SNS COLLEGE OF TECHNOLOGY

Coimbatore-16
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT162 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT V – STATE VARIABLE ANALYSIS

TOPIC 1,2 STATE SPACE REPRESENTATION OF CONTINUOUS TIME SYSTEMS & STATE EQUATIONS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- INTRODUCTION
- STATE-SPACE REPRESENTATION, CONTINUOUS-TIME SYSTEM –NEEDS
- STATE SPACE MODEL
- ACTIVITY
- STATE DIAGRAM
- STATE EQUATIONS
- STATE TRANSITION MATRIX
- SUMMARY

INTRODUCTION

In control engineering, a **state-space representation** is a mathematical model of a physical system as a set of input, output and **state** variables related by first-order differential equations or difference equations.

The **state** of the system **can** be **represented** as a **state** vector within that **space**.

A **continuous-time system** is a **system** in which the signals at input and output are **continuous-time** signals.

NEEDS:

•System description without examining its specific physical meaning.

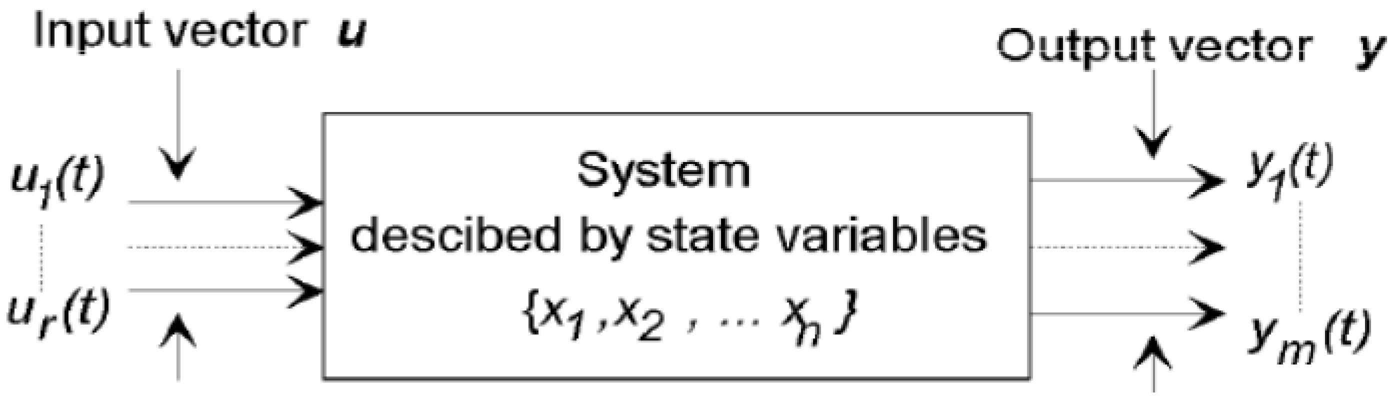
•To understand or to manipulate the property of a system.

INTRODUCTION

- In the classical control theory, the system model is represented by a transfer function.
- The analysis and control tool is based on classical methods such as root locus and Bode plot.
- It is restricted to single input/single output system
- It depends only the information of input and output and it does not use any knowledge of the interior structure of the plant,
- It allows only limited control of the closed-loop behaviour using feedback control is used

INTRODUCTION

- The concept of the state of a dynamic system refers to a minimum set of variables, known as state variables, that fully describe the system and its response to any given set of inputs.



The state variables are an internal description of the system which completely characterize the system state at any time t, and from which any output variables yi(t) may be computed.

STATE MODEL

- A system output is defined to be any system variable of interest.
- A description of a physical system in terms of a set of state variables does not necessarily include all of the variables of direct engineering interest.
- An important property of the linear state equation description is that all system variables may be represented by a linear combination of the state variables xi and the system inputs ui.

