



# **SNS COLLEGE OF TECHNOLOGY**

## **An Autonomous Institution**

### **Coimbatore-35**



Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## **DEPARTMENT OF CIVIL ENGINEERING**

### **23GET276 – VQAR II**

II YEAR/ IV SEMESTER

### **UNIT 2 – QUANTITATIVE ABILITY IV**

### **TOPIC – PERMUTATION AND COMBINATION**



# PERMUTATION AND COMBINATION



## 1. Factorial Notation:

Let  $n$  be a positive integer. Then, factorial  $n$ , denoted  $n!$  is defined as:

$$n! = n(n - 1)(n - 2) \dots 3.2.1.$$

### Examples:

- i. We define  $0! = 1$ .
- ii.  $4! = (4 \times 3 \times 2 \times 1) = 24$ .
- iii.  $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$ .

## 2. Permutations:

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

### Examples:

- i. All permutations (or arrangements) made with the letters  $a, b, c$  by taking two at a time are ( $ab, ba, ac, ca, bc, cb$ ).
- ii. All permutations made with the letters  $a, b, c$  taking all at a time are:  
( $abc, acb, bac, bca, cab, cba$ )



# PERMUTATION AND COMBINATION



## 3. Number of Permutations:

Number of all permutations of  $n$  things, taken  $r$  at a time, is given by:

$${}^n P_r = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

### Examples:

i.  ${}^6 P_2 = (6 \times 5) = 30.$

ii.  ${}^7 P_3 = (7 \times 6 \times 5) = 210.$

iii. Cor. number of all permutations of  $n$  things, taken all at a time =  $n!$ .

## 4. An Important Result:

If there are  $n$  subjects of which  $p_1$  are alike of one kind;  $p_2$  are alike of another kind;  $p_3$  are alike of third kind and so on and  $p_r$  are alike of  $r^{\text{th}}$  kind, such that  $(p_1 + p_2 + \dots + p_r) = n$ .

Then, number of permutations of these  $n$  objects is =  $\frac{n!}{(p_1!)(p_2!) \dots (p_r!)}$



# PERMUTATION AND COMBINATION



## 5. Combinations:

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a **combination**.

### Examples:

1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note: AB and BA represent the same selection.

2. All the combinations formed by  $a, b, c$  taking  **$ab, bc, ca$** .

3. The only combination that can be formed of three letters  $a, b, c$  taken all at a time is  **$abc$** .

4. Various groups of 2 out of four persons A, B, C, D are:

**AB, AC, AD, BC, BD, CD.**

5. Note that  $ab$   $ba$  are two different permutations but they represent the same combination.



# PERMUTATION AND COMBINATION



## 6. Number of Combinations:

The number of all combinations of  $n$  things, taken  $r$  at a time is:

$${}^nC_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2) \dots \text{to } r \text{ factors}}{r!}$$

**Note:**

i.  ${}^nC_n = 1$  and  ${}^nC_0 = 1$ .

ii.  ${}^nC_r = {}^nC_{(n-r)}$

**Examples:**

i.  ${}^{11}C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330$ .

ii.  ${}^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$ .



# PERMUTATION AND COMBINATION



From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

- A. 564
- B. 645
- C. 735
- D. 756
- E. None of these

**Answer:** Option D

**Explanation:**

We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only).

$$\begin{aligned}\therefore \text{Required number of ways} &= ({}^7C_3 \times {}^6C_2) + ({}^7C_4 \times {}^6C_1) + ({}^7C_5) \\ &= \left( \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1} \right) + ({}^7C_3 \times {}^6C_1) + ({}^7C_2) \\ &= 525 + \left( \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6 \right) + \left( \frac{7 \times 6}{2 \times 1} \right) \\ &= (525 + 210 + 21) \\ &= 756.\end{aligned}$$



# PERMUTATION AND COMBINATION



In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

- A. 360
- B. 480
- C. 720
- D. 5040
- E. None of these

**Answer:** Option C

**Explanation:**

The word 'LEADING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter.

Then, we have to arrange the letters LNDG (EAI).

Now, 5 ( $4 + 1 = 5$ ) letters can be arranged in  $5! = 120$  ways.

The vowels (EAI) can be arranged among themselves in  $3! = 6$  ways.

$\therefore$  Required number of ways =  $(120 \times 6) = 720$ .



# PERMUTATION AND COMBINATION



In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

- A. 810
- B. 1440
- C. 2880
- D. 50400
- E. 5760

**Answer:** Option D

**Explanation:**

In the word 'CORPORATION', we treat the vowels OOAIO as one letter.

Thus, we have CRPRTN (OOAIO).

This has 7 (6 + 1) letters of which R occurs 2 times and the rest are different.

