SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT203 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 1 – DISCRETE FOURIER TRANSFORM

TOPIC – Introduction to DFT

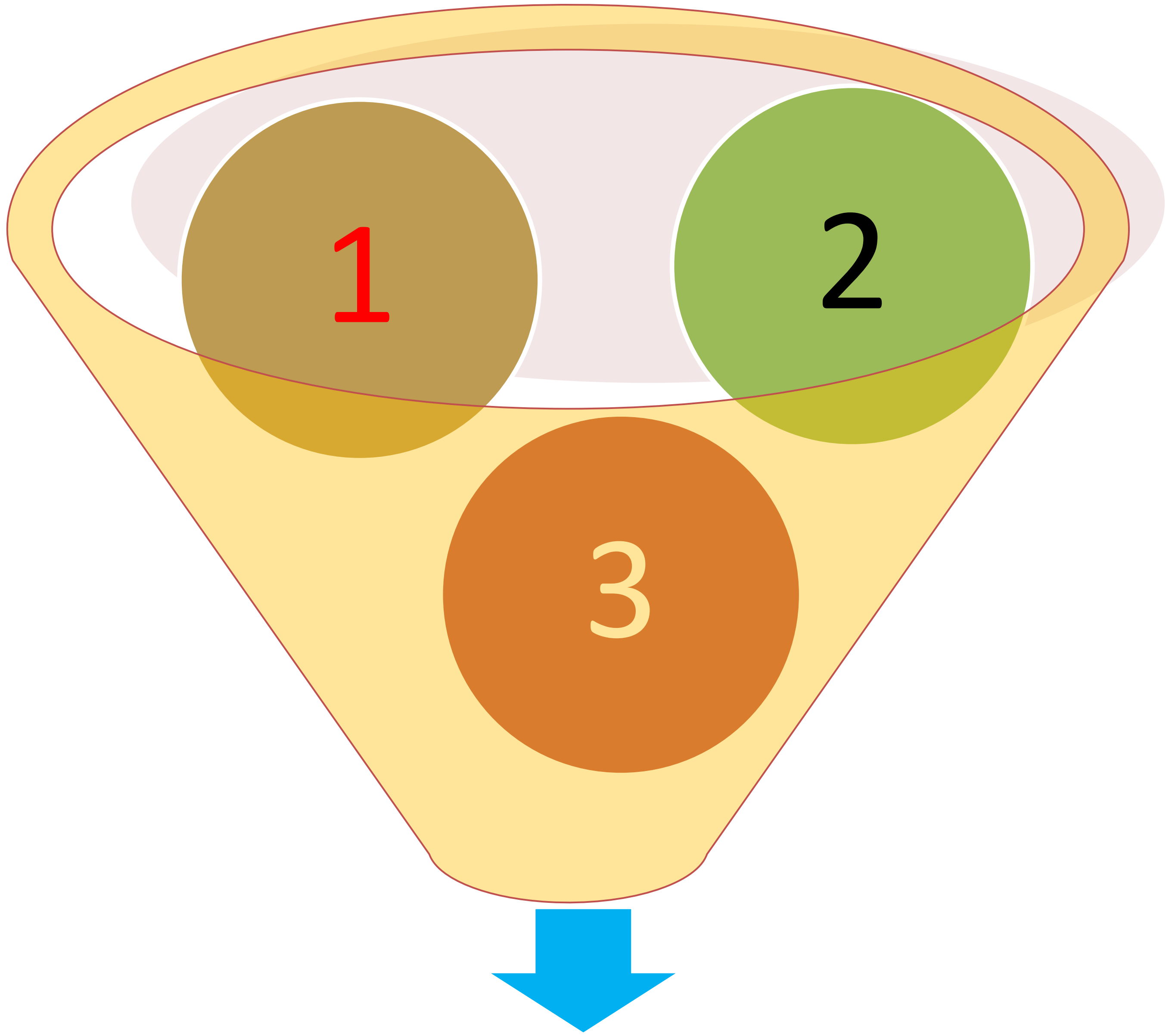