STATE MODEL

- Consider a linear time invariant system with the differential equations represented by,

$$
\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + b_{11}u_1 + \ldots + b_{1r}u_r \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + b_{21}u_1 + \ldots + b_{2r}u_r \\
\vdots \quad & \quad \vdots \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + b_{n1}u_1 + \ldots + b_{nr}u_r
\end{aligned}
$$

- This set of n equations defines the derivatives of the state variables to be a weighted sum of the state variables and the system inputs.

$$
\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & \ldots & b_{1r} \\ b_{21} & & b_{2r} \\ \vdots & & \vdots \\ b_{n1} & \ldots & b_{nr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}
$$

$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}
$$

STATE MODEL

$$y(t) = c_1x_1 + c_2x_2 + \ldots + c_nx_n + d_1u_1 + \ldots + d_ru_r$$

$$\begin{array}{ccccccccccccccc}
y_1 & = & c_{11}x_1 & + & c_{12}x_2 & + & \ldots & + & c_{1n}x_n & + & d_{11}u_1 & + & \ldots & + & d_{1r}u_r \\
y_2 & = & c_{21}x_1 & + & c_{22}x_2 & + & \ldots & + & c_{2n}x_n & + & d_{21}u_1 & + & \ldots & + & d_{2r}u_r \\
\vdots & & \vdots & & & & & & & & & & & & \\
y_m & = & c_{m1}x_1 & + & c_{m2}x_2 & + & \ldots & + & c_{mn}x_n & + & d_{m1}u_1 & + & \ldots & + & d_{mr}u_r
\end{array}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \ldots & c_{1n} \\ c_{21} & c_{22} & \ldots & c_{2n} \\ \vdots & & & \vdots \\ c_{m1} & c_{m2} & \ldots & c_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & \ldots & d_{1r} \\ d_{21} & & d_{2r} \\ \vdots & & \vdots \\ d_{m1} & \ldots & d_{mr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

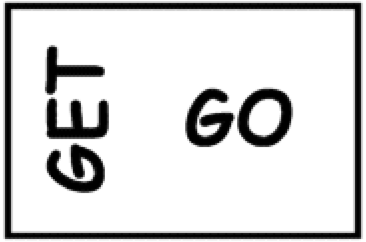
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

ACTIVITY

1.Matt is the fiftieth fastest and the fiftieth slowest runner in his school. Assuming no two runners are the same speed. How many runners are in Matt’s school?

2.
8432: 7
9213: 0
5144: 16
9064: 9
7103 : ?

4.What does this rebus picture means ?



3.Can you guess the Hollywood movie name from the rebus below ?

ACTIVITY-answers

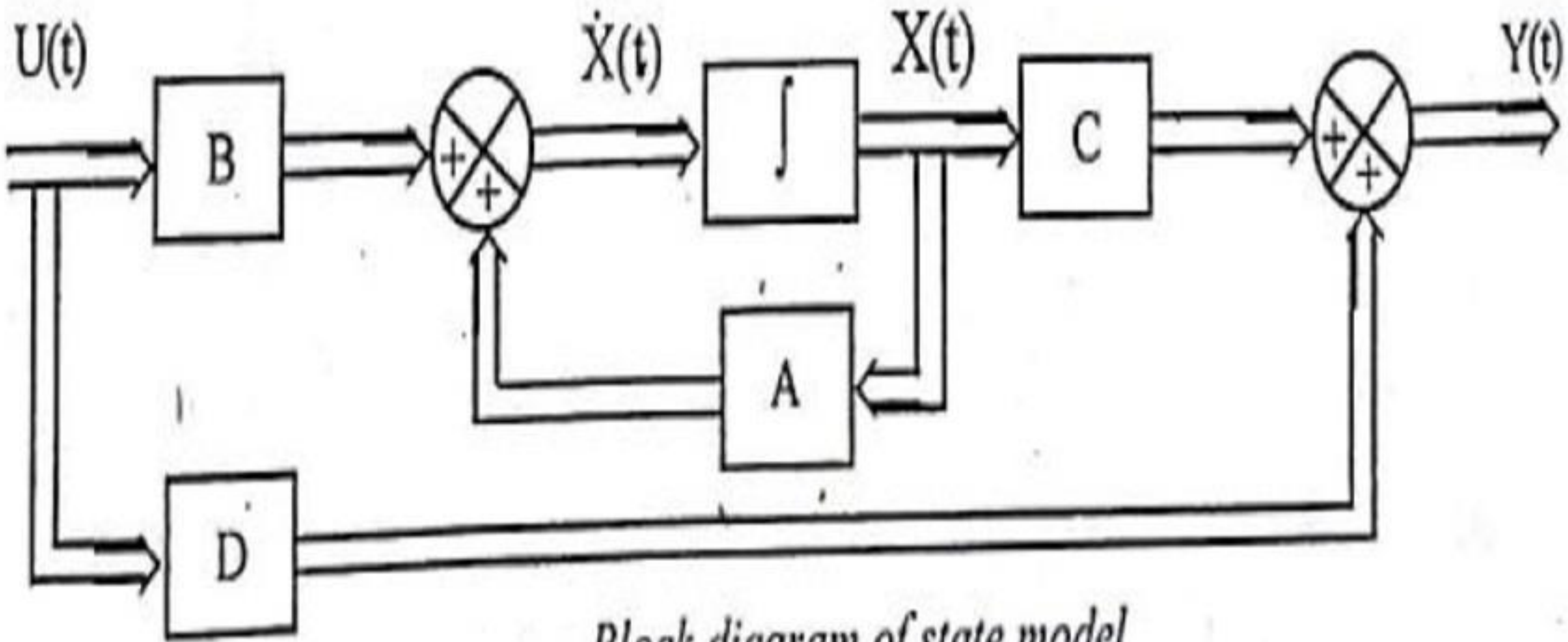
<u>1.Answer</u> – 99 runners in Matt's school.
<u>Explanation-</u>
If Matt is the fiftieth fastest runner, he would be number 50 in the sequence 1, 2, 3…50. To be the fiftieth slowest, he'd have to be number 50 in the sequence 50, 51, 52…99, since there are fifty numbers from 50 to 99 inclusive.