Number of ways arranging these letters =  $\frac{7!}{2!} = 2520$ .

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged

in  $\frac{5!}{3!} = 20$  ways.

∴ Required number of ways = (2520 × 20) = 50400.



# PERMUTATION AND COMBINATION



Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

- A. 210
- B. 1050
- C. 25200
- D. 21400
- E. None of these

**Answer:** Option C

**Explanation:**

Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$$= ({}^7C_3 \times {}^4C_2)$$

$$= \left( \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1} \right)$$

$$= 210.$$

Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging 5 letters among themselves = 5!

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 120.$$

∴ Required number of ways =  $(210 \times 120) = 25200$ .



# PERMUTATION AND COMBINATION



In how many ways can the letters of the word 'LEADER' be arranged?

- A. 72
- B. 144
- C. 360
- D. 720
- E. None of these

**Answer:** Option C

**Explanation:**

The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.

$$\therefore \text{Required number of ways} = \frac{6!}{(1!)(2!)(1!)(1!)(1!)} = 360.$$



# PERMUTATION AND COMBINATION



In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

- A. 159
- B. 194
- C. 205
- D. 209
- E. None of these

**Answer:** Option D

**Explanation:**

We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

$$\begin{aligned}\therefore \text{Required number of ways} &= {}^6C_1 \times {}^4C_3 + {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_4 \\ &= {}^6C_1 \times {}^4C_1 + {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_2 \\ &= (6 \times 4) + \left( \frac{6 \times 5}{2 \times 1} \times \frac{4 \times 3}{2 \times 1} \right) + \left( \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times 4 \right) + \left( \frac{6 \times 5}{2 \times 1} \right) \\ &= (24 + 90 + 80 + 15) \\ &= 209.\end{aligned}$$



# PERMUTATION AND COMBINATION



In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?

- A. 266
- B. 5040
- C. 11760
- D. 86400
- E. None of these

**Answer:** Option C

**Explanation:**

$$\begin{aligned}\text{Required number of ways} &= {}^8C_5 \times {}^{10}C_6 \\ &= {}^8C_3 \times {}^{10}C_4 \\ &= \left( \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \right) \\ &= 11760.\end{aligned}$$



# PERMUTATION AND COMBINATION



A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

- A. 32
- B. 48
- C. 64
- D. 96
- E. None of these

**Answer:** Option C

**Explanation:**

We may have (1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

$$\begin{aligned}\therefore \text{Required number of ways} &= ({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3) \\ &= \left( 3 \times \frac{6 \times 5}{2 \times 1} \right) + \left( \frac{3 \times 2}{2 \times 1} \times 6 \right) + 1 \\ &= (45 + 18 + 1) \\ &= 64.\end{aligned}$$



# PERMUTATION AND COMBINATION



In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

- A. 63
- B. 90
- C. 126
- D. 45
- E. 135

**Answer:** Option A

**Explanation:**

$$\text{Required number of ways} = ({}^7C_5 \times {}^3C_2) = ({}^7C_2 \times {}^3C_1) = \left( \frac{7 \times 6}{2 \times 1} \times 3 \right) = 63.$$



# PERMUTATION AND COMBINATION



How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

- A. 40
- B. 400
- C. 5040
- D. 2520

**Answer:** Option C

**Explanation:**

'LOGARITHMS' contains 10 different letters.

Required number of words = Number of arrangements of 10 letters, taking 4 at a time.

$$= {}^{10}P_4$$

$$= (10 \times 9 \times 8 \times 7)$$

$$= 5040.$$



# PERMUTATION AND COMBINATION



In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together?

- A. 10080
- B. 4989600
- C. 120960
- D. None of these

**Answer:** Option C

**Explanation:**

In the word 'MATHEMATICS', we treat the vowels AEAI as one letter.

Thus, we have MTHMTCS (AEAI).

Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different.

$$\therefore \text{Number of ways of arranging these letters} = \frac{8!}{(2!)(2!)} = 10080.$$

Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.

$$\text{Number of ways of arranging these letters} = \frac{4!}{2!} = 12.$$

$$\therefore \text{Required number of words} = (10080 \times 12) = 120960.$$



# PERMUTATION AND COMBINATION



In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

- A. 120
- B. 720
- C. 4320
- D. 2160
- E. None of these

**Answer:** Option B

**Explanation:**

The word 'OPTICAL' contains 7 different letters.

When the vowels OIA are always together, they can be supposed to form one letter.

Then, we have to arrange the letters PTCL (OIA).

Now, 5 letters can be arranged in  $5! = 120$  ways.

The vowels (OIA) can be arranged among themselves in  $3! = 6$  ways.

$\therefore$  Required number of ways =  $(120 \times 6) = 720$ .



# THANK YOU