EMPATHY

1
- Defects in signals is to identified

2
- Conversion from Time domain to frequency domain takes longer time

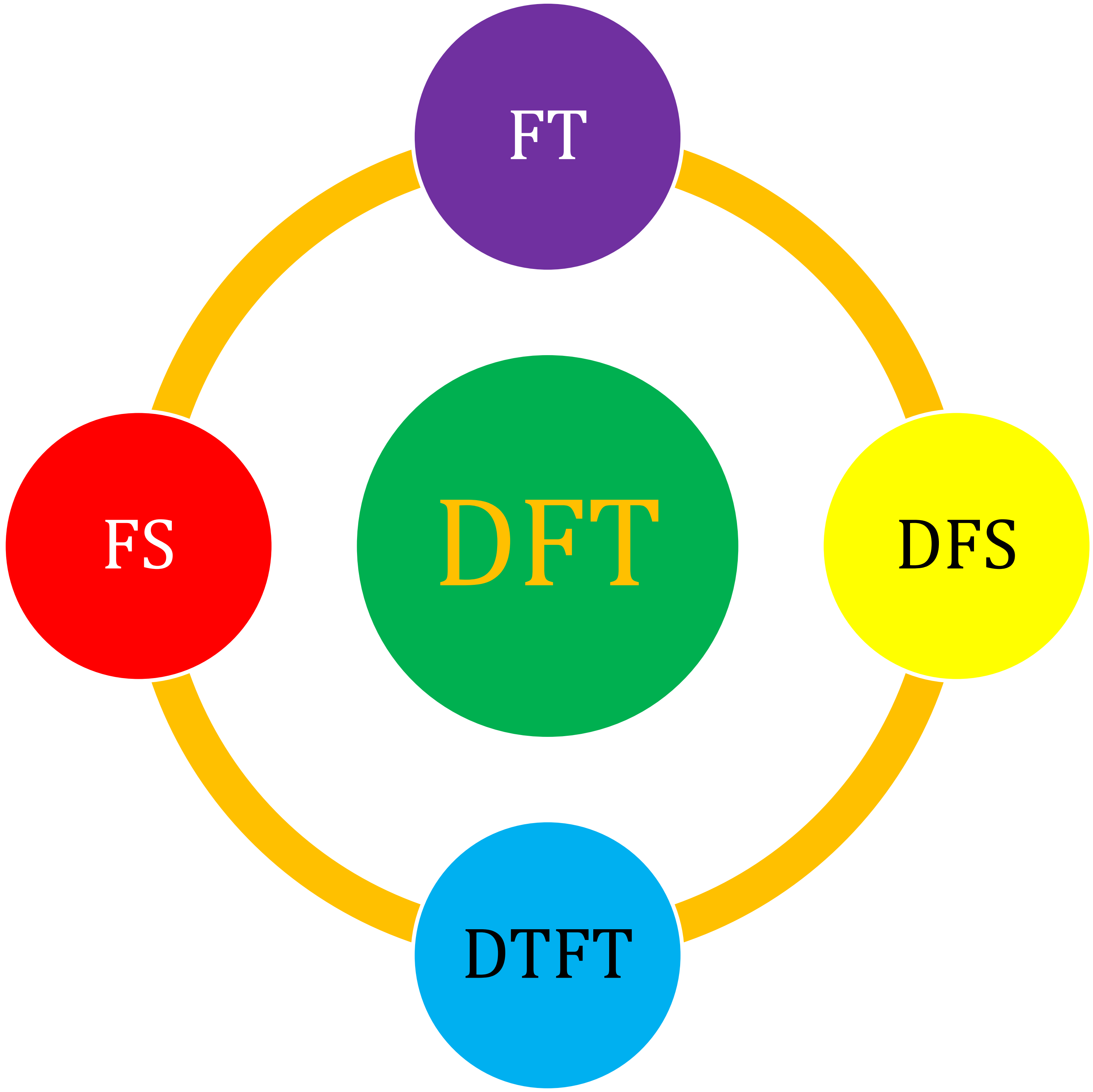
3
- Frequency domain Information must be extracted