2.The answer is 7103: 1
Logic applied is that i have added all the numbers on Left Hand Side.
(i)If the sum is an odd number then the digit at the Unit's place will be the answer.
8+4+3+2 =17 so the answer is 7
9+0+6+4 =19 so the answer is 9

(ii) If the sum is divisible by 2 then add 2 to the answer.
5+1+4+4 =14 so the answer is 14+2=16

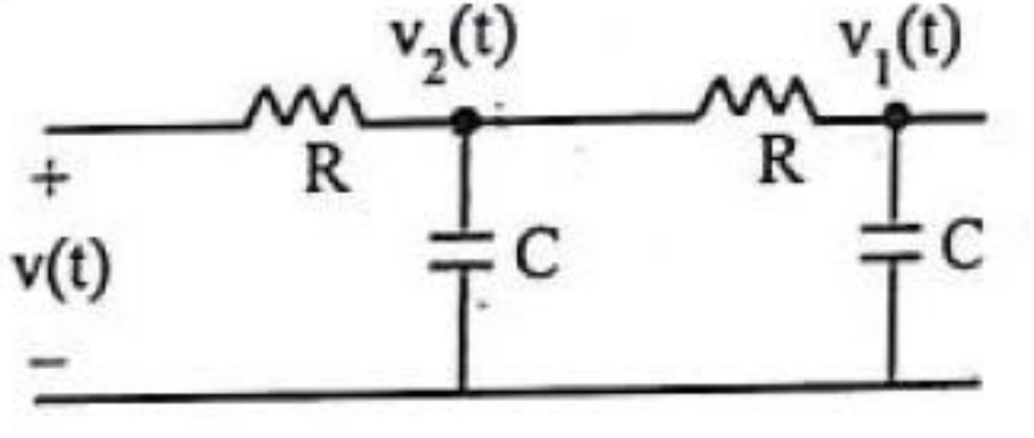
(iii) If the sum is divisible by 5 then the answer will be zero
9+2+1+3 = 15 so the answer is 0
Now, 7+1+0+3 = 11 so the answer will be 1

