AB \end{bmatrix}$$

We will get the product of matrices A and B as,

$$AB = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow Q_c = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

EX:CONTROLLABILITY & OBSERVABILITY

$$|Q_c| = 1 \neq 0$$

Since the determinant of matrix Q_c is not equal to zero, the given control system is controllable

For $n = 2$, the matrix Q_o will be -

$$Q_o = [C^T \quad A^T C^T]$$

Here,

$$A^T = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \quad and \quad C^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

We will get the product of matrices A^T and C^T as

$$A^T C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow Q_o = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

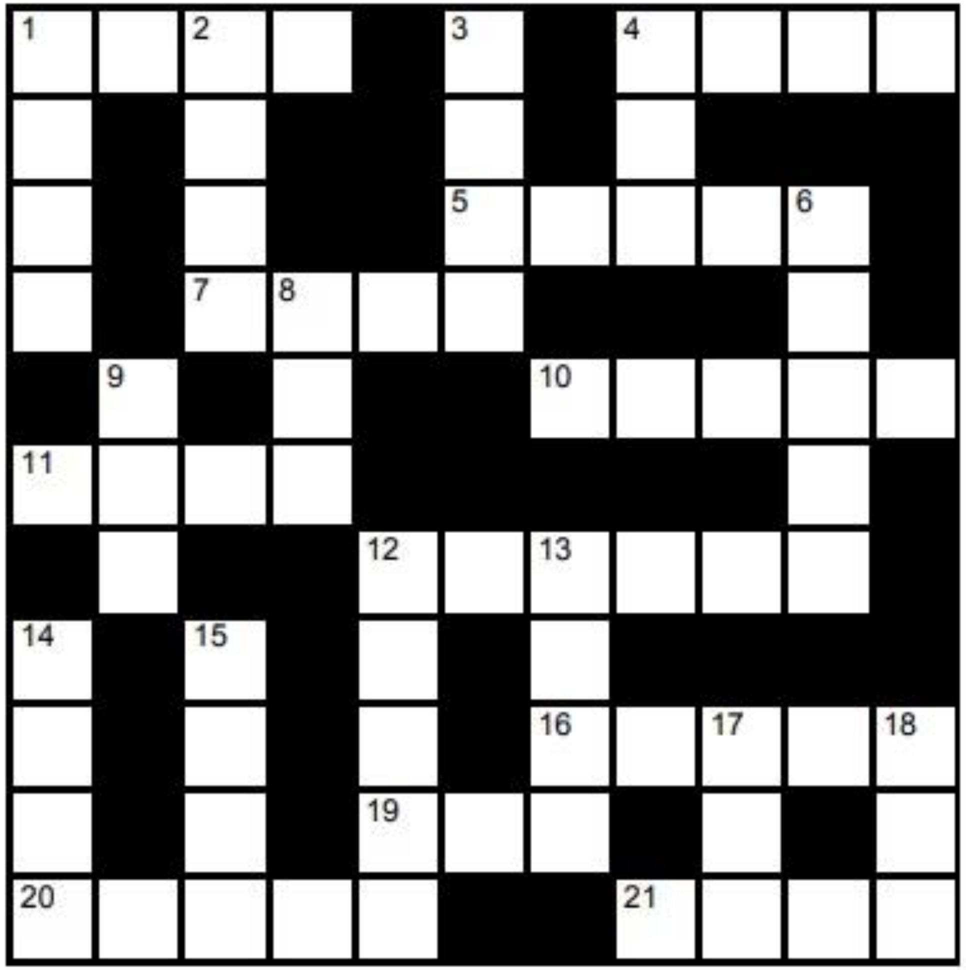
EX:CONTROLLABILITY & OBSERVABILITY

$$\Rightarrow |Qo| = -1 \neq 0$$

Since, the determinant of matrix Qo is not equal to zero, the given control system is observable.

Therefore, the given control system is both controllable and observable.

ACTIVITY-CROSS WORD



Across

1 The front part of your head (4)
4 Your body is covered in this (4)
5 The upper front part of your body (5)
7 It joins the head to the rest of the body (4)
10 The middle part of the body (5)
11 The part in the middle of your leg, where it bends (4)
12 You use this for tasting and speaking (6)
16 One of the five senses (5)
19 You have a big one and a little one on each foot (3)
20 An adult usually has 32 of these (5)
21 Excuse me for not speaking clearly, I've got a ____ in my throat (4)

Down

1 The part of your body at the end of your leg (4)
2 The bottom part of your face, below your mouth (4)
3 You can only see this part of your body in a mirror (4)
4 A blind person cannot ____ (3)
6 One of the five senses (5)
8 Body part used for seeing (3)
9 Fingers are the long thin parts on the ____ of your hand (3)
12 Hard white object inside your mouth used for biting and chewing (5)
13 Body part used for smelling and breathing (4)
14 Plural form of 1 Down (4)
15 One part of the skeleton (4)
17 Body part used for hearing (3)
18 Body part used for standing, walking, running, etc. (3)

ACTIVITY-ANSWERS

ear (3)
end (3)
eye (3)
leg (3)
see (3)
toe (3)
back (4)
bone (4)
chin (4)
face (4)
feet (4)
foot (4)

ACROSS

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CONTROLLABILITY MATRIX

Controllability Matrix

- Consider a single-input system (u ∈ R):

$$\dot{x} = Ax + Bu, \qquad y = Cx \qquad x \in \mathbb{R}^n$$

- The Controllability Matrix is defined as

$$\mathcal{C}(A, B) = [B \mid AB \mid A^2B \mid \dots \mid A^{n-1}B]$$

$$(A, B) \text{ Controllable} \Leftrightarrow rank(C) = n,$$
$$C(A, B) = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

- We say that the above system is controllable if its controllability matrix $C(A, B)$ is invertible.
- As we will see later, if the system is controllable, then we may assign arbitrary closed-loop poles by state feedback of the form $u = -Kx$.
- Whether or not the system is controllable depends on its state-space realization.

OBSERVABILITY MATRIX

Observability Matrix

$$(A, C)\,\text{Observable} \Leftrightarrow rank(V) = n \quad \Leftrightarrow \det(V) \neq 0 \quad \text{if } y \in R$$

$$\text{Observability Matrix } V = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

EXAMPLE 2

Example

- Plant:

$$\dot{x} = Ax + Bu, x \in R^n$$
$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Controllability Matrix $V = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Obervability Matrix $N = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$rank(V) = rank(N) = 2$$

- Hence the system is both controllable and observable.

SUMMARY