DEFINE

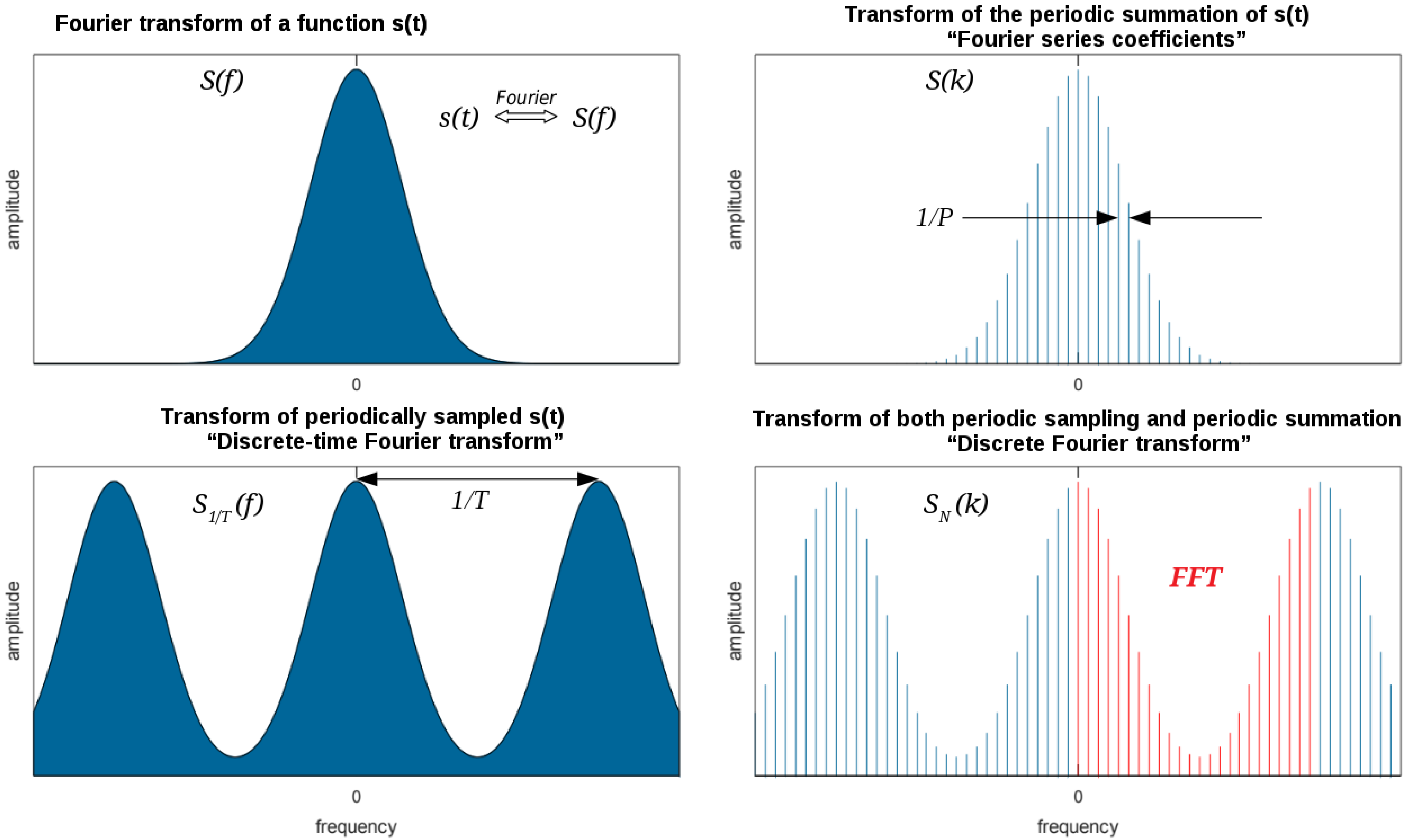


Time to frequency domain conversion and process faster

IDEATE



FOURIER COEFFICIENTS REPRESENTATION



DISCRETE FOURIER TRANSFORM

For a discrete time sequence we define two classes of Fourier Transforms:

- The *DTFT (Discrete Time FT) for sequences having* ***infinite*** *duration,*
- The *DFT (Discrete FT) for sequences having* ***finite*** *duration.*

