



DEPARTMENT OF MATHEMATICS

UNIT-I TESTING OF HYPOTHESIS

TEST OF SIGNIFICANCE OF SMALL SAMPLES!

VARIANCE RATIO TEST (OR) F-TEST FOR EQUALITY OF VARIANCE

Null Hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$

Test statistics: $F = \frac{S_1^2}{S_2^2}$ where $S_1^2 > S_2^2$.

$$\text{where } S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} \text{ or } S_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1} \&$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} \text{ or } S_2^2 = \frac{\sum (x_2 - \bar{x}_2)^2}{n_2 - 1}$$

Degree of Freedom: (v_1, v_2)

where $v_1 = (n_1 - 1)$, $v_2 = (n_2 - 1)$

Note 1:- F Greater than ~~one~~ always.

Note 2 :- Suppose S_2^2 Greater than S_1^2 , then $F = \frac{S_2^2}{S_1^2}$

with degree of freedom, $v_1 = n_2 - 1$, $v_2 = n_1 - 1$

Applications!

'F test' is used to test if the two samples have come from the same population.



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1) two random sample of 11 and 9 items show that the sample standard deviations of their weights as 0.8 & 0.5 respectively. Assuming that the weight distributions are normal, test the hypothesis that the true variances are equal, against the alternative hypothesis that they are not.

Soln:

Given: $n_1 = 11$, $s_1 = 0.8$

$n_2 = 9$, $s_2 = 0.5$

$$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{11(0.8)^2}{11 - 1} = 0.704$$

$$s_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{9(0.5)^2}{9 - 1} = 0.2812$$

$$s_1^2 > s_2^2$$

Step 1 \rightarrow Formulate H_0 & H_1 .

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Step 2 \rightarrow Los at $\alpha = 5\%$.

Step 3 \rightarrow Test statistic, $F = \frac{s_1^2}{s_2^2} = \frac{0.704}{0.2812} = 2.5$

Step 4 \rightarrow Degrees of freedom $(n_1 - 1, n_2 - 1) = (10, 8)$

Critical value, $F_{\alpha} = F_{\alpha} = 3.35$

Step 5 \rightarrow Conclusion: $F = 2.5 < 3.35 = F_{\alpha}$

$\therefore H_0$ is accepted at $\alpha = 5\%$.



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2) Two random samples gave the following results:

sample	size	sample mean	sum of squares of deviation from the means.
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1	12	14	108
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2	10	15	90
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Test whether the samples came from the same population.

Soln:

Given:

$$n_1 = 12, \bar{x}_1 = 14, \sum (x_1 - \bar{x}_1)^2 = 108$$

$$n_2 = 10, \bar{x}_2 = 15, \sum (x_2 - \bar{x}_2)^2 = 90$$

$$s_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{108}{12 - 1} = 9.818$$

$$s_2^2 = \frac{\sum (x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{90}{10 - 1} = 10$$

$$s_1^2 < s_2^2$$

Step 1: Formulate H_0 and H_1 :

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Step 2: Los at $\alpha = 5\%$.

Step 3: Test statistics, $F = \frac{s_2^2}{s_1^2} = \frac{10}{9.818}$

$$F = 1.018$$



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Step 4: Degrees of freedom = (v_1, v_2)
 $= (n_2 - 1, n_1 - 1)$
 $= (9, 11)$

Critical value, $F_\alpha = 2.90$

Step 5: Conclusion:

$$F = 1.018 < 2.90 = F_\alpha$$

$\therefore H_0$ is accepted at 5% LOS.

(ii) t -Test:

Step 1: Formulate H_0 & H_1 :

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Step 2: LOS at 5% = α

Step 3: Test statistic, $t = \frac{\bar{x}_1 - \bar{x}_2}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Here $n_1 = 12, n_2 = 10; \bar{x}_1 = 14, \bar{x}_2 = 15$

$$\text{Now } S^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$= \frac{108 + 90}{12 + 10 - 2} = 9.9$$

$$S = 3.14$$



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$$\therefore t = \frac{14 - 15}{3.14 \sqrt{\frac{1}{12} + \frac{1}{10}}} = -0.744$$

$$|t| = 0.744$$

Step 4: Degrees of freedom, $\nu = n_1 + n_2 - 2$
 $= 12 + 10 - 2$
 $= 20$

$$\therefore t_{\alpha} = 2.086$$

Step 5: Conclusion, $t = 0.744 < 2.086 = t_{\alpha}$

$\therefore H_0$ is accepted at 5% LOS.

3) Test whether the population variances are identical:

Sample I: 10 11 16 12 10 11 12 16

Sample II: 7 9 3 7 9 3 15 at 1% LOS

Soln: Given: $n_1 = 8$, $n_2 = 7$

x_1	$(x_1 - \bar{x}_1)^2$	x_2	$(x_2 - \bar{x}_2)^2$
10	5.0625	7	0.3265
11	1.5625	9	2.0409
16	14.0625	3	20.8944
12	0.0625	7	0.3265
10	5.0625	9	2.0409
11	1.5625	3	20.8944
12	0.0625	15	55.1841
16	14.0625		
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98	41.5	53	101.7143
			$\Sigma(x_2 - \bar{x}_2)^2 = 101.71$

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{98}{8}$$

$$\Sigma(x_1 - \bar{x}_1)^2 = 41.5 \quad \bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{53}{7} = 7.57$$



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$$\therefore S_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{41.5}{7} = 5.9286$$

$$S_2^2 = \frac{\sum (x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{101.7143}{6} = 16.9524$$

$$S_1^2 < S_2^2.$$

Step 1: Formulate H_0 & H_1 :

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Step 2: Los at $\alpha = 1\%$.

Step 3: Test statistic, $F = \frac{S_2^2}{S_1^2}$

$$= \frac{16.9524}{5.9286} = 2.86$$

Step 4: Degrees of freedom: (v_1, v_2)

$$= (n_2 - 1, n_1 - 1)$$

$$= (6, 7)$$

$$\therefore F_\alpha = 7.19$$

Step 5: Conclusion, $F = 2.86 < 7.19 = F_\alpha$

$\therefore H_0$ is accepted at H_0 at 1% Los.