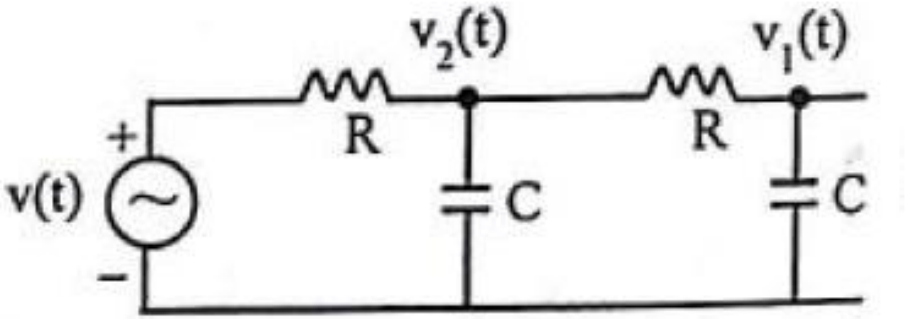
STATE DIAGRAM



Block diagram of state model

STATE EQUATIONS





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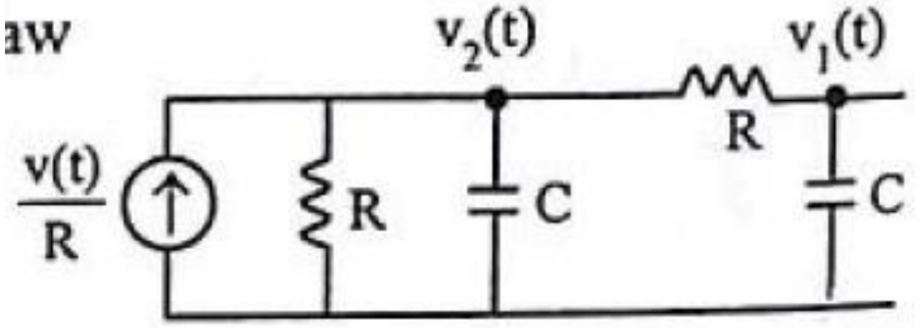

$v_1 = x_1$ and $v_2 = x_2$

$$\frac{v_1 - v_2}{R} + C\frac{dv_1}{dt} = 0$$

$$\frac{v_2 - v_1}{R} + \frac{v_2}{R} + C\frac{dv_2}{dt} = \frac{v(t)}{R}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{RC} \\ \frac{1}{RC} & \frac{-2}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{RC} \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

$\therefore$ The output equation is $y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

STATE TRANSITION MATRIX

We can write the solution of the ***homogeneous*** state equation

$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$ Laplace transform $s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$

$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{x}(0) \Rightarrow \mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0)$

The inverse Laplace transform $\mathbf{x}(t) = \mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]\mathbf{x}(0)$

Note that $(s\mathbf{I} - \mathbf{A})^{-1} = \frac{\mathbf{I}}{s} + \frac{\mathbf{A}}{s^2} + \frac{\mathbf{A}^2}{s^3} + \cdots$

$$\mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}] = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \cdots = e^{\mathbf{A}t}$$

STATE TRANSITION MATRIX

Hence, the inverse Laplace transform of $(s\mathbf{I} - \mathbf{A})^{-1}$

$$\mathscr{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}] = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \cdots = e^{\mathbf{A}t}$$

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0)$$

State-Transition Matrix $\mathbf{x}(t) = \mathbf{\Phi}(t)\mathbf{x}(0)$

where $\mathbf{\Phi}(t)$ is an $n \times n$ matrix and is the unique solution of

$$\dot{\mathbf{\Phi}}(t) = \mathbf{A}\mathbf{\Phi}(t), \qquad \mathbf{\Phi}(0) = \mathbf{I}$$

Where

$\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \mathscr{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]$ Note that $\mathbf{\Phi}^{-1}(t) = e^{-\mathbf{A}t} = \mathbf{\Phi}(-t)$

PROPERTIES OF STATE TRANSITION MATRIX

1. $\Phi(0) = e^{\mathbf{A}0} = \mathbf{I}$

2. $\Phi(t) = e^{\mathbf{A}t} = \left(e^{-\mathbf{A}t}\right)^{-1} = \left[\Phi(\;\;t)\right]^{-1}$ or $\Phi^{-1}(t) = \Phi(\;\;t)$

3. $\Phi(t_1 + t_2) = e^{\mathbf{A}(t_1+t_2)} = e^{\mathbf{A}t_1}e^{\mathbf{A}t_2} = \Phi(t_1)\Phi(t_2) = \Phi(t_2)\Phi(t_1)$

4. $\left[\Phi(t)\right]^n = \Phi(nt)$

5. $\Phi(t_2 - t_1)\Phi(t_1 - t_0) = \Phi(t_2 - t_0) = \Phi(t_1 - t_0)\Phi(t_2 - t_1)$

SUMMARY