DTFT AND INVERSE DTFT

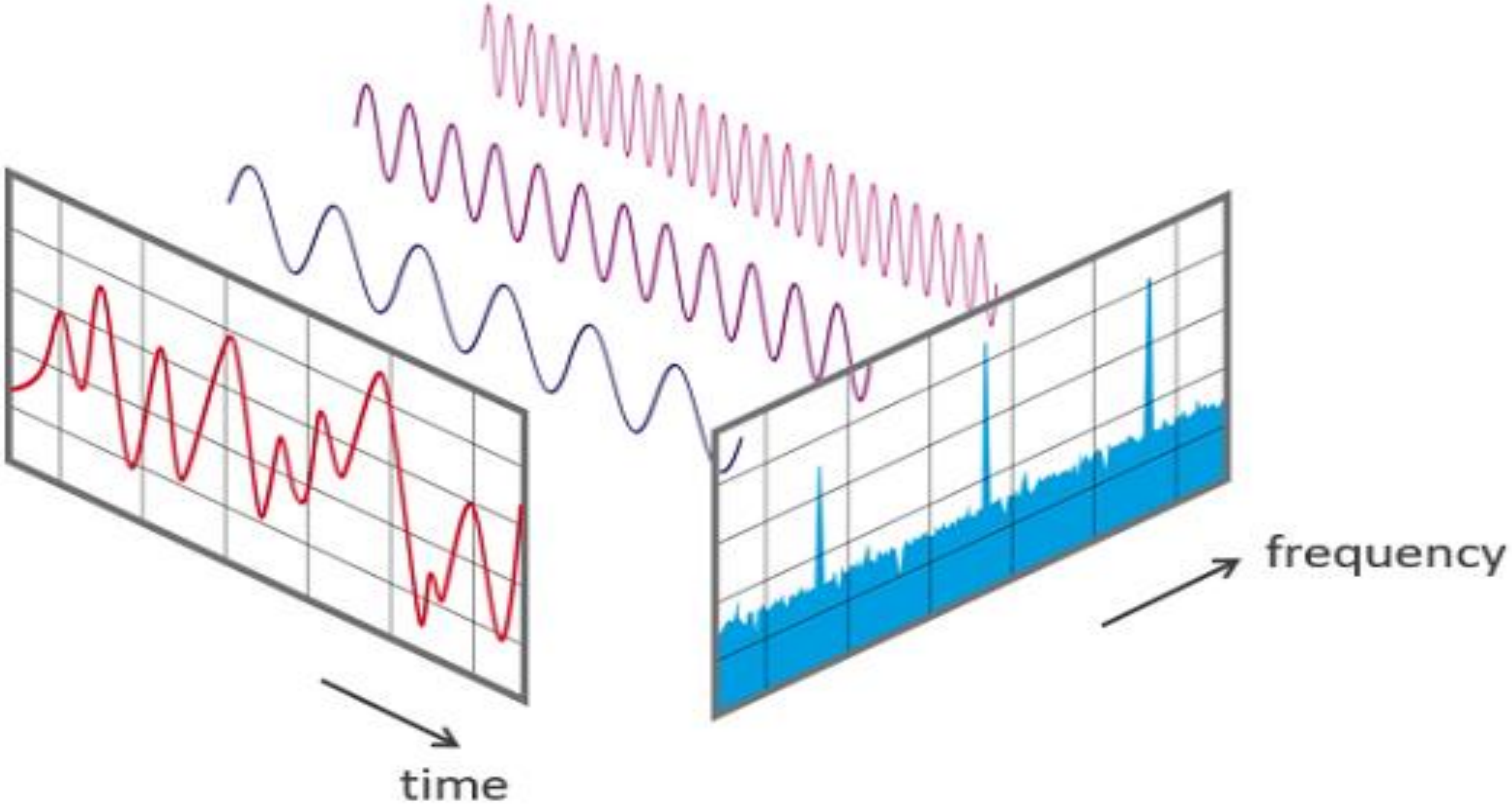
DTFT

$$X(\omega) = DTFT\{x(n)\} = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$$

