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DEPARTMENT OF AEROSPACE ENGINEERING

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UNIT I - FUNDAMENTAL CONCEPTS

Mathematical properties of Fluid Dynamics Equations

Mathematical Properties of Fluid Dynamic Equations – Elliptical, Parabolic and Hyperbolic Equations

The solution procedure of a partial differential equation (PDE) depends upon the type of equation, thus it is important to study the various classifications of PDE's. The imposition of the initial or boundary condition also depends upon the type of PDE.

Linear and Nonlinear PDEs

Linear PDE: In a Linear PDE the dependent variable and its derivative enter the equation linearly, i.e. there is no product of the dependent variable or its derivatives.

Example: One-dimensional wave equation

$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x}$$

Where a is the speed of sound which is assumed constant

Nonlinear PDE: A Nonlinear PDE contains the product of the dependent variable and its derivative.

Example: Inviscid Burgers equation

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x}$$

Second-Order PDE's

To classify the second-order PDE, Consider the following equation.

$$A\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial x\partial y} + C\frac{\partial^2\phi}{\partial y^2} + D\frac{\partial\phi}{\partial x} + E\frac{\partial\phi}{\partial y} + F\phi + G = 0$$

……Equation 1.48 Where the coefficients A, B, C, D, E, F, and G are functions of the independent variables x and y and dependent variable Φ. By the definition, we can express $d\Phi_x$ and $d\Phi_y$ as

$$d\phi_x = \frac{\partial\phi_x}{\partial x}dx + \frac{\partial\phi_x}{\partial y}dy = \frac{\partial^2\phi}{\partial x^2}dx + \frac{\partial^2\phi}{\partial x\partial y}dy$$

$$d\phi_y = \frac{\partial\phi_y}{\partial x}dx + \frac{\partial\phi_y}{\partial y}dy = \frac{\partial^2\phi}{\partial x\partial y}dx + \frac{\partial^2\phi}{\partial y^2}dy$$

The equation 1.48 can be expressed as ……Equation 1.49

$$A\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial x\partial y} + C\frac{\partial^2\phi}{\partial y^2} = H$$

Where,

$$H = -\left(D\frac{\partial\phi}{\partial x} + E\frac{\partial\phi}{\partial y} + F\phi + G\right)$$

……Equation 1.50

Equations 1.49 and 1.50 are solved for Φ, using the cramers rule we get,

$$\frac{\partial^2\phi}{\partial x\partial y} = \frac{\begin{vmatrix} A & H & C \\ dx & d\phi_x & 0 \\ 0 & d\phi_y & dy \end{vmatrix}}{\begin{vmatrix} A & B & C \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix}}$$

Since it is possible to have the discontinuous in the second-order derivatives of the dependent variable across the characteristics, these derivatives are indeterminate. Thus setting the denominator equal to zero.

$$\begin{vmatrix} A & B & C \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix} = 0$$

Yields the equation

$$A\left(\frac{dy}{dx}\right)^2 - B\left(\frac{dy}{dx}\right) + C = 0$$

……Equation

1.51 Solving this quadratic equation yields the equation of the characteristic in physical space.

$$\left(\frac{dy}{dx}\right)_{\alpha,\beta} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

……Equation

1.52 Depending on the value of B^2-4AC the characteristic curves are real or imaginary. They are classified as,

(a) elliptic if $B^2 - 4AC < 0$

(b) parabolic if $B^2 - 4AC = 0$

(c) hyperbolic if $B^2 - 4AC > 0$

Elliptical Equations

A partial differential equation is elliptical in a region if B^2-4AC is less than zero at all points in the region. An elliptic PDE has no real characteristic curves. A disturbance is propagated instantly in all directions within the region. The domain of the solution of an elliptical equation is a closed region.

Example:

Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

……Equation 1.53

Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = f(x,y)$$

……Equation 1.54

Parabolic Equations

A partial differential equation is parabolic in a region if B^2-4AC is equal to zero at all points in the region. A parabolic PDE has a solution domain as an open region. A parabolic PDE has one real characteristic curve.

Example:

Unsteady heat conduction in one dimension

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

……Equation 1.55

Diffusion of viscosity equation

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

……Equation 1.56

Hyperbolic Equations

A partial differential equation is called hyperbolic if B^2-4AC is greater than zero at all points in the region. A hyperbolic PDE has two real characteristic curves.

Example: Second-order wave equation

$$\frac{\partial^2 \phi}{\partial t^2} = a^2 \frac{\partial^2 \phi}{\partial x^2}$$