Random Variable:

Random variable is a real valued function that assigns a numerical value to each possible outcome of an experiment.

Eg:

consider an experiment, tossing a coin twice.

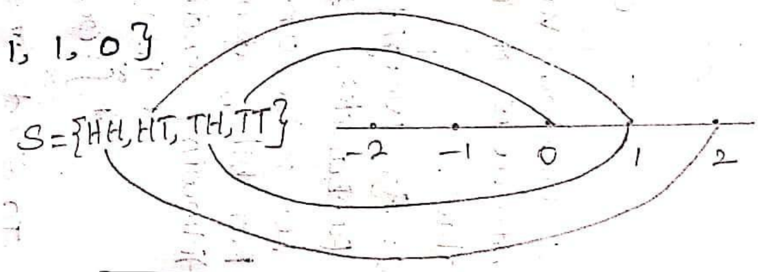
The sample space is

$$S = \{HH, HT, TH, TT\}$$

let x be the random variable such that

$x(s) = x$(No. of heads)

$\therefore x(s) = \{2, 1, 1, 0\}$



Types of Random variable:

* Discrete Random variable
* Continuous Random variable

Note:

* $P(X \geq x) = 1 - P(X < x)$
* $P(X \leq x) = 1 - P(X > x)$

Discrete

* A Random variable x is said to be discrete if it assumes either finite or countably infinite.

Eg: No. of students present

* probability Mass Function $P(x)$
* Let x be discrete random variable then probability mass function satisfies the following conditions.

$$* \; P(x_i) \geq 0$$
$$* \; \sum_{i=1}^{n} P(x_i) = 1$$

* To find constant, $\sum_{i=1}^{n} P(x_i) = 1$
* To find cumulative distribution function.

$$F(x) = P[X \leq x]$$

* cumulative distribution is given, then find PMF

$$P(x) = F(x_i) - F(x_{i-1})$$

* To find mean: $E(x) = \sum_{i=1}^{n} x_i P(x_i)$; $E(x^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$

variance $V(x) = E(x^2) - (E(x))^2$

Continuous

* A Random Variable X is said to be continuous if it takes only an interval values.

Eg: Age, Weight

* Probability density function $f(x)$
* Let x be continuous random variable. Th[e] probability density function satisfies the following conditions

$$* \; f(x) \geq 0$$
$$* \; \int_{-\infty}^{\infty} f(x)\,dx = 1$$

* To find constant, $\int_{-\infty}^{\infty} f(x)\,dx = 1$
* To find cumulative distributive functio[n]

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

* cumulative distribution is given, then find

$$f(x) = \frac{d}{dx} F(x)$$

* To find mean: $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$; $E(x^2) = \int_{-\infty}^{\infty} x^2$

variance : $V(x) = E(x^2) - (E(x))^2$

7). A random variable X has the following probability function

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

i). Find K ii). $P(X<6)$, $P(X\geq 6)$, $P(0<X<5)$

iii). Distributive or cumulative function

iv). $P(1/2 < X < 5/2 / X > 1)$

v). Find the smallest value of x such that $P(X\leq x) > \frac{1}{2}$.

Soln.

i). Find K:

$$\sum_{i=0}^{n} P(x_i) = 1$$

$$\sum_{i=0}^{7} P(x_i) = 1$$

$$0+K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$$

$$10K^2+9K-1 = 0$$

$$K = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \begin{array}{l} a\to 10 \\ b\to 9 \\ c\to -1 \end{array}$$

$$= \frac{-9\pm\sqrt{81-4(10)(-1)}}{2(10)}$$

$$= \frac{-9\pm\sqrt{81+40}}{20} = \frac{-9\pm\sqrt{121}}{20}$$

$$= \frac{-9\pm 11}{20} = -1, \frac{1}{10}$$

$$K = -1, \frac{1}{10}$$

Here $K = 0.1$ ($\because K=-1$ is not possible)

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x	0	1	2	3	4	5	6	7
$P(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

ii). $P(X<6) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0+0.1+0.2+0.2+0.3+0.01$

$P(X<6) = 0.81$

$P(X\geq 6) = 1-P(X<6)$

$= 1-0.81$

$= 0.19$

$P(0<X<5) = P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 0.1+0.2+0.2+0.3$

$= 0.8$

iii). Distributive or cumulative function:

$F(x) = P(X\leq x)$

x	0	1	2	3	4	5	6	7
$P(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17
$F(x)$	0	0.1	0.3	0.5	0.8	0.81	0.83	1

iv). $P(1/2 < X < 5/2 \,/\, X>1)$

$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

$$\therefore P\left(\tfrac{1}{2} < X < 5/2 \,/\, X>1\right) = \frac{P\left(\frac{1}{2}<X<\frac{5}{2} \cap X>1\right)}{P(X>1)}$$

$$= \frac{P(0.5<X<2.5 \cap X>1)}{P(X>1)}$$

$$= \frac{P[1<X<2.5]}{1-P(X\leq 1)}$$

$$= \frac{P(X=2)}{1-[P(X=0)+P(X=1)]}$$

$$= \frac{0.2}{1-[0+0.1]}$$

$$= \frac{0.2}{1-0.1}$$

$$= \frac{0.2}{0.9}$$

$$= 0.222$$

v). $P(X \le x) > 1/2$

$\Rightarrow F(x) > 1/2$

The values are 4, 5, 6, 7

The smallest value is 4.

2). A random variable X as the probability function

x	0	1	2	3	4	5	6	7	8
$P(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i). Find a ii). $P(X<3)$, $P(X \ge 3)$, $P(0<X<6)$

iii). Distribution function.

Soln.

i). Find a:

$$\sum_{i=0}^{n} p(x_i) = 1$$

$$\Rightarrow \sum_{i=0}^{8} p(x_i) = 1$$

$$a+3a+5a+7a+9a+11a+13a+15a+17a=1$$

$$81a = 1 \Rightarrow a = 1/81 = 0.012$$

$\therefore \; a = 0.012$

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.012	0.036	0.060	0.084	0.108	0.132	0.156	0.180	0.204

ii) $P(X<3)$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= 0.012 + 0.036 + 0.060 = 0.108$

$P(X \geq 3) = 1 - P(X<3)$

$= 1 - 0.108$

$= 0.892$

$P(0<X<6) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0.036 + 0.060 + 0.084 + 0.108 + 0.132$

$= 0.420$

iii). Distribution Function:

$F(x) = P(X \leq x)$

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.012	0.036	0.060	0.084	0.108	0.132	0.156	0.180	0.204
$F(x)$	0.012	0.048	0.108	0.192	0.300	0.432	0.588	0.768	0.972