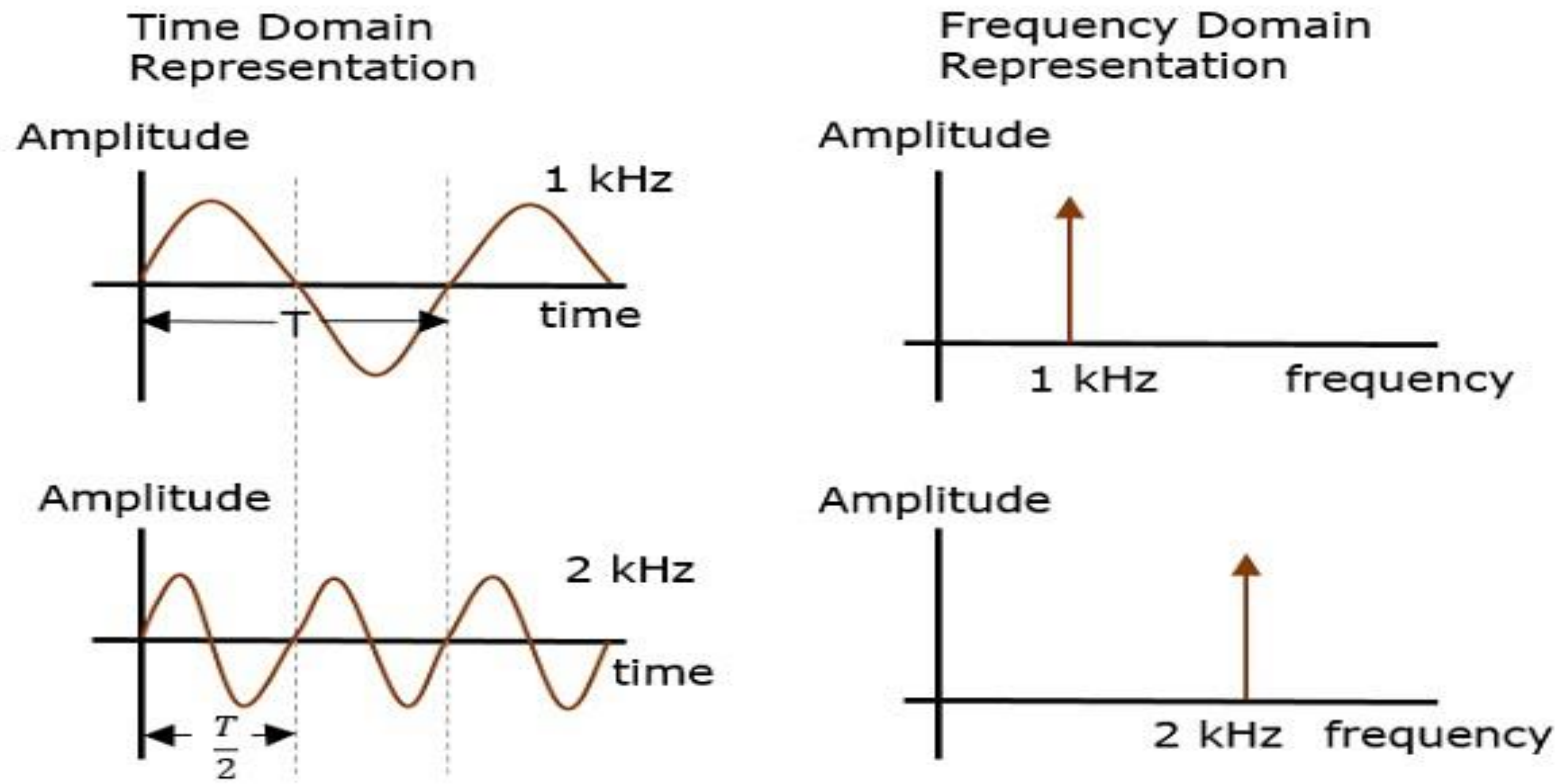
Inverse DFT

$$x(n) = IDTFT\{X(\omega)\} = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\omega)e^{j\omega n}d\omega$$

DISCRETE FOURIER TRANSFORM



REPRESENTATION OF SIGNALS



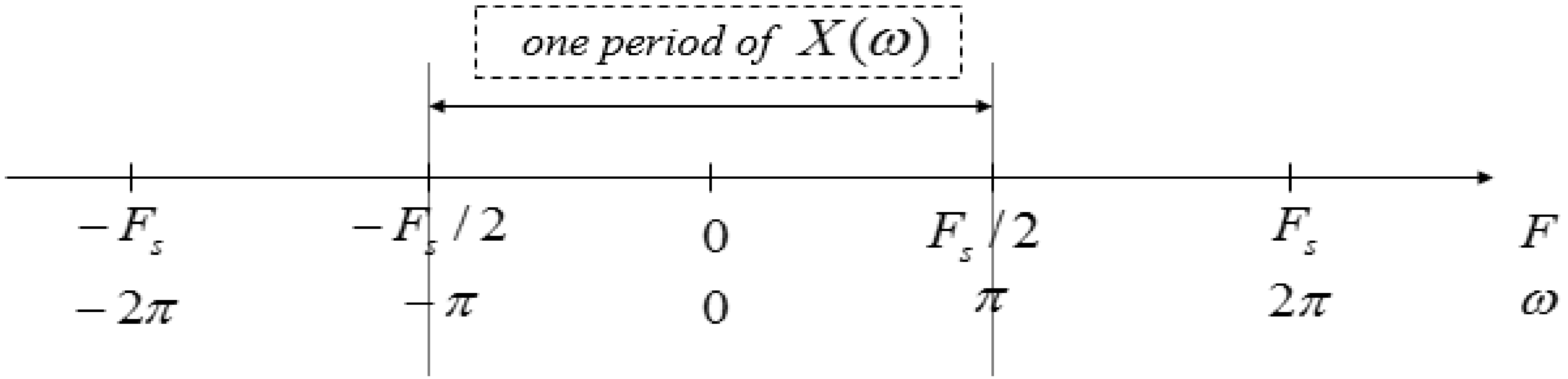
DISCRETE FOURIER TRANSFORM

- *The DTFT* ***X(ω)*** *is periodic with period* ***2π***
- *The frequency* ***ω*** *is the digital frequency and therefore it is limited to the interval*

$$-\pi < \omega < +\pi$$

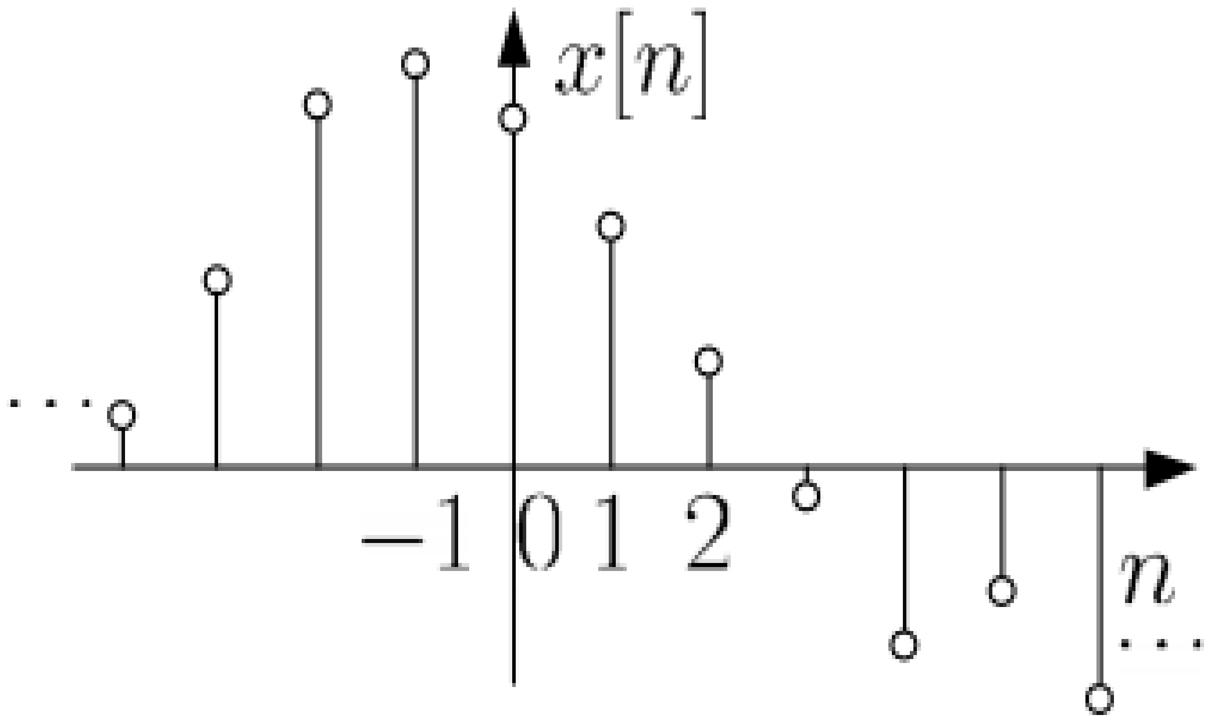
- *The digital frequency* ***ω*** *is a normalized frequency relative to the sampling frequency, defined as*

$$\omega = 2\pi \frac{F}{F_s}$$

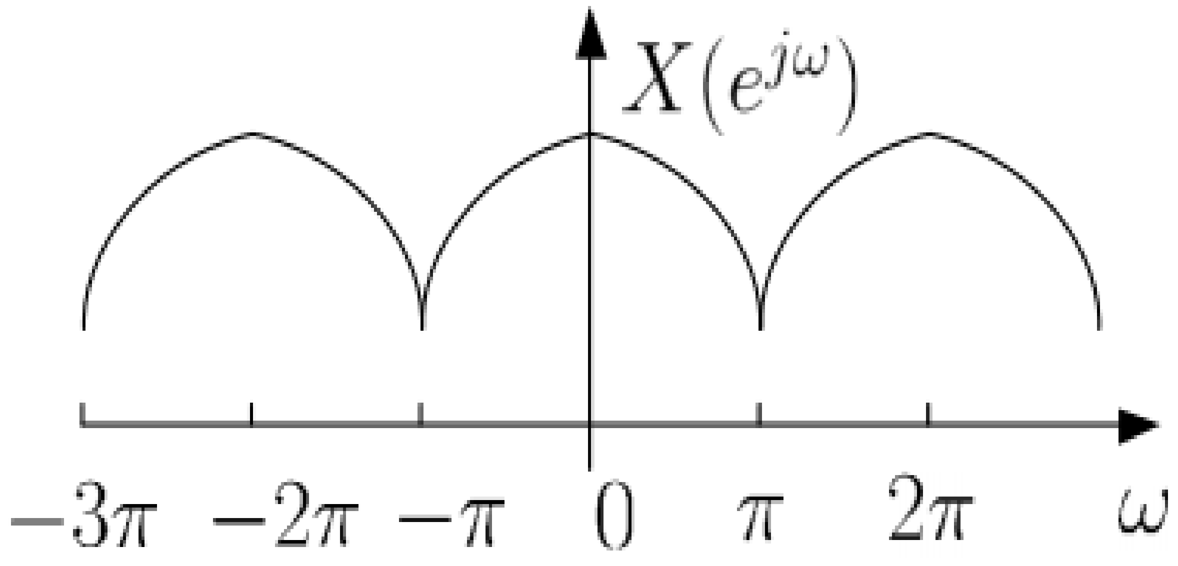


DFT REPRESENTATION

Time Domain



Freq Domain



DISCRETE FOURIER TRANSFORM

In Discrete Fourier Transform, Given a finite sequence

$$x = [x(0), x(1), \ldots, x(N-1)]$$

its Discrete Fourier Transform (DFT) is a finite sequence

$$X = DFT(x) = [X(0), X(1), \ldots, X(N-1)]$$

Where

$$X(k) = \sum_{n=0}^{N-1} x(n) w_N^{kn}, \quad w_N = e^{-j2\pi/N}$$

INVERSE DISCRETE FOURIER TRANSFORM

In Inverse Discrete Fourier Transform, Given a sequence

$$X = [X(0), X(1), \ldots, X(N-1)]$$

its Inverse Discrete Fourier Transform (IDFT) is a finite sequence

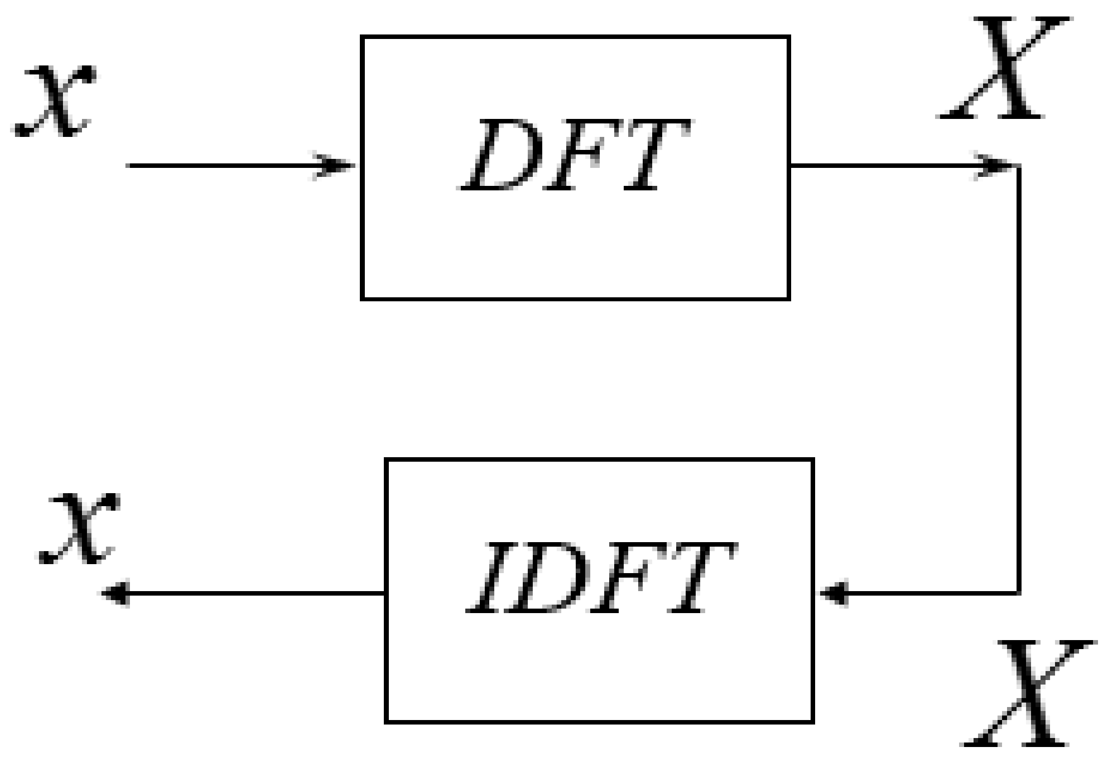
$$x = IDFT(X) = [x(0), x(1), \ldots, x(N-1)]$$

Where

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) w_N^{-kn}, \quad w_N = e^{-j2\pi/N}$$

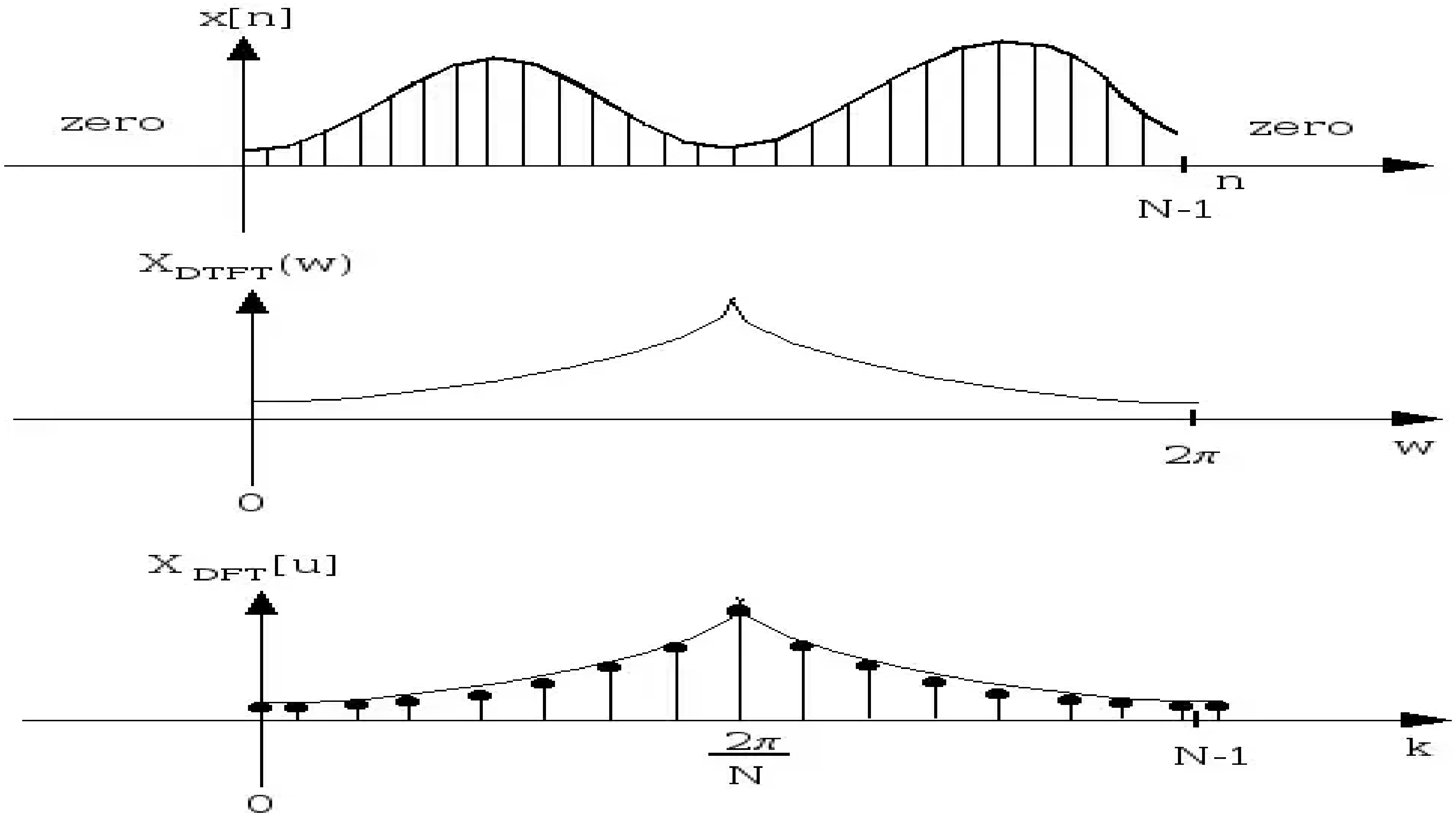
DISCRETE FOURIER TRANSFORM PAIR

The DFT and the IDFT form a transform pair.



The DFT is a numerical algorithm, and it can be computed by a digital computer.

REPRESENTATION OF DTFT & DFT



PROPERTIES OF DFT

Property	Time Domain	Frequency Domain
1. Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
2. Time-shifting	$x[n-m]$	$e^{-j2\pi km}X(k)$
3. Frequency-shifting (modulation)	$e^{-j2\pi k_0 n/N}x[n]$	$X(k-k_0)$
4. Time reversal	$x[-n]$	$X(-k)$
5. Conjugation	$x^*[n]$	$X^*(-k)$
6. Time-convolution	$x_1[n] \otimes x_2[n]$	$X_1[k]X_2[k]$
7. Frequency-convolution	$x_1[n]x_2[n]$	$\frac{1}{N}X_1[k] \otimes X_2[k]$

APPLICATIONS OF DFT

1. Spectral Analysis
2. Image Processing
3. Signal Processing

Other Applications:

1. Sound Filtering
2. Data Compression
3. Partial Differential Equations
4. Multiplication of large integers

DIFFERENCE B/W DFT & IDFT

DFT (Analysis transform)	IDFT (Synthesis transform)
DFT is finite duration discrete frequency sequence that is obtained by sampling one period of FT.	IDFT is inverse DFT which is used to calculate time domain representation (Discrete time sequence) form of x(k).
DFT equations are applicable to causal finite duration sequences.	IDFT is used basically to determine sample response of a filter for which we know only transfer function.
Mathematical Equation to calculate DFT is given by $$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\prod kn/N}$$	Mathematical Equation to calculate IDFT is given by $$x(n) = 1/N \sum_{n=0}^{N-1} X(k) e^{j2\prod kn/N}$$
Thus DFT is given by $X(k) = [W_N][x_n]$	In DFT and IDFT difference is of factor 1/N & sign of exponent of twiddle factor. Thus $x(n) = 1/N\, [W_N]^{-1}[X_K]$

ASSESSMENT

1. Define DFT

2. What is meant by IDFT.

3. Give some applications of Fourier Transform.

4. Define DFT Pair.

5. *The DTFT (Discrete Time FT) for sequences having -------------------- duration*

6. *Determine DFT of $x(n) = \{1,0,1,0\}$*

THANK YOU